**EXISTENCE OF SOLUTIONS FOR A
GENERALIZED YANG-MILLS THEORY**

By

Yisong Yang

IMA Preprint Series # 562

August 1989

EXISTENCE OF SOLUTIONS FOR A GENERALIZED YANG-MILLS THEORY

YISONG YANG^{*†}

Abstract. The main result of this note is a proof of a conjecture of Burzlaff, O'sé and Tchrakian [1] concerning the existence of a certain kind of finite action solutions of the seven-dimensional generalized Yang-Mills-Higgs model derived from an eight-dimensional $SO(8)$ Yang-Mills theory.

The elegance of the higher dimensional generalizations of the Euclidean Yang-Mills (YM) theory introduced by Tchrakian [2] is that the theory allows duality operations and finite action self-dual fields, also called instantons, may be characterized by some higher order Chern or Pontryagin classes. In $4n(n \geq 1)$ dimensions, spherically symmetric instantons have explicitly been found [3,4] which contain the classical Belavin-Polyakov-Schwartz-Tyupkin solution [5,6] and the cylindrically symmetric self-dual gauge field configurations of the Witten type [7] have been discussed [8].

For the eight-dimensional case and $G = SO(8)$, the action density of Tchrakian's model is

$$\mathcal{L} = \frac{1}{2} \text{Tr} (F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}), \quad \mu, \nu, \rho, \sigma = 1, \dots, 8,$$

where the four-tensor

$$F_{\mu\nu\rho\sigma} = \{F_{\mu\nu}, F_{\rho\sigma}\} - \{F_{\mu\rho}, F_{\nu\sigma}\} - \{F_{\mu\sigma}, F_{\rho\nu}\},$$

the curvature

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu],$$

and the potential A_μ lies in the Lie algebra $so(8)$.

Under the assumption that the gauge fields are independent of the coordinate x_8 and introducing a Higgs field $\phi = A_8$ (in the adjoint representation of $SO(8)$), we see that the model becomes a seven-dimensional generalized Yang-Mills-Higgs (YMH) theory with vanishing Higgs potential. This procedure is known as the Julia-Zee correspondence [9,6]. For this model, if one specifies the spin representation of $SO(8)$ and takes the spherically symmetric ansatz:

$$\phi(x) = h(r) \hat{x}_i L_{i8}, \quad A_i(x) = \frac{k(r) + 1}{r} \hat{x}_j L_{ij}, \quad i, j = 1, \dots, 7,$$

^{*}Institute for Mathematics and its Applications, University of Minnesota,
Minneapolis, MN 55455, USA

[†]Current address: Department of Mathematics and Statistics, University of New Mexico,
Albuquerque, NM 87131, USA

where h, k are real scalar functions, $r = |x|$, $\hat{x}_i = x_i/r$, and the group generators $L_{\mu\nu}$ are as defined in [1], various field strengths can be written:

$$F_{ij} = \left(\frac{k'}{r} + \frac{k^2 - 1}{r^2} \right) \left(\hat{x}_i L_{jl} - \hat{x}_j L_{il} \right) \hat{x}_l + \frac{k^2 - 1}{r^2} L_{ij},$$

$$D_i \phi = F_{i8} = \left(h' + \frac{hk}{r} \right) \hat{x}_i \hat{x}_j L_{j8} - \frac{hk}{r} L_{i8},$$

and, up to a positive constant factor, the corresponding YMH action and the YMH equations are reduced to the functional

$$(1) \quad A = \int_0^\infty dr \left\{ \frac{r^2}{3} ([(1 - k^2) h]')^2 + 4(1 - k^2)^2 k'^2 + \frac{3(1 - k^2)^4}{r^2} + 4k^2 h^2 (1 - k^2)^2 \right\}$$

and the associated variational equation

$$(2) \quad \delta A = 0,$$

respectively. A finite action solution of (2) satisfying the boundary condition

$$(3) \quad \lim_{r \rightarrow 0} h(r) = 0, \quad \lim_{r \rightarrow \infty} h(r) = 1, \quad \lim_{r \rightarrow 0} k(r) = 1, \quad \lim_{r \rightarrow \infty} k(r) = 0$$

gives rise to a finite action solution of the original seven-dimensional YMH equations with a nontrivial monopole charge expressed in terms of the Higgs field and the curvature form.

