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EQUILIBRIUM PRICE DISTRIBUTIONS

Abstract

Equilibrium price distributions (for a homogeneous product) consistent with individual incentives are investigated. They arise in informationally imperfect markets in which the only primitive datum is the distribution of search costs. It is shown that single, multi- and continuous price distributions are all viable long-run phenomena depending on the nature of search costs. A method for computing equilibrium price distributions is also provided.

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1. Introduction

Jevons' law of one price has been a time-honored dictum in economic theory. In a classroom presentation, it still serves (sometimes implicitly) as a major premise in many micro-models of market interaction. The would-be-professional is exposed to the idea that at a point in time only one price can prevail in a competitive market for a certain commodity. The well-known logical foundation of this law is a simple arbitrage argument. If commodities sold at two trade posts have identical characteristics, masses of rational consumers would shift, in response to the slightest discrepancy, from the high to the low-priced seller driving the former out of business. Any price differences are, therefore, instantaneously wiped out. Certain examples of markets behaving in accordance with the law can be cited. A case in point is the organized exchange. In futures markets, for example, certain economic agents, the arbitrageurs, actually earn their incomes by exploiting and thereby eliminating price differentials. Indeed, if consumers are perfectly informed, the logic of this idea is compelling. The forces of competition ensure that price multiplicity, at least in equilibrium, is impossible.

Yet, one's day-to-day experience reveals that looking for superior deals may well be a rewarding activity. The prospects of eventually hitting a bargain and the resulting price reduction would more than compensate for the cost incurred during the search process. Moreover, the existence of price dispersion appears to be an empirically stable phenomenon. Snapshots of markets for relatively homogeneous products reveal substantial and recurring price differentials. Stigler [22] and Pratt, Wise and Zackhauser [14], for example, report some evidence supporting this claim.

An important article by Stigler [22] focussed economists' attention on the crucial role played by the informational structure of markets. It gave rise to the study of the economics of information, which in turn shed new light on the above-described discrepancy between theory and the empirically valid phenomenon of price dispersion. Current wisdom is that in informationally imperfect

markets multiple prices are consistent with consumers' rational behavior. This is so even if the mobility of buyers is not hampered by any legal or institutional restraints. If it costs (time, lost opportunities, etc.) something to secure a price quotation, a buyer will stop looking for bargains as soon as the anticipated price reduction falls short of his (subjective) search costs. Many consumers will thus consciously buy at prices which are not the lowest obtainable. Sellers posting high prices can still realistically expect to serve some customers whose time is relatively precious, for example. A price distribution is thus a viable possibility, so far as the behavior of consumers is concerned. As Rothschild [17] forcefully argued, however, a price dispersion can be sustained as an equilibrium phenomenon only if it is fully consistent with the optimizing micro-behavior of all economic agents. In particular, sellers charging different prices must find their decisions to be best responses to other price announcements and to the market demand they are facing. If we assume that these sellers are identical in all respects, it is not clear (even if we do take buyers' imperfect information into account) why more than a single price is ever observed.

The purpose of this paper is to analyse equilibrium issues within the canonical search model. More specifically, our primary objectives are the existence and the nondegeneracy of an equilibrium price distribution. The analysis below considers an isolated market for a narrowly defined commodity. Our is thus a partial equilibrium approach where price variability which is due to product differentiation is excluded by assumption. In our set-up, risk-neutral consumers inelastically demand one unit of a durable commodity. The model pertains therefore to a one-shot purchase of an indivisible commodity such as a car or a house. Buyers wish to minimize the cost of purchasing the product. It consists of the price they end up paying for the item plus the cost of finding a price which they consider "acceptable." The search procedure they employ is optimal inasmuch as the search proceeds sequentially. At each stage, in other words, they decide whether to buy at the lowest price they have discovered so far (the perfect recall assumption) or whether it is worth their while to bear the cost of obtaining an additional price quotation (see Weitzman [23] for a general treatment of this problem). We finally assume that unit search costs are fixed, rather than rising for each buyer. A crucial premise in our structure

is the heterogeneity of buyers with regard to their search costs. This variability of sampling costs generates, in turn, the demand side of the market. That is to say, the density of consumer types indexed by search costs together with price announcements of competitors determine the demand curve facing each seller. We shall see momentarily exactly how the primitive data, i.e., the distribution of types, induces a demand relationship. We shall also show that the viability of multiple-price equilibria crucially hinges on the nature of this distribution.

Regarding sellers, we assume that they are all identical. More concretely, all have access to the same zero marginal cost of production technology. The identity of producers assumption is intentionally employed to eliminate other technologically-based reasons for price dispersion, as in Reinganum [15]. The zero marginal cost of production is just a convenient working assumption and can be easily replaced by any other constant without altering the basic structure or the results (see Axell [2]).

The equilibrium notion we use is of the Nash variety. Producers make price decisions which determine their profit rate and market share based on the market demand they are facing. They take as given prices charged by other sellers and consumers' demand induced by these prices and the postulated search behavior. Given the zero costs of production, each seller is maximizing his expected revenues. In a stable equilibrium sellers are just indifferent among all announced prices. Put differently, these prices generate the same revenues (= profits) while other (un-announced) prices generate lower profits. No seller wishes then to deviate from his price decision. Furthermore, common profits equal the fixed cost of entering this market. The industry is thus in a long-run equilibrium.

The section below surveys some of the literature on this subject, putting the present paper in its right perspective. I will, in particular, elaborate on the reasons why a general existence proposition and a multiplicity of equilibrium prices emerge in my structure. Within the basic sequential search these results are, to the best of my knowledge, novel. In Section 3, I formulate the model I have just discussed, and I formally analyse it in Section 4. Section 5 concludes the paper.

