**DYNAMIC PHASE TRANSITIONS:
A CONNECTION MATRIX APPROACH**

By

Konstantin Mischaikow

IMA Preprint Series # 584

October 1989

DYNAMIC PHASE TRANSITIONS: A CONNECTION MATRIX APPROACH*

KONSTANTIN MISCHAIKOW†

Abstract. Travelling wave solutions between liquid and vapor phases in a van der Waals fluid are shown to exist. The emphasis, however, is on the techniques, namely the Conley index and connection matrix. In particular it is suggested that these methods provide a natural approach for solving problems of this nature.

1. Introduction. In this paper we shall discuss the existence of travelling wave solutions between liquid and vapor phases in a van der Waals fluid. The particular equations to be studied were derived by Marshall Slemrod [S11], and are meant to model an elastic fluid with pressure given by the van der Waals equations of state and possessing a higher order correction term given by Korteweg's theory of capillarity. A complete description of the analysis of shocks and phase transitions in such a fluid would have to begin with a discussion of a set of partial differential equations of mixed type. However, since we are only interested in the travelling wave solutions, we shall begin with the reduced problem, namely, a system of ordinary differential equations. The interested readers should consult [S11], [S12], [HS], [G1], [G2], and [G3] for a discussion of the full problem.

It must be emphasized that most of the results which we shall describe were obtained by Slemrod [S11] and M. Grinfeld [G1], [G3]. The purpose for reproducing these results is to demonstrate the power of recent developments in the Conley index. Both Slemrod and Grinfeld made extensive use of this index. However, the techniques which we shall use, most prominently, the connection matrix, were not available when their work was being done. It is our contention that these new techniques simplify the computations sufficiently to warrant this presentation. Furthermore, this problem is typical of a wide variety of phase transition problems and these techniques should prove equally useful in these other applications.

Before considering the specific equations let us consider four important characteristics of this problem. First, the system of ordinary differential equations is n -dimensional (in this case $n = 3$), and phase plane techniques are difficult if not impossible to apply in $n \geq 3$. Second, the number of critical points is large (i.e. greater than or equal to 4). Third, the goal is to find heteroclinic orbits, i.e. solutions which asymptote to different critical points in forward and backward time. These three characteristics make the problem "challenging". Finally, there exists a Lyapunov function. This of course simplifies the problem.

*Partially supported by an AURIG from Michigan State University.

†Department of Mathematics, Michigan State University, East Lansing, MI 48824. Current address: School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332.

We hope to convince the reader that for any problem with the above characteristics, the Conley index theory is a natural approach to adopt. This approach can be summarized in three steps:

1. Identify the critical points and their Conley indices.
2. Show that the set of bounded solutions is compact and compute its index.
3. Apply the connection matrix theory.

This last step is, for the most part, just a matter of simple matrix computations.

The equations we shall study are

$$\begin{aligned} w' &= v \\ \mathbf{P}_U \quad Av' &= -Uv - U^2(w - w_-) - p(w, \theta) + p_- \\ \theta' &= \frac{u}{\alpha} U \left\{ -(\varepsilon(w, \theta) - \varepsilon_-) - (w - w_-)p_- + \frac{U^2}{2}(w - w_-)^2 + \frac{Av^2}{2} \right\} \end{aligned}$$

where:

1. The pressure p is given by the van der Waals equation of state

$$p(w, \theta) = \frac{R\theta}{w - b} - \frac{a}{w^2} \quad \text{for } 0 < b < w < \infty$$

where a, b , and R are all positive constants, w is the specific volume, and θ is the absolute temperature.

2. The specific internal energy function is given by

$$\varepsilon(w, \theta) = -\frac{a}{w^2} + F(\theta) - \theta F'(\theta)$$

where F is any function for which $F''(\theta) < 0$ and $\lim_{\theta \rightarrow \infty} \frac{F(\theta)}{\theta} = -\infty$. The most natural choice of F is $F^*(\theta) = -c_v \theta \ln \theta + \text{constant}$. We shall study the problem with the more general internal energy function only because it demonstrates some of the power of the index theory. In particular, for $F = F^*$ there are at most 4 critical points, two of which are of particular importance since they correspond to the liquid and vapor phases of the fluid. For a general F , there may be more critical points though again there are only two physically relevant ones. Using the index theory we shall show that there is a natural way to ignore the irrelevant critical points. In particular, under appropriate conditions (see the end of §3) treating the general F is no more difficult than studying $F = F^*$.

3. U is the wave speed of the travelling wave and as indicated above will be used as a parameter. One only expects to find traveling wave solutions for a small set of wave speeds.
4. α and μ are positive constants corresponding to the coefficient of thermal conductivity and the viscosity coefficient, respectively.

5. w_- and θ_- are the values of the specific volume and absolute temperature at the critical point corresponding to the liquid phase, $p_- = p(w_0, \theta_-)$ and $\varepsilon_- = \varepsilon(w_-, \theta_-)$.

Since we are studying travelling waves, we are interested in those solutions to $\mathbf{P}_U$ which satisfy certain boundary conditions, the explanation of which requires the following digression concerning the function p . In Figure 1.1, two θ isoclines of p have been plotted (clearly, fixing θ defines p as a function only of w). Choose θ_- small enough that the function $p(w, \theta_-)$ has a local minimum and a local maximum. In the diagram the location of this minimum and maximum are labelled by w_α and w_β . Let $w_- \in (b, w_\alpha)$. This of course defines p_- and ε_- . One can now easily check that $(w, v, \theta) = (w_-, 0, \theta_-)$ is a critical point of $\mathbf{P}_U$.

Figure 1.1

We can now formally state the problem which needs to be solved. Prove the existence of wave speeds U for which there exist solutions $(w(t), v(t), \theta(t))$ to $\mathbf{P}_U$ satisfying the boundary conditions

$$\lim_{t \rightarrow -\infty} (w(t), v(t), \theta(t)) = (w_-, 0, \theta_-)$$

and

$$\lim_{t \rightarrow \infty} (w(t), v(t), \theta(t)) = (w_+, 0, \theta_+)$$

where $w_+ > w_\beta$. Solutions of this form shall be denoted by $\mathbf{w}_- \rightarrow \mathbf{w}_+$. If one defines $\mathcal{U} = \{U > 0 | \mathbf{w}_- \rightarrow \mathbf{w}_+ \text{ exists} \}$ then the problem can be restated as: show that $\mathcal{U} \neq \emptyset$.