To solve Eq.(2) subject to (3), it suffices to find a minimum point of A under the constraint (3). Hence the following additional restrictions on the admissible functions h and k may be made without any loss of generality:

$$(4) \quad h(r) \geq 0; \quad 0 \leq k(r) \leq 1.$$

Now the functional A can be put into the following workable form

$$(5) \quad A(H, K) = \int_0^\infty dr \left\{ \frac{r^2}{3} H'^2 + 4K'^2 + \frac{3}{r^2} (1 - 4 \cos^2 \alpha(K))^4 + 16H^2 \cos^2 \alpha(K) \right\},$$

under the transform

$$(6) \quad H = (1 - k^2)h, \quad K = k - \frac{1}{3}k^3, \quad h \geq 0, \quad 0 \leq k \leq 1.$$

Here, and in what follows, $\alpha(K) = \frac{1}{3}(\pi + \cos^{-1}[\frac{3}{2}K])$. Note that the boundary condition (3) becomes

$$(7) \quad \lim_{r \rightarrow 0} H(r) = 0, \quad \lim_{r \rightarrow \infty} H(r) = 1, \quad \lim_{r \rightarrow 0} K(r) = \frac{2}{3}, \quad \lim_{r \rightarrow \infty} K(r) = 0.$$

In [1], Burzlaff, O'sé and Tchakian proved using the method of Tyupkin, Fateev and Shvarts [10] that the action (5) has a minimizer (H, K) satisfying $0 \leq K(r) \leq \frac{2}{3}$. In order to invert the transform (6) and find a regular solution of our original problem, one needs to achieve the strict inequality

$$(8) \quad 0 < K(r) < \frac{2}{3}, \quad r > 0.$$

This is the conjecture made in [1]. Besides, Burzlaff et al. were unable to recover, for their solution, the boundary behavior $H(r) \rightarrow 0$ as $r \rightarrow 0$ in (7).

The main purpose of the present note is to prove the existence of a minimizer of the action (5) which satisfies the inequality (8) and the full boundary condition (7). Moreover, we can show the solution (H, K) enjoys the property that H (K) is strictly increasing (decreasing) over $r > 0$ and H, K separate long and short range interaction, respectively.

For our purpose, define the set of admissible functions as

$$\mathcal{X} = \left\{ (H, K) \mid \begin{array}{l} H \text{ and } K \text{ are absolutely continuous on any compact subinterval} \\ \text{of } (0, \infty) \text{ and satisfy (7), } -\frac{2}{3} \leq K \leq \frac{2}{3}, \text{ and } A(H, K) < \infty \end{array} \right\}.$$

Consider the minimization problem

$$(9) \quad m = \min_{(H, K) \in \mathcal{X}} A(H, K).$$

Let $\{(H_n, K_n)\}$ be a minimizing sequence. The structure of the functional A and the set $\mathcal{X}$ tells us that we may assume $0 \leq H_n \leq 1, K_n \geq 0$.

LEMMA 1. *The sequence $\{H_n\}$ can be modified so that the following properties hold:*

- (i) H_n is nondecreasing for $r > 0$;
- (ii) H_n solves the equation

$$(10) \quad (r^2 H_n')' = 48 \cos^2 \alpha(K_n) H_n, \quad r > \frac{1}{n}, \quad n = 1, 2, \dots.$$

Proof. Define

$$\rho_n(r) = \begin{cases} 0, & r \leq \frac{1}{n}, \\ 1, & r \geq \frac{2}{n}, \\ n\left(r - \frac{1}{n}\right), & \frac{1}{n} < r < \frac{2}{n}. \end{cases}$$

Then $\{(\rho_n H_n, K_n)\}$ is also a minimizing sequence because

$$\int_0^\infty r^2 H_n^2 \rho_n'^2 dr \leq n^2 \int_{\frac{1}{n}}^{\frac{2}{n}} r^2 dr = \frac{7}{3n} \rightarrow 0, \quad n \rightarrow \infty.$$

This consideration allows us to assume for convenience that $H_n(r) = 0, r \leq \frac{1}{n}$.