2. Review of the Literature

Utilizing the conventional approach to microtheory one could usefully divide the literature into two general categories. First, there is the individual maximization problem of choosing an optimal search procedure given the uncertainty stemming from a postulated price distribution, i.e., from the fact that consumers know the relative frequencies of prices but not which store charges which price. This issue has been satisfactorily worked out in the literature (McCall [12], Kohn-Shavell [9], Weitzman [23]). At this level most authors consider sequential search to be the "correct" procedure (with the notable exception of Burdett and Judd [5]). It is clearly consistent both with the basic maximization hypothesis, i.e., consumers' rationality, and the common real-life situation where only one price can be observed at a time.

Second there is the problem of market interaction and the endogenous determination of equilibrium prices. This is also the question that will concern us here. Early contributions to the subject suggested that when the behavior of sellers is taken into account, equilibrium price distributions, if they exist at all, collapse to the unique monopoly price (see Diamond [7], Rothschild [17] and Arrow and Rothschild [1]). This trivial equilibrium lends support to the notion that imperfect information allows firms to act as monopolists in their own market areas since mobility of consumers is constrained by the cost of finding better deals. To see how this degeneracy is brought about consider a firm charging a price below the monopoly price. As long as individual search costs uniformly exceed a positive constant, the firm could raise its price by an amount smaller than this constant without the fear of losing any customers. The latter just find this price increase too negligible to induce them to start searching elsewhere. The firm therefore stands to gain from this alternative price decision and the postulated original price is not an optimal choice in the first place. With search costs bounded away from zero, the only possible equilibrium is therefore at the monopoly price. This result is unsatisfactory since it implies that all steady-state distributions are degenerate—contradicting the empirical evidence we have previously cited.

Faced with this difficulty, later writers modified the basic search model so that nontrivial price configurations could emerge in an equilibrium. In most of these contributions the standard sequential search set-up is altered by introducing a simpler information gathering technology. One common assumption in this literature is that each consumer can obtain perfect information about market prices and, in particular, about where the lowest price is to be found. He does so by paying a fixed fee, e. g., by purchasing a newspaper that lists all store prices. By assuming, in addition, individualized costs of observing this information, one is led to the conclusion that only consumers with low search cost choose to become informed. Given the ignorance of other buyers, a nondegenerate price distribution is a viable possibility. This approach is taken by Salop-Stiglitz [21] and Braverman [4] who analyse the nature of equilibrium and its welfare properties. It is shown, on the one hand, that only two prices can be candidates for a single-price equilibrium—the competitive price at the minimum of the average cost curve and the monopolistically competitive one. If a nontrivial distribution exists, on the other hand, it must involve again at most two prices. Finally, it is shown that it is perfectly possible that no Nash equilibrium exists. Braverman explores also the similarity between imperfect information and product variety as alternative foundations of a monopolistic competition framework. It is shown, for example, that the excess capacity proposition generally holds in models where consumers are searching for the best alternative.

At the other end, there are models where individual search is replaced altogether by an alternative random information generator such as advertising. Butters [6] introduced a model where firms choose both prices and advertising intensity. Consumers, on the other hand, act passively by going to the lowest priced store they know of (through ads they receive). The resulting Nash equilibrium is one where high-priced stores advertise more to enhance the demand they are facing. They do this to offset the negative price effect and to generate higher revenues compensating them for their higher advertising expenditures. Butters shows, in addition, that the total advertising activity induced by the market

mechanism is identical with what a social planner, operating under the same informational constraints, would dictate. His set-up confirms thus the fundamental theorem of Welfare Economics when informational rigidities are present.

The model we adopt in this paper and which we described in the introductory section is very much in the spirit of Axell's work [2]. In his paper equilibrium issues were analysed within the framework of a sequential search model. In particular, the existence of an equal-profits equilibrium price dispersion was investigated. More specifically, he characterized search-cost distributions which give rise to an absolutely continuous equilibrium price distribution. His analysis is, however, limited to the extent that equal profits are required over the entire range of positive prices. This unrealistic and stringent requirement gave rise to very restrictive existence conditions which are rarely fulfilled by any reasonable density of consumer types. The necessary and sufficient proposition he proves can be therefore regarded, it seems to me, as a generic nonexistence theorem. The analysis below shows that by requiring price announcements to lie within a bounded interval (as they do in the real world) one could prove a much more general existence result.

As we shall see from several examples, equilibrium price distributions may involve multiple or even a continuum of price quotations. This feature, as we have already indicated, is not shared by most of the papers we have surveyed (with the exception of [2]). The reason for this discrepancy is twofold. First we maintain in our structure the sequential search procedure as the basis for individual consumer behavior. Our model and results are thus applicable to markets where price quotations are obtained seriatim and with short time lags between successive searches. In contrast, the Salop-Stiglitz-Braverman framework is more realistic when price information is obtained from a specialist or a newspaper. Second we assume that there are always consumers with search costs arbitrarily close to zero. These buyers produce an external economy from which others benefit. Their extensive search activity ensures that prices will not be driven to the unique monopoly price.

3. Formulation

In the model below we assume that a continuum of risk-neutral sellers produces a homogeneous commodity at a zero cost. Each is thus attempting to maximize expected revenues by appropriately selecting a price, p . Let the resulting distribution function of quoted prices be $F(p)$. We, similarly, assume a continuum of buyers with differential search costs, s . Let $q(s)$, $Q(s)$ be the density and the c.d.f. respectively of these costs, defined over $[0, \infty)$ and suppose that $q(\cdot)$ is a smooth function with $0 < q(0) < \infty$. This structure should be interpreted as the limiting case of a finite (in the number of sellers and buyers) market. We shall momentarily elaborate on the exact nature of the limiting argument and on the determination of the buyer/seller ratio.