The Lyapunov function for the system $\mathbf{P}_U$ is given by

$$V(w, v, \theta) = R \ln(w - b) - F'(\theta) + \frac{1}{2} \left\{ -(\varepsilon(w, \theta) - \varepsilon_-) - (w - w_-)p_- + \frac{U^2}{2}(w - w_-)^2 + \frac{Av^2}{2} \right\}$$

It is a simple computation to check that for the system $\mathbf{P}_U$,

$$\begin{aligned} \frac{dV}{dt} &= -\frac{U}{\theta} w^2 - \frac{\mu U}{\alpha \theta^2} \left\{ -(\varepsilon(w, \theta) - \varepsilon_-) - (w - w_-)p_- + \frac{U^2}{2}(w - w_-)^2 \right\}^2 \\ &\leq 0. \end{aligned}$$

The rest of the paper is organized as follows. We begin in §2 with a brief review of the relevant aspects of the Conley index theory, with a special emphasis on the connection matrix and transition matrix techniques. In §3 we analyze the existence, and compute the index, of the critical points. (Notice that it is not clear from the above discussion that the critical point $(w_+, 0, \theta_+)$ exists.) Our analysis is quite similar to that of Grinfeld [G1]. §4 is concerned with the set of bounded solutions. In §5 we compute the necessary connection matrices. And, finally in §6 we state and prove theorems guaranteeing the existence of the desired travelling waves.

A final comment is in order. As was indicated earlier the main goal of this paper is to show how the connection matrix theory can be applied to dynamic phase transition problems. Thus we are emphasizing the technique rather than the results. This is most apparent in §3 where certain necessary assumptions are made, and in §6 where the results are stated. In the first case we state the assumptions in the most convenient form for the connection matrix theory. We justify this approach by acknowledging that the work required to verify the assumptions can be found in [G3]. With regard to §6, we state three theorems, only two of which we prove. As the reader will see the proofs are trivial (given the connection matrix machinery). The theorem we do not prove is due to Grinfeld and requires a more subtle approach. We state it for two reasons: (one) to acknowledge that the connection matrix cannot do everything, and (two) because we feel that it would be desirable, and perhaps even possible, to extend Conley index techniques to handle such problems directly.

Acknowledgements. I would like to thank Marshall Slemrod and Michael Grinfeld for calling this problem to my attention.

2. Conley Index Theory. This section presents a brief review of the Conley index, connection matrices, and transition matrices. While it is assumed that the reader is familiar with the Conley index (see [C], [S], [Sm]), the basic properties of the connection matrices and transition matrices are described in sufficient detail to allow the reader to mechanically

apply the theory. For a complete presentation of this material see [F], [Mc], [M1], [M2], and [R].

Throughout this paper S will denote an isolated invariant set and $\mathbf{h}(S)$, the Conley index of S . Recall that $\mathbf{h}(S)$ is the homotopy type of a pointed topological space, and therefore one can define a homology version of the Conley index by

$$CH_*(S; \mathbf{R}) \equiv H_*(\mathbf{h}(S); \mathbf{R})$$

where the latter expression denotes the singular homology of the topological space $\mathbf{h}(S)$ relative the special point with coefficients in a ring $\mathbf{R}$. The reason for using the homology index is that in general homology theory is more computable than homotopy theory. To simplify the computations even further we shall choose $\mathbf{R} = \mathbf{Z}_2$. Of course this implies that $CH_*(S) = CH_*(S; \mathbf{R})$ is a graded vector space.

PROPOSITION 2.1. *If S is a hyperbolic critical point with an unstable manifold of dimension k , then*

$$CH_n(S) \approx \begin{cases} \mathbf{Z}_2 & \text{if } n = k \\ 0 & \text{otherwise} \end{cases}.$$

If $S = \emptyset$, then $CH_(S) \approx (0, 0, 0, \dots)$.*

DEFINITION 2.2. Given two isolated invariant sets, S and S' , the set of *connections* from S to S' is denoted by $C(S, S') = \{x | \omega(x) \subset S' \text{ and } \omega^*(x) \subset S\}$ where $\omega(x)$ and $\omega^*(x)$ denote the omega and alpha limit sets of x , respectively.

Using this language, our problem can be restated as; show that $C(w_-, w_+) \neq \emptyset$. Before considering how one decomposes isolated invariant sets we need to introduce some notation.

A partially order set, $(P, >)$, consists of a finite set P along with a strict partial order relation, $>$, which satisfies:

- (i) $i > i$ never holds for $i \in P$, and
- (ii) if $i > j$ and $j > k$, then $i > k$ for all $i, j, k \in P$.

An interval in $(P, >)$ is a subset, $I \subset P$, such that given $i, j \in I$, if $i < k < j$ then $k \in I$.

DEFINITION 2.3. A *Morse decomposition* of S , denoted by $\mathbf{M}(S) = \{M(i) | i \in (P, >)\}$, is a collection of mutually disjoint isolated invariant subsets of S , indexed by $(P, >)$, such that given $x \in S$, then either $x \in M(i)$ for some i , or there exists $i, j \in P$ such that $i > j$ and $x \in C(M(i), M(j))$. The invariant sets, $M(i)$, are called *Morse sets*.

Recall that in our problem we have a Lyapunov function, V . Hence, a natural Morse decomposition of S , the set of all bounded solutions, would consist of all the critical points of the system. Furthermore, an ordering on this Morse decomposition could be given by $i > j$ if $V(M(i)) > V(M(j))$.

Let I be an interval in P , then one can define a new isolated invariant set by

$$M(I) = \left(\bigcup_{i \in I} M(i) \right) \cup \left(\bigcup_{ji \in I} C(M(j), M(i)) \right)$$

Since $M(I)$ is an isolated invariant set $CH_*(M(I))$ is defined.

Turning to the definition of a connection matrix, let

$$\Delta : \oplus_{i \in P} CH_*(M(i)) \rightarrow \oplus_{i \in P} CH_*(M(i))$$

be a linear map, then Δ can be written as a matrix of maps

$$\Delta = (\Delta(i, j))$$

where $\Delta(ij) : CH_*(M(i)) \rightarrow CH_*(M(j))$. For any interval I in P , let

$$\Delta(I) = (\Delta(i, j))_{i, j \in I}.$$

DEFINITION 2.4. Δ is called a *connection matrix* for $\mathbf{M}(S)$ if the following four properties are satisfied:

- (i) If $i \not\geq j$ then $\Delta(i, j) = 0$. (*strict upper triangularity*)
- (ii) $\Delta(i, j)(CH_k(M(i))) \subset CH_{k-1}(M(j))$. (*degree -1 map*)
- (iii) $\Delta \circ \Delta = 0$.
- (iv) $CH_*(M(I)) = \frac{\text{kernel } \Delta(I)}{\text{image } \Delta(I)}$ for every interval I . (*rank condition*)

Some of the basic theorems concerning connection matrices are as follows.