For the time being note that a typical buyer with search costs s will stop searching once he hits (or "beats") an optimal reservation value $r(s)$. The latter is determined as the root r to the equation:

$$\int_0^r (r-x)dF(x) = \int_0^r F(x)dx - rF(0) = s \quad . \quad (1)$$

(See Weitzman [23].) It is quite clear that no seller will ever charge a zero price since it ensures zero profits. We may thus disregard the term involving $F(0)$. Consider then the function

$$H(r; F) \equiv \int_0^r F(x)dx \quad . \quad (2)$$

The dependence of H on F will be suppressed and we will simply write $H(r)$. Let us note in passing that

Lemma 1: $H(\cdot)$ is an increasing, convex, absolutely continuous function, $H(r) < r$ and $H'(r) = F(r) \leq 1$ whenever H' exists.

This proposition ensures the existence and uniqueness of a root $r(s) = H^{-1}(s)$ to equation (1) for any $s > 0$. For $s = 0$ we take $r(0)$ to be the lower end of the

Figure 1

support of F , i.e., $r(0) = \text{Sup}\{x \mid F(x) = 0\}$. $r(s)$ is then the reservation value of buyer s and one clearly has $r(s) > s$. See Figure 1.

Denote by R the random variable corresponding to the distribution of reservation values, $r(s)$. Given F and Q we find the distribution $G(\cdot)$ of these cutoff points

$$G_R(r) = \Pr(R \leq r) = \Pr(S \leq H(r)) = Q(H(r)) \quad .$$

Thus,

$$g(r) = q \left(\int_0^r F(x) dx \right) F(r) \quad . \quad (3)$$

Let us immediately remark that $G(\cdot)$ is larger, in the sense of first order stochastic dominance, than $Q(\cdot)$ and that G is determined once F and q are given.

Consider now a finite representation of the above market in which there are N buyers and M sellers where N, M are to be thought of as "large" numbers and $\lambda = N/M$ is the buyer/seller ratio. Assume that $M-1$ of these sellers have

already chosen their prices so that F and G are (almost) determined, and consider the best response problem of the remaining seller. To this end, we wish to find the quantity $x(p_0)$ he expects to sell if he announces a price p_0 . It is clear first that only buyers with reservation values $r \geq p_0$ will ever buy from him. If we take a representative buyer, r , of this sort, it is natural to assume that he is equally likely to purchase the item from any seller posting a price $p \leq r$. Since there are (approximately) $MF(r)$ such sellers, the consumer will buy one unit of commodity from the merchant quoting p_0 with the probability $\frac{1}{MF(r)}$. Divide now the R axis into N subintervals $[r_i, r_{i+1})$, $i = 0, 1, \dots, N-1$ so that

$$0 = r_0 < r_1 < r_2 < \dots < r_N < \infty$$

and
$$g(r_i)\Delta r_i = 1/N \quad .$$

Define binomial random variables as follows:

$$B_i \equiv \begin{cases} 1 & \text{w.p. } \frac{1}{MF(r_i)} \quad , \quad i = 1, \dots, N \\ 0 & \text{w.p. } 1 - \frac{1}{MF(r_i)} \quad . \end{cases}$$

The quantity sold by the merchant is then the sum of some of these random variables where summation extends over all buyers i with reservation values $r_i \geq p_0$. We thus have:

$$S = \sum_{[i | r_i \geq p_0]} B_i$$

$$x(p_0) \equiv E(s) = \sum_{[i | r_i \geq p_0]} \frac{1}{MF(r_i)} = \frac{N}{M} \sum \frac{g(r_i)\Delta r_i}{F(r_i)}$$

which, for large N, M , is a good approximation of

$$\lambda \int_p^\infty \frac{g(r)}{F(r)} dr \quad ,$$

i.e., when we pass to the limit as N, M increase while the ratio $\lambda = N/M$ is held fixed. Using (3) we obtain

$$x(p) = \lambda \int_p^\infty q\left(\int_0^r F(x)dx\right) dr \quad (4)$$

We see that a merchant in a "nonatomic" market selects p to maximize

$$\pi(p; H) \equiv p \int_p^\infty q(H(r))dr \quad (5)$$

The maximization of equation (5) with respect to p summarizes then the best response problem of a typical seller given the price distribution F , i.e., given the price choices of all other sellers. Our working assumption is therefore that traders are Nash-players. The concept of equilibrium is accordingly one where profits are equalized and maximal across all quoted prices so that each seller is indifferent between these price announcements. Formally we introduce Definition 1: F is an equilibrium distribution if, for some constant K , $\pi(p) \leq K$ for all p while equality holds for all p in the support of F .

If the fixed cost of entry into this market is given by c (the cost of foregone opportunities by committing oneself to this activity), then the buyer-seller ratio λ , in a long-run equilibrium, is given by $\lambda^* = c/K$ where K is the common level of profits in the industry, given by Definition 1. This is, of course, the ex-ante zero profits condition.

Our model is now well-specified and we may proceed to study its properties.