THEOREM 2.5. (Franzosa [F]) *Given a Morse decomposition there exists at least one connection matrix.*

THEOREM 2.6. (Franzosa [F]) *Let $\{i, j\}$ be an interval in P and assume that $\Delta(i, j) \neq 0$, then $C(M(i), M(j)) \neq \emptyset$.*

THEOREM 2.7. (McCord [Mc]) *Let $\mathbf{M}(S) = \{M(1), M(0) | 1 > 0\}$ be a Morse decomposition consisting of hyperbolic critical points. Assume for $i = 0, 1$, that*

$$CH_n(M(i)) = \begin{cases} \mathbb{Z}_2 & \text{if } n = k + i \\ 0 & \text{otherwise} \end{cases}.$$

Furthermore, let $C(M(1), M(0))$ consists of exactly q heteroclinic orbits which arise as the intersection of the stable and unstable manifolds of $M(0)$ and $M(1)$, respectively. Then $\Delta(1, 0) = q \bmod 2$.

Up to this point, our discussion has been based on a given flow. However, the problem we wish to study consists of a parameterized family of flows. Thus we need to compare

different connection matrices corresponding to the flows at different parameter values. With this in mind, consider the family of differential equations

$$x' = f(x, \lambda)$$

where $x \in \mathbb{R}^n$ and $\lambda \in \Lambda = \mathbb{R}$. The first assumption that needs to be made is that there exists an isolated invariant set S which continues over Λ (see [Sa]). For a fixed λ this isolated invariant set will be denoted by S^λ . Let $\mathbf{M}(S^0) = \{M^0(i) | i \in (P^0, >^0)\}$ and $\mathbf{M}(S^1) = \{M^1(i) | i \in (P^1, >^1)\}$ denote Morse decompositions of S^λ at $\lambda = 0$ and 1 . Let Δ^0 and Δ^1 be connection matrices for these Morse decompositions.

DEFINITION 2.8. A *transition system* consists of an infinite sequence of systems of the form

$$\begin{aligned} x' &= f(x, \lambda) \\ \lambda' &= \varepsilon_n \lambda (\lambda - 1) \end{aligned}$$

where $1 \gg \varepsilon_n > \varepsilon_{n+1}$ and $\lim_{n \rightarrow \infty} \varepsilon_n = 0$.

For each ε_n there exists an isolated invariant set with a Morse decomposition

$$\mathbf{M} = \{\bar{M}(k) | k \in P^0 \cup P^1 \text{ and } \bar{M}(k) = M^i(k) \text{ for } k \in P^i\}$$

and with the partial order relation

$$\begin{aligned} j &> i && \text{for } j \in P^1 \text{ and } i \in P^0 \\ i &>^0 i' && \text{for } i, i' \in P^0 \\ j &>^1 j' && \text{for } j, j' \in P^1. \end{aligned}$$

PROPOSITION 2.9. $CH_n(\bar{M}(i)) \approx CH_n(M^0(i))$

$$CH_n(\bar{M}(j)) \approx CH_{n-1}(M^1(j))$$

PROPOSITION 2.10. ([R], [M2]) A connection matrix for $\mathbf{M}$ takes the form

$$\begin{pmatrix} \Delta^0 & T^{10} \\ 0 & \Delta^1 \end{pmatrix}$$

where the entries Δ^0 and Δ^1 are the connection matrices for $\mathbf{M}(S^0)$ and $\mathbf{M}(S^1)$.

Remark 2.11. Notice, by Proposition 2.9 that the homology indices of $\bar{M}(j)$ are not in exact agreement with those of $M^1(j)$. However, the difference is only in the grading. Thus the Δ^1 referred to in Proposition 2.10 is obtained by shifting the grading of the connection matrix for $\mathbf{M}(S^1)$ up by one.

DEFINITION 2.12. T^{10} is called the *transition matrix* from $\lambda = 1$ to $\lambda = 0$.

Returning to the transition system. Notice that in the limit one obtains the parameterized system

$$\begin{aligned} x' &= f(x, \lambda) \\ \lambda' &= 0. \end{aligned}$$

The following theorem indicates that entries in the transition matrix correspond to connections for the system $x' = f(x, \lambda)$ for $\lambda \in (0, 1)$.

THEOREM 2.13. (Reineck [R]) *Let $T(i^1 j^0) : CH * (M^1(i)) \rightarrow CH * (M^0(j))$ be an entry of the transition matrix T^{10} . If $T(i^1 j^0) \neq 0$, then there exists a finite sequence $1 \geq \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k \geq 0$ of parameter values and corresponding $k_r \in P^{\lambda_r}$ such that $C(M(k_r), M(k_{r+1})) \neq \emptyset$ under the flow defined by $x' = f(x, \lambda_r)$.*

Finally, the following theorem indicates that certain entries in the transition matrix can be determined by understanding how the individual Morse sets continue.

THEOREM 2.12. (Mischaikow [M1]) *Let the Morse set $M(i)$ continue over $[0, 1]$ as an attractor or repeller. Then $T(i^1, i^0)$ is an isomorphism.*

3. The Critical Points. As was indicated in the introduction, the first step in applying the connection matrix techniques is to study the critical points of the system. Obviously, one obtains the critical points of $\mathbf{P}_U$ by solving the following system of equations:

$$\begin{aligned} (3.1) \quad & 0 = v \\ & 0 = -Uv - U^2(w - w_-) - p(w, \theta) + p_- \\ & 0 = \frac{\mu}{\alpha} U \left\{ -(\varepsilon(w, \theta) - \varepsilon_-) - (w - w_-)p_- + \frac{U^2}{2}(w - w_-)^2 + \frac{Av^2}{2} \right\}. \end{aligned}$$

Substituting in the definitions of $p(w, \theta)$ and $\varepsilon(w, \theta)$ one gets

$$(3.2a) \quad 0 = -U^2(w - w_-) - \frac{R\theta}{w - b} + \frac{a}{w^2} + p_-$$

$$(3.2b) \quad 0 = \frac{a}{w} - F(\theta) + \theta F'(\theta) + \varepsilon_- - (w - w_-)p_- + \frac{U^2}{2}(w - w_-)^2.$$

Define

$$f_1(w) = \frac{w - b}{R} \left\{ -U^2(w - w_-) + \frac{a}{w^2} + p_- \right\}$$

and

$$f_2(w) = \frac{a}{w} + \varepsilon_- - (w - w_-)p_- + \frac{U^2}{2}(w - w_-)^2.$$

Clearly then (3.2) is equivalent to

$$(3.3a) \quad \theta = f_1(w)$$

$$(3.3b) \quad F(\theta) - \theta F'(\theta) = f_2(w).$$

Notice that $f_2'(w) = -\frac{R\theta}{w-b}f_1(w)$. Thus, differentiating (3.3b) with respect to w leads to

$$-\theta F''(\theta) \frac{d\theta}{dw} = -\frac{R\theta}{w-b} f_1(w)$$

or

$$(3.4) \quad \frac{d\theta}{dw} = \frac{R}{w-b} \frac{1}{\theta F''(\theta)} f_1(w).$$

Hence, for $f_1(w) > 0$, one has that $\frac{d\theta}{dw} < 0$, i.e. the graph of the curve defined by (3.2b) is monotone decreasing as long as $f_1(w)$ is positive. In particular, (3.2b) defines θ as a function of w for all $\theta > 0$. From now on this function will be denoted by g_2 , i.e. $\theta = g_2(w)$.

Recall, from the introduction, that θ_- was chosen in such a manner as to insure that $p(w, \theta_-) = p_-$ has three solutions. Thus, setting $U = 0$ implies that the graph of $f_1(w)$ is as in Figure 3.1. Choosing $F = F^*$ and c_v large one obtains Figure 3.1(a). In particular, there are three values of w which correspond to critical points of $\mathbf{P}_U$ and the third one, w_+ , is greater than w_β . If we consider arbitrary F (though insisting that $F'' < 0$) then it is possible to have the situation described in Figure 3.1(b). Notice, however, that w_+ is still well defined. On the other hand if one chooses $U > 0$, but small then one obtains the diagrams of Figure 3.2. Again, (a) corresponds to $F = F^*$ and (b) represents a possible graph for a more general F . And again, it is worth emphasizing that the critical point $(w_+, 0, \theta_+)$ is uniquely defined in both cases.

(a)

(b)

Figure 3.1

(a)

(b)

Figure 3.2

Before continuing we wish to introduce some notation. As was mentioned before we are primarily interested in the critical points $(w_-, 0, \theta_-)$ and $(w_+, 0, \theta_-)$. To simplify the notation we shall denote these critical points by $\mathbf{w}_\pm$. However, the other critical points shall also play a role in our analysis. If $F = F^*$ then we need only concern ourselves with the points $(w_i, 0, \theta_i)$ for $i = 1, 2$, however, for general F there may be more critical points. We can however define the Morse sets

$$\mathbf{W}_1 = \left(\bigcup_{w_\alpha < w_i < w_\beta} (w_i, 0, \theta_i) \right) \cup \left(\bigcup_{w_\alpha < w_i, w_j < w_\beta} C((w_1, 0, \theta_1), (w_j, 0, \theta_j)) \right)$$

and

$$\mathbf{W}_2 = \left(\bigcup_{w_+ < w_i} (w_1, 0, \theta_i) \right) \cup \left(\bigcup_{w_+ < w_i, w_j} C((w_i, 0, \theta_i), (w_j, 0, \theta_j)) \right)$$

As was indicated in the introduction U is treated as a free parameter. Thus the information of Figures 3.1 and 3.2 needs to be presented in terms of U and w . We begin with the following definitions. Let

$$U_0 = \min\{U \geq 0 | w_+ \text{ exists} \}$$

and

$$U_1 = \max\{U \geq 0 | w_+ \text{ exists} \}.$$

As is indicated in Figure 3.1, it is possible for $U_0 = 0$.

To understand how the curves in Figures 3.1 and 3.2 change as a function of U , consider f_1 and f_2 as functions of U and w , i.e. (3.3) becomes

$$(3.5a) \quad \theta = f_1(w, U)$$

$$(3.5b) \quad F(\theta) - \theta F'(\theta) = f_2(w, U).$$

Now differentiating by U gives

$$\frac{\partial \theta}{\partial U} = -2(w - b)(w - w_-) \frac{U}{R} > 0$$

and

$$\begin{aligned} -\theta F''(\theta) \frac{\partial \theta}{\partial U} &= U(w - w_-)^2 \\ \frac{\partial \theta}{\partial U} &= -\frac{U(w - w_-)^2}{\theta F''(\theta)} > 0 \end{aligned}$$

for $w > w_-$. Thus, as U is increased the curve corresponding to f_1 moves down pointwise and g_2 moves up pointwise.

LEMMA 3.1. *There exists $U_1 \in (0, \infty)$.*

Proof. Consider $f_1(w, U) = 0$ as a function of U . This is equivalent to solving

$$-w^4 + \left[w_- + b + \frac{p_-}{U^2} \right] w^3 - b \left(w_- + \frac{p_-}{U^2} \right) w^2 + \frac{a}{U} w - \frac{ab}{U} = 0.$$

If we let $U \rightarrow \infty$, then we have that the zeroes are given by $0, b$, and w_- . Furthermore for $w > w_1$, $f(w, \infty) < 0$. Thus the right local maxima of f_1 tends to zero as $U \rightarrow \infty$. Therefore, there exists U^* such that the right local maximum of f_1 is tangent to g_2 . Since f_1 is moving down pointwise and g_2 is moving up pointwise as a function of U for all $U > U^*$, w_1 does not exist. $\square$

If we set $F = F^*$, then the above arguments can be summarized by the bifurcation diagrams of Figures 3.3 and Figures 3.4. (For general F one might have more wiggles).

Figure 3.3

Figure 3.4

Our next step is to determine the Conley indices of the critical points. This is most easily done by assuming that $F = F^*$, linearizing about the critical points, and computing the characteristic equation. Doing this simple calculation yields:

$$\lambda^3 + \left(\frac{\mu U c_v}{\alpha} + \frac{U}{A} \right) \lambda^2 + \left(\frac{\mu U^2 c_v}{A\alpha} - \frac{R}{A(w-b)} f_1'(w) \right) \lambda - \frac{\mu U c_v}{\alpha} \frac{R}{A(w-b)} (f_1'(w) - f_2'(w)) = 0.$$

Solving for the roots explicitly one gets that if $f_1'(w) - f_2'(w) > 0$, then there exists a unique positive solution and if $f_1'(w) - f_2'(w) < 0$, then all the solutions have negative real parts. Therefore, we have the following lemma.