4. Analysis

4.1. Single-Price Equilibrium

Our first step is to examine the possibility of one-price or degenerate equilibria. We will characterize in this section situations where Jevons' law of one price is valid. To this end let us return to the finite market set-up where $M-1$ sellers are announcing a price, $p_0 > 0$ and consider the last seller's profits when he selects a price, p , $p \geq p_0$. Suppose, first, that $p < p_0$. Then for $p < r < p_0$ we have $H(r) = \frac{r-p}{M}$. Announcing p will thus bring in (with certainty) all customers with search costs $s \leq \frac{p_0-p}{M}$, i.e., $\lambda M Q\left(\frac{p_0-p}{M}\right)$ of them. In addition, this seller will get his proportional share $(1/M)$ of the remaining buyers, i.e., those with $s > \frac{p_0-p}{M}$. So his total sales are given by

$$\begin{aligned} x(p) &= \lambda M \left\{ Q\left(\frac{p_0-p}{M}\right) + \frac{1}{M} \left[1 - Q\left(\frac{p_0-p}{M}\right) \right] \right\} \\ &= \lambda(p_0-p) \frac{Q\left(\frac{p_0-p}{M}\right) - Q(0)}{\frac{p_0-p}{M}} + \lambda - \lambda Q\left(\frac{p_0-p}{M}\right), \end{aligned}$$

assuming $Q(0) = 0$. If $Q(0) > 0$, on the other hand, then $x(p) = \infty$.

Letting $M, N \rightarrow \infty$ (while $\lambda = N/M$ remains constant) we see that

$$\begin{aligned} x(p) &= \lambda(p_0-p)q(0) + \lambda = \lambda \left[1 + (p_0-p)q(0) \right], \quad Q(0) = 0 \\ &= \infty, \quad \text{if } Q(0) > 0 \end{aligned} \tag{6}$$

This equation can be obtained alternatively as follows. Denote:

$$D_{p_0}(p) = \begin{cases} 0 & p < p_0 \\ 1 & p \geq p_0 \end{cases} \tag{7}$$

i.e., start out from the "limit economy" where all but one seller post the same price, p_0 . Then

$$H(r; D_{p_0}) = \begin{cases} 0 & r < p_0 \\ r - p_0 & r \geq p_0 \end{cases}$$

$$q(H(r)) = \begin{cases} q(0) & r < p_0 \\ q(r - p_0) & r \geq p_0 \end{cases}$$

Therefore, for $p < p_0$

$$\begin{aligned} \frac{x(p)}{\lambda} &= \int_p^\infty q(H(r)) dr = \int_p^{p_0} q(0) dr + \int_{p_0}^\infty q(r - p_0) dr \\ &= (p_0 - p)q(0) + \int_0^\infty q(z) dz = (p_0 - p)q(0) + 1 . \end{aligned} \quad (8)$$

which is the same as (6).

Similarly for $p > p_0$, all buyers with $s \leq \frac{M-1}{M} (p - p_0)$ will purchase from sellers announcing p_0 so that sales of the seller charging $p > p_0$ are given by

$$\lambda M \left\{ \frac{1}{M} \left[1 - Q \left(\frac{M-1}{M} (p - p_0) \right) \right] \right\} = \lambda \left[1 - Q \left(\frac{M-1}{M} (p - p_0) \right) \right]$$

and, in the limit,

$$x(p) = \lambda \left[1 - Q(p - p_0) \right] . \quad (9)$$

We can, again, obtain the same expression directly from (4) and (7). For $p = p_0$ we see that $\pi(p_0) = \lambda p_0$.

According to Definition 1, D_{p_0} is therefore an equilibrium distribution if and only if:

$$p \left[1 + (p_0 - p)q(0) \right] \leq p_0 , \quad p < p_0$$

$$p \left[1 - Q(p - p_0) \right] \leq p_0 , \quad p > p_0$$

which, in turn, is equivalent to

$$p_0 - p \geq p(p_0 - p)q(0) \quad , \quad p < p_0$$

$$\frac{p_0}{p} \geq 1 - Q(p - p_0) \quad , \quad p > p_0$$

or

$$1 \geq pq(0) \quad , \quad p < p_0$$

$$Q(p - p_0) \geq 1 - \frac{p_0}{p} \quad , \quad p > p_0 \quad .$$

To sum up:

Lemma 2: D_{p_0} with $p_0 > 0$ is an equilibrium distribution if and only if

$$(i) \quad Q(0) = 0$$

$$(ii) \quad q(0) \leq 1/p_0 \quad \text{and}$$

$$(iii) \quad Q(s) \geq 1 - \frac{p_0}{p_0 + s} = \frac{s}{p_0 + s} \quad , \quad s \geq 0 \quad .$$

Observe now that when $Q(0) > 0$ there can be no single price equilibrium. Indeed, if $p_0 > 0$ then we see from (6) that undercutting, $p < p_0$, by an arbitrarily small amount is always profitable while if $p_0 = 0$ profits are zero and each seller can benefit by a small price increase. Thus

Lemma 3: No single price equilibrium exists if $Q(0) > 0$.

Assume therefore that $Q(0) = 0$ and examine condition (iii) of Lemma 2:

$$\frac{Q(s) - Q(0)}{s} \geq \frac{s}{(p_0 + s)s} = \frac{1}{p_0 + s} \rightarrow \frac{1}{p_0} \quad .$$

By passing to the limit on both sides of this inequality, as $s \rightarrow 0$, we get

$$(ii') \quad q(0) \geq 1/p_0$$

which together with (ii) of Lemma 2 shows that there is at most one single price equilibrium at $p_0 = 1/q(0)$. We may finally state:

Theorem 1*: The only possible one-price equilibrium is $p_0 = 1/q(0)$. This is an equilibrium if and only if

$$a) \quad Q(0) = 0, \quad \text{and}$$

$$b) \quad Q(s) \geq 1 - \frac{p_0}{p_0 + s} = \frac{sq(0)}{1 + sq(0)}, \quad s \geq 0.$$

To interpret this result, recall that it is the consumers of low search costs who arbitrage away price differences. This is so because their low costs enable them to search extensively, reject high price offers and thus lower the profits of high price sellers. This, in turn, forces the latter to lower their prices and thus tends to wipe out price differentials. Given this logic a reasonable conjecture is that when these low-cost types are sufficiently numerous a single price equilibrium emerges. This is exactly the intuition behind Theorem 1. Indeed, given $q(0)$, the "benchmark" Pareto distribution on the right hand side of (b) is determined. The first order stochastic dominance condition (b) captures the idea that Q puts sufficient mass on the left hand tail, i.e., that in comparison with this benchmark distribution, there are many buyers with low sampling costs.