LEMMA 3.2. $CH_*(\mathbf{w}\pm) \approx (0, Z_2, 0, 0, \dots)$ and $CH_*(\mathbf{W}_i) \approx (Z_2, 0, 0, \dots)$.

Before considering this lemma in the context of more general F 's, we shall make the following three assumptions.

A1 $V(\mathbf{w}_-) > V(\mathbf{w}_+)$

A2 If $\mathbf{w}^*$ is a critical point in $\mathbf{W}_i$ then $V(\mathbf{w}\pm) > V(\mathbf{w}^*)$.

A3 For $U \in [U^*, U^{**}]$, $\theta_2 > 0$.

Since we are interested in obtaining connections of the form $\mathbf{w}_- \rightarrow \mathbf{w}_+$ and since V is a Lyapunov function for $(\mathbf{P})$ it is obvious that **A1** is a necessary condition. For a proof that this assumption can be satisfied, at least in the case of $F = F^*$, we refer the reader to [G3]. Since it holds for F^* it must also hold for F near F^* .

The second assumption is considerably stronger than what is necessary, but clearly holds for $F = F^*$ and hence for F near F^* . It guarantees two things, first, that $\mathbf{W}_i$ are Morse sets and, two, that $CH_*(\mathbf{W}_i)$ remains the same as in Lemma 3.2. Therefore, Lemma 3.2 remains true for all F for which assumption **A2** holds.

Finally, **A3** is necessary on physical grounds since θ represents absolute temperature, and hence, must be positive.

4. The Set of Bounded Solutions. The second step in applying the Conley theory, is to study the set of bounded solutions. In our case this means understanding the set of bounded solutions for the parameter values $U^* < U < U^{**}$ and for which $w > b$ and $\theta > 0$. Thus in this section we define an appropriate isolating neighborhood, i.e. one which contains all the orbits of interest to us, and then compute the index of the isolated invariant.

Notice that one can think of the Lyapunov function V as a function of the parameter value. To emphasize this we shall write $V = V_U$. Define

$$K = \max_{U \in [U^*, U^{**}]} V_U(w_-, 0, \theta_-).$$

Let $N = \{(w, v, \theta) | w \geq \lambda, \theta > 0, V(w, v, \theta) \leq 2K\}$ and let S_U be the maximal invariant set in N under $\mathbf{P}_U$.

PROPOSITION 4.1. *There exists $\lambda \in (b, w_-)$ such that N is an isolating neighborhood for $\mathbf{P}_U$ for all $U \in [U^*, U^{**}]$.*

Proof. The first step is to show that N is compact. This follows from the following limits which hold for $w > b$, and $\theta > 0$;

$$\begin{aligned} \lim_{w \rightarrow \infty} V(w, v, \theta) &= \infty \\ \lim_{|v| \rightarrow \infty} V(w, v, \theta) &= \infty \\ \lim_{\theta \rightarrow 0} V(w, v, \theta) &= \infty \end{aligned}$$

and

$$\lim_{\theta \rightarrow \infty} V(w, v, \theta) = \infty.$$

Next one needs to show that there exists λ such that $S \cap \partial N = \emptyset$. Let $(w, v, \theta) \in \partial N$ with $w > \lambda$. Since $\frac{dV}{dt} < 0$ for these points, $(w, v, \theta) \notin S$. Thus we need only consider

those points $(w, v, \theta) \in \partial N$ for which $w = \lambda$. If $v > 0$, then $(\lambda, v, \theta) \cdot [-t, 0] \not\subset N$ and if $v < 0$, then $(\lambda, v, \theta) \cdot [0, t] \not\subset N$. So assume $(\lambda, v, \theta) \in \partial N$ and $v = 0$. If $v' < 0$, then one has an external tangency. Let $(\lambda, 0, \bar{\theta}) \in \partial N$ for which $v' = 0$, then $\lim_{\lambda \rightarrow b} V(\lambda, 0, \bar{\theta}) = \infty$, i.e. there exists a λ such that $v' \neq 0$. One easily checks that in fact $v' < 0$. $\square$

PROPOSITION 4.2. $CH_*(S) \approx (0, 0, 0, \dots)$.

Proof. Notice that N is a contractible region. Furthermore $L = \{(\lambda, v, \theta) \in \partial N | v \leq 0\}$ is contractible and (N, L) is an index pair for S . $\square$

In the following sections S_U will be the total isolated invariant set and its Morse decomposition will consist of the Morse sets $\mathbf{w} \pm$, and $\mathbf{W}_i, i = 0, 1$ with the ordering induced by V .

5. The Connection Matrices. We are now in the position to begin step three in the application of the Conley index theory, namely the computations of the connection matrices.

Recall that we are attempting to show that $\mathcal{U} \neq \emptyset$. So choose $U^1, U^2, U^3 \notin \mathcal{U}$ such that $0 \leq U^* < U^1 < U_0 < U^2 < U_1 < U^3 < U^{**}$ (obviously, if $U_0 = 0$, then U^1 cannot be defined). Let Δ^i denote the connection matrix for the isolated invariant set S_{U^i} .

LEMMA 5.1.

$$\Delta^1 = \begin{matrix} CH_0(\mathbf{w}_2) & CH_1(\mathbf{w}_-) \\ CH_0(\mathbf{w}_2) & CH_1(\mathbf{w}_-) \end{matrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

and

$$\Delta^3 = \begin{matrix} CH_0(\mathbf{w}_1) & CH_1(\mathbf{w}_-) \\ CH_0(\mathbf{w}_1) & CH_1(\mathbf{w}_-) \end{matrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Proof. Consider Δ^1 . In this case the Morse decomposition consists of $\{\mathbf{W}_2, \mathbf{w}_-\}$. Thus, $\Delta^1 : CH_*(\mathbf{w}_2) \oplus CH_*(\mathbf{w}_-) \rightarrow CH_*(\mathbf{w}_2) \oplus CH_*(\mathbf{w}_-)$. By Lemma 3.2, Δ^1 can be considered as a map on $CH_0(\mathbf{w}_2) \oplus CH_1(\mathbf{w}_-)$. Since a connection matrix is a degree -1 map Δ^1 must take the form

$$\begin{pmatrix} 0 & * \\ 0 & 0 \end{pmatrix}$$

where $*$ remains to be determined. By the rank condition, $*$ must be an isomorphism, which in $\mathbf{Z}_2$ coefficients implies that $*$ = 1.

The proof for Δ^3 is similar. $\square$

The same sorts of arguments, which rely only on the definition of the connection matrix give the following result.