Note also that the single-price equilibrium, provided it exists, is between the competitive (zero) and the monopolistic (infinity) prices. This is not too surprising, of course. Given the notion that buyers' imperfect information endows the sellers with a limited monopoly power, one would certainly anticipate a monopolistically competitive price outcome of this sort. Moreover, if we regard $q(0)$ as a measure of the degree of competition we see that equilibrium prices are inversely related to this measure. That is again very plausible. The larger is the number of consumers with low search costs (the higher is $q(0)$), the stronger is the downward pressure they exert on prices.

Let us now illustrate this result by means of three simple

Examples:

$$1. \text{ (Exponential)} \quad q(s) = e^{-s}, \quad Q(s) = 1 - e^{-s}, \quad q(0) = 1.$$

* I am grateful to an anonymous referee for pointing out to me this stronger version of the theorem.

In accordance with Theorem 1, we need first to ensure that

$$1 - e^{-s} \geq 1 - \frac{p_0}{p_0 + s},$$

which certainly holds for any $p_0 \geq 1$. Second, one must have

$$1 = q(0) \leq 1/p_0$$

which is true only for $p_0 \leq 1$. Thus the unique degenerate price equilibrium obtains when all sellers set $p_0 = 1$. Their common profits, π_0 , are then $\pi_0 = 1$ and the buyer/seller ratio must therefore be $\lambda^* = c$.

One can similarly establish that the unique single-price equilibrium for the density, $q(s) = \gamma e^{-\gamma s}$ is $p_0 = 1/\gamma$ and $\pi_0 = 1/\gamma$ so that $\lambda^* = c\gamma$.

$$2. \text{ (Pareto)} \quad q(x) = \frac{\theta}{(x+1)^{\theta+1}}, \quad q(0) = \theta, \quad Q(x) = 1 - \frac{1}{(x+1)^\theta}.$$

From condition (ii) we know that $p_0 \leq 1/\theta$ while the requirement in condition (iii) is that

$$(s+1)^\theta \geq 1 + \frac{s}{p_0}, \quad s \geq 0$$

which is satisfied iff $\theta \geq 1$ and $1/p_0 \leq \theta$. We thus conclude that a single-price equilibrium exists only if $\theta \geq 1$. In that case, $p_0 = \pi_0 = 1/\theta$ and $\lambda^* = c\theta$.

$$3. \text{ (Uniform)} \quad q(x) = 1/\theta, \quad q(0) = 1/\theta, \quad Q(x) = \frac{1}{\theta}x, \quad \text{for } 0 \leq x \leq \theta.$$

Applying the same reasoning, $p_0 = \theta$ is an equilibrium iff

$$\frac{1}{\theta}s \geq \frac{s}{\theta+s}, \quad s \geq 0$$

which is obviously satisfied. We have $\pi_0 = p_0 = \theta$ and $\lambda^* = c/\theta$.

4.2. Existence of Multiple Price Equilibria

From Theorem 1 we observe that all search-cost distributions Q which are not dominated, in the sense of first-order stochastic dominance, by the Pareto

distribution, $Q^p(s) = \frac{sq_0}{1+sq_0}$, do not admit a single-price equilibrium. Since this condition is rather stringent, it is expected that single-price equilibria do not generically exist. Put differently, inasmuch as $Q(0) = 0 = Q^p(0)$, any initial Q distribution admitting a single-price equilibrium can be "slightly perturbed" to yield a Q for which no single-price equilibrium exists. The degeneracy of an equilibrium distribution is thus a knife-edge property. The next natural step therefore is to consider the possibility of multiple-price equilibrium. Our aim, in other words, is to obtain a general existence result, i.e., to show that an equilibrium price distribution exists under rather weak assumptions.

We start this investigation by focussing on multiple but finite price equilibria. That is, we look at price distributions of the form

$$F(x) = \sum_{i=1}^n \delta_i D_{p_i}(x) \quad (11)$$

where

$$\delta_i \geq 0, \quad \sum \delta_i = 1$$

$$0 \leq p_1 < p_2 < \dots < p_n < \infty.$$

For these distributions we initially demonstrate the existence of approximate equilibria. This will be the key step towards proving the main result concerning exact equilibria. To this end we adopt the following

Definition 2: F (as in (11)) is an ϵ -equilibrium, if for some constant K ,

$$\pi(p_i) \leq K \quad i = 1, \dots, n$$

$$\pi(p_i) = K \quad \text{for all } i \text{ such that } \delta_i > 0$$

$$\text{and} \quad \pi(p) \leq K + \epsilon \quad \text{for } p \notin \{p_i\}_{i=1}^n.$$

F is an approximate equilibrium inasmuch as a seller deviating from any of the "viable" prices $\{p_i\}_{i=1}^n$ can gain no more than an ϵ amount. Figure 2 illustrates such an equilibrium.

Figure 2

Price announcements are some of those points along the p axis where the graph of π crosses or touches the line $\pi(p) = K$. We also see that π never rises above $K + \epsilon$. We now show that an appropriate choice of prices will give rise to the profit curve we have diagrammed, i.e., to an ϵ -equilibrium.

Lemma 4: Assume $q(\cdot)$ and $\frac{1-Q(\cdot)}{q(\cdot)}$ are bounded functions on $[0, \infty)$ and $0 < q(0) < \infty$. Then an ϵ -equilibrium F distribution exists whose support is contained in $[m, M]$ where $m < 1/q(0)$ and $M = \sup_{x \geq 0} \frac{1-Q(x)}{q(x)}$.

Proof: Denote

$$\bar{q} \equiv \sup_{x \geq 0} q(x), \quad B \equiv 1 + \bar{q}(M - m)$$

and let $m = p_1 < p_2 < \dots < p_{n+1} = M$ be a partition of $[m, M]$ for which

$$\Delta \equiv \max_{1 \leq i \leq n} (p_{i+1} - p_i) \leq \epsilon/B.$$

A theorem due to Pratt, Wise and Zeckhauser [14] tells us that a "restricted" equilibrium, $\{\delta_1, \dots, \delta_n\}$, exists, i.e., that for some K , $\pi(p_i) = K$ for $i = 1, \dots, n$. This theorem does not specify, however, what happens to profits off this domain, i.e., what is $\pi(p)$ at $p \notin \{p_i\}_{i=1}^n$. We therefore consider these prices now.

From the definition of π in equation (5) we see that

$$\pi'(p) = \int_p^{\infty} q(H(r))dr - pq(H(p)) \quad .$$

For $p \leq m$ we have:

$$\pi'(p) = \int_p^m q(0)dr + \int_m^{\infty} q(H(r))dr - pq(0) = (m - 2p)q(0) + \int_m^{\infty} q(H(r))dr \quad .$$

This expression attains its minimum at $p = m$ where

$$\pi'(m) = -mq(0) + \int_m^{\infty} q(H(r))dr > \int_m^{\infty} q(H(r))dr - 1 > 0$$

by the choice of m ($m < 1/q(0)$) and the fact that

$$\int_m^{\infty} q(H(r))dr > 1$$

which follows from Lemma 1. Therefore: $\pi'(p) > 0$ for $p \leq m$.

Next consider $p \geq M$:

$$\begin{aligned} \pi'(p) &= \int_p^{\infty} q(I + r - M)dr - pq(I + p - M) \\ &= \int_{I+p-M}^{\infty} q(z)dz - pq(I + p - M) = 1 - Q(I + p - M) - pq(I + p - M) \end{aligned}$$

where $I \equiv \int_m^M F(x)dx$ and $H(r) = I + r - M$ for $r > M$.

$\pi'(p)$ is therefore negative for $p > M$ iff

$$\frac{1 - Q(I+p-M)}{q(I+p-M)} < p$$

which is true since M is the supremum of the left-hand side and $p > M$.

Finally, for $m \leq p \leq M$, we have

$$\begin{aligned} \pi'(p) &\leq \int_p^\infty q(H(r))dr = \int_p^M q(H(r))dr + \int_M^\infty q(H(r))dr \\ &\leq \int_m^M q(H(r))dr + \int_I^\infty q(z)dz \leq (M-m)\bar{q} + 1 - Q(I) \leq B \end{aligned}$$

Therefore

$$\left| \pi(p) - \pi(p_i) \right| \leq \sup_{p_i \leq c \leq p_{i+1}} \pi'(c) \cdot \Delta \leq \Delta B \leq \epsilon, \quad \text{for } p_i \leq p < p_{i+1}$$

which implies, in turn, that

$$\pi(p) \leq K + \epsilon, \quad m \leq p \leq M.$$

The proof is complete. $\square$

F is an approximate equilibrium, as we have already indicated, in the sense that the "variation" of π over the interval $J \equiv [m, M]$ does not exceed ϵ , i.e., that $d(H) \equiv \sup_{p \in J} \pi(p) - \inf_{p \in J} \pi(p) \leq \epsilon$. Armed with this result we are ready to prove our main

Theorem 2: Under the assumptions of the lemma, there exists an exact equilibrium price distribution.

Proof: Let ϵ_n be a sequence of positive scalars so that $\epsilon_n \downarrow 0$. By Lemma 4 we can find distributions F_n and associated H_n functions such that $d(H_n) \leq \epsilon_n$. Note that H_n is a family of equicontinuous and uniformly bounded functions on $[0, M+1]$ which uniformly satisfies a Lipschitz condition, $|H_n(r) - H_n(z)| \leq |r - z|$. We apply then the Arzela-Ascoli theorem and select a subsequence converging to a function H . This H clearly inherits some of the properties of the sequence $\{H_n\}$. It is, in particular, increasing, convex and satisfies the Lipschitz condition

$$|H(r) - H(z)| \leq |r - z|. \quad \text{Also, since } H_n(r) \equiv 0, \quad 0 \leq r \leq m, \quad n \geq 1, \quad \text{it follows that}$$

$$H(r) \equiv 0 , \quad 0 \leq r \leq m \quad (*)$$

and $H_n(r) = I_n + r - M \rightarrow H(r)$ as $n \rightarrow \infty$. Thus $I_n \rightarrow I$ and

$$H(r) = I + r - M , \quad r \geq M . \quad (**)$$

We can now find an $F \in L_1(0, M+1)$ so that

$$H(r) = \int_0^r F(x) dx .$$

This F is increasing since H is convex.

$$F(x) \equiv 0 , \quad 0 \leq x \leq m \quad \text{by } (*)$$

$$F(x) \equiv 1 , \quad x \geq M \quad \text{by } (**) .$$

By redefining F , if necessary, we can ensure that F is continuous from the right. F is therefore a c.d.f. of price quotations. We next show that it is an equilibrium price distribution.