LEMMA 5.2.

$$\Delta^2 = \begin{matrix} & CH_0(\mathbf{w}_1) & CH_0(\mathbf{w}_2) & CH_1(\mathbf{w}_+) & CH_1(\mathbf{w}_-) \\ \begin{matrix} CH_0(\mathbf{w}_1) \\ CH_0(\mathbf{w}_2) \\ CH_1(\mathbf{w}_+) \\ CH_1(\mathbf{w}_-) \end{matrix} & \begin{pmatrix} 0 & 0 & a & b \\ 0 & 0 & c & d \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

and $ad + bc = 1$.

Since $U^i \notin \mathcal{U}$, we shall use the transition matrices to show that for some value of U between U^1 and U^3 there exists an element of $\mathcal{U}$. The transition matrices we shall use are those which relate Δ^1 to Δ^2 and Δ^2 to Δ^3 and we shall denote these by T^{12} and T^{23} respectively.

LEMMA 5.3.

$$\begin{pmatrix} \Delta^2 & T^{12} \\ 0 & \Delta^1 \end{pmatrix} = \begin{matrix} & CH_0(\mathbf{W}_1) & CH_0(\mathbf{W}_2) & CH_1(\mathbf{w}_+) & CH_1(\mathbf{w}_-) & CH_1(\mathbf{W}_2) & CH_2(\mathbf{w}_-) \\ \begin{matrix} CH_0(\mathbf{W}_1) \\ CH_0(\mathbf{W}_2) \\ CH_1(\mathbf{w}_+) \\ CH_1(\mathbf{w}_-) \\ CH_1(\mathbf{W}_2) \\ CH_2(\mathbf{w}_-) \end{matrix} & \begin{pmatrix} 0 & 0 & a & b & 0 & 0 \\ 0 & 0 & c & d & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

where $a\delta + b = 0$ and $c\delta + d + 1 = 0$.

Proof. From the discussion of §2 it is clear that only the T^{12} entries need to be determined. Because T^{12} is a degree -1 map, one immediately obtains the fact that

$$T^{12} = \begin{pmatrix} \alpha & 0 \\ \beta & 0 \\ 0 & \delta \\ 0 & \gamma \end{pmatrix}.$$

To determine the values of the entries α, β, γ , and δ require a more careful analysis of the flow. Notice that $\mathbf{w}_-$ continues as a Morse set over the interval $[U^*, U^{**}]$ and furthermore, is a repeller for all parameter values U . Thus $\gamma = 1$. Similarly, $\mathbf{W}_2$ continues as an attractor over the interval $[U^1, U^2]$ and hence, $\beta = 1$ and $\alpha = 0$. The equalities involving the entries of the matrix follow from the condition that connection matrices square to zero. $\square$

A similar argument results in:

LEMMA 5.4.

$$\begin{pmatrix} \Delta^2 & T^{23} \\ 0 & \Delta^3 \end{pmatrix} = \begin{matrix} & CH_0(\mathbf{W}_1) & CH_0(\mathbf{W}_2) & CH_1(\mathbf{w}_+) & CH_1(\mathbf{w}_-) & CH_1(\mathbf{W}_1) & CH_2(\mathbf{w}_-) \\ \begin{matrix} CH_0(\mathbf{W}_1) \\ CH_0(\mathbf{W}_2) \\ CH_1(\mathbf{w}_+) \\ CH_1(\mathbf{w}_-) \\ CH_1(\mathbf{W}_1) \\ CH_2(\mathbf{w}_-) \end{matrix} & \begin{pmatrix} 0 & 0 & a & b & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

where $a_\gamma + b + 1 = 0$ and $c_\gamma + d = 0$.

Notice that $\gamma = 0$ or 1 and $\delta = 0$ or 1 . Thus we can solve the systems of equations

$$\begin{pmatrix} \Delta^2 & T^{12} \\ 0 & \Delta^1 \end{pmatrix}^2 = 0$$

and

$$\begin{pmatrix} \Delta^2 & T^{23} \\ 0 & \Delta^3 \end{pmatrix}^2 = 0$$

to obtain additional constraints on a, b, c , and d . Doing so one obtains the following lemma

LEMMA 5.5.

$$\Delta^2 = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

or

$$\Delta^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Remark 5.6. The result of Lemma 5.5 is independent of the fact that $U_0 > 0$. this follows from the fact that we can continue the system with $U_0 = 0$ to a system with $U_0 > 0$. Thus, given a connection matrix Δ^2 for $U_0 = 0$ it must be related to a Δ^2 for $U_0 > 0$ by a transition matrix, which only allows for the possibilities stated in Lemma 5.5.

6. Results. We are now ready to present the results. We remark that Theorems 6.1 and 6.4 can be found in [G3] while 6.2 is new. Hopefully the brevity and simplicity of the *proofs* will convince the reader that the previous sections were worth reading.

THEOREM 6.1. Assume **A1** through **A3**, and that $0 \leq U^* < U_0 < U_1 < U^{**}$, then $\mathcal{U} \neq \emptyset$, i.e. there exists $\bar{U} \in [U_0, U_1]$ for which there exists a heteroclinic orbit from $(w_-, 0, \theta_-)$ to $(w_+, 0, \theta_+)$ solving $\mathbf{P}_{\bar{U}}$.

Proof. Recall from Lemmas 5.3 and 5.4 that $\delta a + b = 0$ and $\gamma a + b + 1 = 0$. If $\mathcal{U} = \emptyset$, then $\delta = \gamma = 0$ and hence, $b = 0 = 1$. Therefore $\mathcal{U} \neq \emptyset$. $\square$

THEOREM 6.2. Assume **A1** through **A3**, that $0 = U^* = U_0 < U_1 < U^{**}$, and that for some $U \in (0, U_1)$ there exists $\mathbf{w}_- \rightarrow \mathbf{W}_2$, then $\mathcal{U} \neq \emptyset$.