Note that $\pi(p; H)$ in equation (5) is jointly continuous in the pair (p, H) . If we let $\pi_M(H) \equiv \sup_{p \in J} \pi(p, H)$, then the maximum principle [3, p. 116] tells us that $\pi_M(H)$ is continuous. We therefore have:

$$\lim_{n \rightarrow \infty} \pi_M(H_n) = \pi_M(H) \quad (12)$$

Let $\epsilon > 0$ be arbitrarily given. By construction we can find an N_1 so that

$$|\pi(p, H_n) - \pi_M(H_n)| \leq d(H_n) < \epsilon/2 , \quad n \geq N_1 .$$

By (11) we can also find an N_2 so that

$$|\pi_M(H_n) - \pi_M(H)| < \epsilon/2 , \quad n \geq N_2 .$$

Therefore,

$$n \geq \text{Max}\{N_1, N_2\} \Rightarrow |\pi(p, H_n) - \pi_M(H)| < \epsilon .$$

We see that

$$\pi(p, H_n) \rightarrow \pi_M(H) , \quad p \in [m, M]$$

and
$$\pi(p, H_n) \rightarrow \pi(p, H) \quad p \in [m, M]$$

so that

$$\pi(p, H) \equiv \pi_M(H) , \quad p \in [m, M] .$$

Also, $\pi(p, H)$ is increasing on $[0, m]$ and decreasing on $[M, \infty)$ since this is the case for all $\pi(p, H_n)$. The claim follows. ■

One immediate implication of this proposition is that an equilibrium exists for all search cost distributions possessing a uniformly positive density over an interval $[0, A]$ with $0 < A < \infty$. This boundedness assumption on consumers' types is, in turn, quite innocuous so that the theorem is applicable to many standard cases. In any event the existence conditions we have stated are much weaker than those of Axell [2, p. 35] and an equilibrium with prices in a bounded interval is accordingly anticipated to exist for a wide class of distributions. Note also that this distribution is nondegenerate provided $Q(s) < \frac{sq(0)}{1+sq(0)}$ for some $s > 0$.

Let us return to the examples in the previous section and see whether the existence conditions are satisfied for them.

1. $\frac{1-Q(s)}{q(s)} \equiv 1 , \quad q(s) \leq \lambda$
2. $\frac{1-Q(s)}{q(s)} = \frac{s+1}{\theta} \uparrow \infty , \quad q(s) \leq \theta .$

We see that the boundedness condition on the hazard ratio is violated for this Paretian distribution for all values of the parameter θ . The fact that an equilibrium nevertheless exists whenever $\theta \geq 1$ is just an indication that our conditions are only sufficient. The economic reason that a degenerate equilibrium does not exist when $\theta < 1$ is that the underlying distribution induces a less than unitary price elasticity of demand. The loss of customers generated by a price rise is, in other words, less than offset by the increase in the per unit profits. This, in

turn, is caused by the fat upper tail of the Paretian distribution when $\theta < 1$, i.e., by the fact that there are many consumers with high search costs so that the loss of buyers is "very" gradual.

$$3. \quad \frac{1-Q(s)}{q(s)} = \theta - s \leq \theta, \quad q(s) = 1/\theta.$$

Examples with nondegenerate equilibria will be considered in the next section.

4.3. Price Dispersions

Having explored the possibility of a single-price equilibrium, we turn now to the other polar case, namely to price distributions where a continuum of prices is announced in equilibrium. Technically, these are absolutely continuous distributions possessing a positive density over a nontrivial interval(s). I was, regrettably, unable to obtain a complete characterization of this case so I will start out by stating a Diamond-type condition which must be satisfied in order for such highly-dispersed equilibria to exist. The purpose of this condition (which is, in turn, due to Arrow and Rothschild [1]) is to rule out "uncontrolled" price escalation phenomena. We have already remarked that when all buyers have uniformly positive search costs ($s \geq A > 0$), prices will be driven up to the monopoly price. If no upper bound on prices is imposed, as is the case in our structure, then prices will just increase forever.

Theorem 3: No absolutely continuous equilibrium on a segment $[p_1, p_2]$ exists if the density q vanishes on some interval $[0, A)$ with $A > 0$.

Proof: Assume, per absurdum, that a solution to the functional equation

$$\int_p^\infty \phi(r) dr \equiv \int_p^\infty q \left(\int_0^r F(x) dx \right) dr = K/p \quad (13)$$

on $[p_1, p_2]$ exists where K is some constant and $p_2 > p_1$. Since K/p is strictly convex we know (see [8, p. 230]) that ϕ must be strictly decreasing on $[p_1, p_2]$. But then q must be decreasing on $[s_1, s_2]$ where $s_i = \int_0^{p_i} F(x) dx$, $i = 1, 2$, which,

in turn, implies that $s_i \geq A > 0$. By Lemma 1, we can find a $p' \leq p_1$ so that $H(p') = A$ and we obviously have $F(p') > 0$. Therefore, some seller must quote a price $p_0 < p'$.

Now: $H(p) < A$, $p_0 < p < p'$

and thus: $q(H(p)) = 0$, $p_0 < p < p'$.

Consequently

$$\int_{p_0}^{\infty} q(H(p)) dp = \int_{p'}^{\infty} q(H(p)) dp$$

$$\text{and } \pi(p_0) = p_0 \int_{p_0}^{\infty} q(H(p)) dp < p' \int_{p'}^{\infty} q(H(p)) dp = \pi(p') .$$

The trader posting p_0 can thus do better by announcing the higher price p' so that F is not a Nash equilibrium after all. $\blacksquare$

The next result will enable us to check whether a given search cost density can give rise to a dispersed (absolutely continuous) price distribution. Moreover, it will generate this equilibrium distribution if the answer is in the affirmative.

Theorem 4: If a solution F to the functional equation (13) exists and is absolutely continuous on $[p_1, p_2]$, then it must be of the form

$$F(p; K) = \frac{-2K}{q' \left(q^{-1}(K/p^2) \right) p^3} , \quad p_1 \leq p \leq p_2 , \quad (14)$$

where K is the common level of profits.