Proof. Lemma 5.5 determines the set of possible connection matrices for $\mathbf{P}_U$ with $U \in (0, U_1)$. In particular, either there exists a $\mathbf{w}_- \rightarrow \mathbf{W}_2$ or there exists a $\mathbf{w}_- \rightarrow \mathbf{W}_1$. Furthermore, since $\mathbf{W}_2$ is an attractor, the existence of a $\mathbf{w}_- \rightarrow \mathbf{W}_i$ orbit for $i = 1, 2$ implies that the corresponding connection matrix entry is 1. (If $\mathbf{W}_i$ is a single attracting critical point then this follows from Theorem 2.7. Even if $\mathbf{W}_i$ is more complicated the proof of the result still applies.) Thus if we assume that $\mathbf{w}_- \rightarrow \mathbf{W}_2$ exists and we let Δ^2 denote the corresponding connection matrix, then by Lemma 5.4, $\gamma = 1$. therefore, there exists $\bar{U} \in (U, U_1)$ for which $\mathbf{w}_- \rightarrow \mathbf{w}_+$ exists. $\square$

Remark 6.3. Since $\mathbf{w}_-$ has a one dimensional unstable manifold and since $\mathbf{W}_2$ is an attractor, checking the existence of a $\mathbf{w}_- \rightarrow \mathbf{W}_2$ orbit is numerically tractable.

We finish with a theorem which cannot be proved directly with our techniques. However, we feel it is important to generalize or extend the Conley index theory to the point that it can be applied to problems of this nature.

THEOREM 6.4. Assume **A1** through **A3**, that $0 = U^* = U_0 < U_1 < U^{**}$, that $F = F^*$, that c_v is sufficiently large, and that μ is sufficiently small, then $\mathcal{U} \neq \emptyset$.

REFERENCES

- [C] C. CONLEY, *Isolated Invariant Sets and the Morse Index*, Conf. Board Math. Sci. 38, AMS, Providence (1978).
- [F] R. FRANZOSA, The connection matrix theory for Morse decompositions, Trans. AMS, Vol. 311, No. 2, Feb. 1989, pp. 781-803.
- [HS] R. HAGEN AND M. SLEMROD, The viscosity-capillarity admissibility criterion for shocks and phase transitions, Arch. for Rat. Mech. and Anal, 83, pp. 333-361.
- [G1] M. GRINFELD, *Topological Techniques in Dynamic Phase Transitions*, Ph.D. Thesis, Rensselaer Polytechnic Institute (1986).
- [G2] M. GRINFELD, Dynamic phase transitions: existence of "cavitation" waves, Proc. of the Royal Soc. of Edin., 107A (1987), pp. 153-163.
- [G3] M. GRINFELD, Nonisothermal dynamic phase transitions, Quart. of Applied Math., Vol 47, No. 1 (March 1989), pp. 71-84.
- [Mc] C. MCCORD, *The connection map for attractor-repeller pairs*, To appear in Trans. A.M.S.
- [M1] K. MISCHAIKOW, *Existence of generalized homoclinic orbits for one-parameter families of flows*, Proc. A.M.S., May 1988.
- [M2] K. MISCHAIKOW, *Transition systems*, Proceedings of the Royal Society of Edinburgh, 112 A (1989), pp. 155-175.

- [R] J. REINECK, *Connecting orbits in one-parameter families of flows*, J. Erg. thy. and Dyn. Sys., Vol 8* (1988), pp. 359-374.
- [S] D. SALAMON, *Connected simple systems and the Conley index of isolated invariant sets*, Trans. AMS, 291(1) (1985).
- [Sl1] M. SLEMROD, *Admissibility criteria for propagating phase boundaries in a van der Waals fluid*, Arch. Rat. Mech. Anal., Vol. 81, No 4, pp. 301-315.
- [Sl2] M. SLEMROD, *Dynamic phase transitions in a van der Waals fluid*, Journal of Differential Equations, Vol. 52, No. 1, pp. 1-23.
- [Sm] J. SMOLLER, *Shock Waves and Reaction Diffusion Equations*, Springer-Verlag, New York, 1983.

#	Author/s	Title
516	P. Singh, Ph. Caussignac, A. Fortes, D.D. Joseph and T. Lundgren,	Stability of Periodic Arrays of Cylinders Across the Stream by Direct Simulation
517	Daniel D. Joseph,	Generalization of the Foscolo-Gibilario Analysis of Dynamic Waves
518	A. Narain and D.D. Joseph,	Note on the Balance of Energy at a Phase Change Interface
519	Daniel D. Joseph,	Remarks on inertial radii, persistent normal stresses, secondary motions , and non-elastic extensional viscosities
520	D. D. Joseph,	Mathematical Problems Associated with the Elasticity of Liquids
521	Henry C. Simpson and Scott J. Spector,	Some Necessary Conditions at an Internal Boundary for Minimizers in Finite Elasticity
522	Peter Gritzmann and Victor Klee,	On the 0-1 Maximization of Positive Definite Quadratic Forms
523	Fu-Cho Pu and D.H. Sattinger,	The Yang-Baxter Equations and Differential Identities
524	Avner Friedman and Fernando Reitich,	A Hyperbolic Inverse Problem Arising in the Evolution of Combustion Aerosol
525	E.G. Kalnins, Raphael D. Levine and Willard Miller, Jr.,	Conformal Symmetries and Generalized Recurrences for Heat and Schrödinger Equations in One Spatial Dimension
526	Wang Jinghua and Gerald Warnecke,	On Entropy Consistency of Large Time Step Godunov and Glimm Schemes
527	C. Guillopé and J.C. Saut,	Existence Results for the Flow of Viscoelastic Fluids with a Differential Constitutive Law
528	H.L. Bodlaender, P. Gritzmann, V. Klee and J. Van Leeuwen	Computational Complexity of Norm-Maximization
529	Li Ta-tsien (Li Da-qian) and Yu Xin,	Life-Span of Classical Solutions to Fully Nonlinear Wave Equations
530	Jong-Shenq Guo,	A Variational Inequality Associated with a Lubrication Problem
531	Jong-Shenq Guo,	On the Semilinear Elliptic Equation $\Delta w - \frac{1}{2}y \cdot \nabla w + \lambda w - w^{-\beta} = 0$ in R^n
532	Andrew E. Yagle,	Inversion of the Bloch transform in magnetic resonance imaging using asymmetric two-component inverse scattering
533	Bei Hu,	A Fiber Tapering Problem
534	Peter J. Olver,	Canonical Variables for BiHamiltonian Systems
535	Michael Renardy,	A Well-Posed Boundary Value Problem for Supercritical Flow of Viscoelastic Fluids of Maxwell Type
536	Michael Renardy,	Ill-Posedness Resulting from Slip As a Possible Explanation of Melt Fracture
537	Michael Renardy,	Compatibility Conditions at Corners Between Walls and Inflow Boundaries for Fluids of Maxwell Type
538	Rolf Rees,	The Spectrum of Restricted Resolvable Designs with $r = 2$
539	D. Lewis and J.C. Simo,	Nonlinear stability of rotating pseudo-rigid bodies
540	Robert Hardt and David Kinderlehrer,	Variational Principles with Linear Growth
541	San Yih Lin and Yisong Yang,	Computation of Superconductivity in Thin Films
542	A. Narain,	Pressure Driven Flow of Pure Vapor Undergoing Laminar Film Condensation Between Parallel Plates
543	P.J. Vassiliou,	On Local Equivalence for Vector Field Systems
544	Brian A. Coomes,	On Conditions Sufficient for Injectivity of Maps
545	Yanchun Zhao,	A Class of Global Smooth Solutions of the One Dimensional Gas Dynamics System
546	H. Holden, L. Holden and N.H. Risebro,	Some Qualitative Properties of 2×2 Systems of Conservation Laws of Mixed Type
547	M. Slemrod,	Dynamics of Measured Valued Solutions to a Backward-Forward Heat Equation
548	Avner Friedman and Jürgen Sprekels,	Steady States of Austenitic–Martensitic–Domains in the Ginzburg–Landau Theory of Shape Memory Alloys
549	Avner Friedman and Bei Hu,	Degenerate Hamilton–Jacobi–Bellman Equations in a Bounded Domain
550	E.G. Kalnins, Willard Miller, Jr., and M.V. Tratnik,	Families of Orthogonal and Biorthogonal Polynomials on the N-Sphere
551	Heinrich Freistühler,	On Compact Linear Degeneracy
552	Matthew Witten,	Quantifying the Concepts of Rate and Acceleration/Deceleration of Aging
553	J.P. Albert and J.L. Bona,	Total Positivity and the Stability of Internal Waves in Stratified Fluids of Finite Depth
554	Brian Coomes and Victor Zurkowski,	Linearization of Polynomial Flows and Spectra of Derivations

Recent IMA Preprints (Continued)

#	Author/s	Title
555	Yuriko Renardy , A Couette-Poiseuille Flow of Two Fluids in a Channel	
556	Michael Renardy , Short wave instabilities resulting from memory slip	
557	Daniel D. Joseph and Michael Renardy , Stokes' first problem for linear viscoelastic fluids with finite memory	
558	Xiaxi Ding , Superlinear Conservation Law with Viscosity	
559	J.L. Ericksen , Liquid Crystals with Variable Degree of Orientation	
560	F. Robert Ore, Jr. and Xinfu Chen , Electro-Optic Modulation in an Arbitrary Cross-Section Waveguide	
561	M.V. Tratnik , Multivariable biorthogonal continuous-discrete Wilson and Racah polynomials	
562	Yisong Yang , Existence of Solutions for a Generalized Yang-Mills Theory	
563	Peter Gritzmann, Laurent Habsieger and Victor Klee , Good and Bad Radii of Convex Polygons	
564	Martin Golubitsky, Martin Krupa and Chjan. C. Lim , Time-Reversibility and Particle Sedimentation	
565	G. Yin , Recent Progress in Parallel Stochastic Approximations	
566	G. Yin , On H-Valued SA: Finite Dimensional Approximations	
567	Chien-Cheng Chang , Accurate Evaluation of the Effect of Diffusion and Conductivity in Certain Equations	
568	Chien-Cheng Chang and Ruey-Ling Chern , The Effect of Viscous Diffusion in Discrete Vortex Dynamics for Slightly Viscous Flows	
569	Li Ta-Tsien (Li Da-qian) and Zhao Yan-Chun , Global Existence of Classical Solutions to the Typical Free Boundary Problem for General Quasilinear Hyperbolic Systems and its Applications	
570	Thierry Cazenave and Fred B. Weissler , The Structure of Solutions to the Pseudo-Conformally Invariant Nonlinear Schrödinger Equation	
571	Marshall Slemrod and Athanasios E. Tzavaras , A Limiting Viscosity Approach for the Riemann Problem in Isentropic Gas Dynamics	
572	Richard D. James and Scott J. Spector , The Formation of Filamentary Voids in Solids	
573	P.J. Vassiliou , On the Geometry of Semi-Linear Hyperbolic Partial Differential Equations in the Plane Integrable by the Method of Darboux	
574	Jerome V. Moloney and Alan C. Newell , Nonlinear Optics	
575	Keti Tenenblat , A Note on Solutions for the Intrinsic Generalized Wave and Sine-Gordon Equations	
576	P. Szmolyan , Heteroclinic Orbits in Singularly Perturbed Differential Equations	
577	Wenxiong Liu , A Parabolic System Arising In Film Development	
578	Daniel B. Dix , Temporal Asymptotic Behavior of Solutions of the Benjamin-Ono-Burgers Equation	
579	Michael Renardy and Yuriko Renardy , On the nature of boundary conditions for flows with moving free surfaces	
580	Werner A. Stahel , Robust Statistics: From an Intellectual Game to a Consumer Product	
581	Avner Friedman and Fernando Reitich , The Stefan Problem with Small Surface Tension	
582	E.G. Kalnins and W. Miller, Jr. , Separation of Variables Methods for Systems of Differential Equations in Mathematical Physics	
583	Mitchell Luskin and George R. Sell , The Construction of Inertial Manifolds for Reaction-Diffusion Equations by Elliptic Regularization	
584	Konstantin Mischaikow , Dynamic Phase Transitions: A Connection Matrix Approach	
585	Philippe Le Floch and Li Tatsien , A Global Asymptotic Expansion for the Solution to the Generalized Riemann Problem	
586	Matthew Witten, Ph.D. , Computational Biology: An Overview	
587	Matthew Witten, Ph.D. , Peering Inside Living Systems: Physiology in a Supercomputer	
588	Michael Renardy , An existence theorem for model equations resulting from kinetic theories of polymer solutions	
589	Daniel D. Joseph and Luigi Preziosi , Reviews of Modern Physics: Addendum to the Paper "Heat Waves"	
590	Luigi Preziosi , An Invariance Property for the Propagation of Heat and Shear Waves	
591	Gregory M. Constantine and John Bryant , Sequencing of Experiments for Linear and Quadratic Time Effects	
592	Prabir Daripa , On the Computation of the Beltrami Equation in the Complex Plane	
593	Philippe Le Floch , Shock Waves for Nonlinear Hyperbolic Systems in Nonconservative Form	
594	A.L. Gorin, D.B. Roe and A.G. Greenberg , On the Complexity of Pattern Recognition Algorithms On a Tree-Structured Parallel Computer	
595	Mark J. Friedman and Eusebius J. Doedel , Numerical computation and continuation of invariant manifolds connecting fixed points	
596	Scott J. Spector , Linear Deformations as Global Minimizers in Nonlinear Elasticity	