Proof: Differentiation of (13) over (p_1, p_2) gives us

$$q \left(\int_0^p F(x) dx \right) = \frac{K}{p^2} , \quad p_1 < p < p_2 . \quad (15)$$

From the proof of Theorem 3 we know that q is strictly decreasing over $[s_1, s_2]$. We can thus write

$$\int_0^p F(x)dx = q^{-1}(K/p^2) \quad . \quad (16)$$

Differentiating once more gives the desired result. ■

To test for existence of dispersed equilibrium we simply look at the right-hand side of (14) and ask whether it qualifies as a cumulative distribution function over some nondegenerate segment $p, \bar{p}$. It must, in particular, be increasing and right continuous. The determination of the endpoints $p, \bar{p}$ and the level of common profits, K , is via certain boundary conditions (which are, in turn, dictated by the data of the problem). More specifically,

$$F(p) = 0 \quad , \quad F(\bar{p}) = 1 \quad (17a)$$

$$\pi(p) = \pi(\bar{p}) = K \quad . \quad (17b)$$

Direct applications of this idea are given in the following

Examples:

$$1. \quad q(s) = \gamma e^{-\gamma s} \quad .$$

If we look at (16), we see that an absolutely continuous F must be of the form

$$F(p) = \frac{d}{dp} \left\{ \frac{-\log(K/\gamma p^2)}{\gamma} \right\}$$

where the term in the brackets is just $q^{-1}(K/p^2)$. A straightforward computation shows that the RHS = $2/\gamma p$. Inasmuch as this function is diminishing throughout, it is clear that no dispersed equilibrium exists for the exponential distribution of consumer types.

$$2. \quad Q(s) = \begin{cases} 1 - \frac{1}{(s+1)^\theta} & s < \bar{s} \\ 1 & s \geq \bar{s} \end{cases}$$

where $0 < \theta < 1$.

Q is obtained from the Pareto distribution by truncating and adding a mass point at $\bar{s}$. We already know that an equilibrium distribution for Q exists (Section 4.2) but that this solution is nondegenerate (Section 4.1). We can exploit now equation (14) to obtain explicitly the desired price distribution. This equation yields:

$$F(p;K) = \frac{2(\theta/K)^{\frac{1}{\theta+1}}}{\theta+1} p^{\frac{1-\theta}{1+\theta}}$$

Recalling that K is the common level of profits and that F is a c.d.f., two auxiliary conditions determine F completely. Namely,

$$F(\bar{p};K) = 1$$

$$\bar{p} \int_{\bar{p}}^{\infty} q(I+p-\bar{p})dp = \bar{p}[1-Q(I)] = K$$

where
$$I = \int_0^{\bar{p}} F(x)dx = \bar{s}.$$

The analytic solution of these equations seems a rather nontrivial task. Numerical methods could be utilized, nevertheless, to approximate the desired root $(\bar{p}, K)$. The corresponding profit function is shown in Figure 3.

Figure 3

3. Let

$$q(x) = \begin{cases} \frac{K}{(\sqrt{2x} + 1)^2} & 0 \leq x \leq .5 \\ \frac{qK}{32} - \frac{K}{16} x & .5 \leq x \leq 4.5 \\ 0 & x \geq 4.5 \end{cases}$$

where $K = 1/\log 2$.

A similar application of Theorem 4 shows that the uniform distribution of prices on $[1, 2]$ serves as an equilibrium distribution, i.e.,

$$F(p) = \begin{cases} 0 & p \leq 1 \\ p - 1 & 1 \leq p \leq 2 \\ 1 & p \geq 2 \end{cases}$$

Direct computation shows that $\pi(p) = K$ for $p \in [1, 2]$ and that π is increasing over $[0, 1]$, diminishing over $[2, 6]$ and vanishes for $p > 6$ (see Figure 4). F is therefore an equilibrium distribution according to Definition 1.

Figure 4

We see, in summary, that Theorem 4 provides an algorithm for computing absolutely continuous equilibrium solutions. Equation (14) together with the boundary conditions (17) will, in principle, give us a price distribution provided certain

admissibility requirements (monotonicity, etc.) are met. To check that this F is indeed an equilibrium distribution one must, in addition, verify that the induced profit function π (see equation (5)) satisfies $\pi(p) \leq K$ for $p \notin [p, \bar{p}]$.

5. Conclusion

This paper presents a systematic investigation of the price dispersion phenomenon which is due to the fact that consumers must pay something to secure a price quotation. This informational imperfection gives sellers a limited monopoly power to the extent that buyers shift to other sellers only in response to significant price rises. The idea of this construction is that buyers are self-selecting. More specifically, buyers with high search costs conduct a less extensive search and buy, on the average, at higher prices. The Edgeworth instability is thus circumvented and a monopolistic competition type interaction is generated. It is well known that product differentiation can give rise to price dispersion. Inasmuch as the model analysed here is characterized by product homogeneity, this structure provides an alternative rationale for the price multiplicity phenomenon. Furthermore, the latter is compatible also with the profit maximizing behavior of identical sellers. It cannot be regarded, therefore, as a transitory or a disequilibrium adjustment process.

I have analysed the canonical sequential search set-up which is consistent with the standard postulate of consumers' rational choice. The results obtained within this framework are as follows:

1. Characterization of buyers' search cost distribution giving rise to a single-price equilibrium. The main property of this distribution is that there are sufficiently many consumers with low sampling costs. Their extensive search activity wipes out all price differentials.
2. An existence proposition is proved under rather weak assumptions. It is argued that a multitude of prices will be generically associated with an equilibrium distribution.
3. An explicit formula for computing equilibrium distributions with a whole continuum of market prices is provided and illustrated with several examples.

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