For fixed $n = 1, 2, \dots$, consider the minimization problem

$$(11) \quad m_n = \min_{H \in \mathcal{H}_n} A(H, K_n)$$

where

$$\mathcal{H}_n = \left\{ H \mid \begin{array}{l} H \text{ is absolutely continuous in any compact subinterval} \\ \text{of } (0, \infty), H(r) = 0 \text{ for } r \leq \frac{1}{n}, H(\infty) = 1, \text{ and } A(H, K_n) < \infty \end{array} \right\}.$$

Let $\{h_j\}$ be a minimizing sequence of (11). Similarly, one may assume $0 \leq h_j(r) \leq 1$. It is not hard to prove that $\{h_j\}$ has a subsequence which is still numerated by the same indices for simplicity and there is an $\tilde{H}_n \in \mathcal{H}_n$ so that $h_j \rightarrow \tilde{H}_n$ (as $j \rightarrow \infty$) weakly in $W^{1,2}(a, b)$ for any $0 < a < b < \infty$ and $A(\tilde{H}_n, K_n) = m_n$.

$\tilde{H}_n(r)$ must be nondecreasing in $(0, \infty)$. Suppose otherwise there are $\frac{1}{n} < r_1 < r_2 < \infty$ such that $\tilde{H}_n(r_1) > \tilde{H}_n(r_2)$. Since $\tilde{H}_n(\frac{1}{n}) = 0$, one may find an $r_0 : \frac{1}{n} \leq r_0 < r_2$ satisfying $\tilde{H}_n(r_0) = \tilde{H}_n(r_2)$ and $\tilde{H}_n(r) > \tilde{H}_n(r_0)$ for $r \in (r_0, r_2)$.

Construct a new function

$$H(r) = \begin{cases} \tilde{H}_n(r_0), & r \in (r_0, r_2), \\ \tilde{H}_n(r), & r \notin (r_0, r_2). \end{cases}$$

Then $H \in \mathcal{H}_n$ and $A(H, K_n) < A(\tilde{H}_n, K_n) = m_n$. This is impossible.

Consequently, $\tilde{H}_n(r)$ verifies both (i) and (ii) and the lemma follows.

Suppose $\{(H_n, K_n)\}$ is a minimizing sequence of the problem (9) satisfying the properties stated in Lemma 1. It is easily seen that for any pair of numbers $0 < a < b$, $\{(H_n, K_n)\}$ is a bounded sequence in $W^{1,2}(a, b)$. A diagonal subsequence argument shows that there exist a pair of functions H and K in $W_{\text{loc}}^{1,2}(0, \infty)$ and a suitable subsequence of $\{(H_n, K_n)\}$ (say $\{(H_n, K_n)\}$ to simplify the notation) so that $H_n \rightarrow H, K_n \rightarrow K$ weakly in $W^{1,2}(a, b)$ and strongly in $C[a, b]$ for any $0 < a < b < \infty$. Fatou's lemma and the continuity of the function $\alpha(K)$ in $K \in [-\frac{2}{3}, \frac{2}{3}]$ then enable us to reach the correct comparison $A(H, K) \leq m$.

It remains to prove that $(H, K) \in \mathcal{X}$ and hence $A(H, K) = m$.

Since $H_n(r)$ is nondecreasing and $0 \leq H_n(r) \leq 1$ for $r > 0$, so is $H(r)$. Therefore the limit

$$\lim_{r \rightarrow 0} H(r) = \eta$$

exists and $\eta \in [0, 1]$. We must show $\eta = 0$ which is not simply implied by the finite action condition.

LEMMA 2. $H(r)$ satisfies the equation

$$(12) \quad (r^2 H')' = 48 \cos^2 \alpha(K) H, \quad r > 0.$$

REMARK. At this moment, we have not established the fact $(H, K) \in \mathcal{X}$. So Eq.(12) which is formally the H -component of the variational equation $\delta A = 0$ of the functional (5) over $\mathcal{X}$ is not automatically ensured and needs to be justified.

Proof of Lemma 2. By virtue of Lemma 1, H_n satisfies Eq.(10). On the other hand, one knows that $H_n \rightarrow H, K_n \rightarrow K$ uniformly on any given interval $[a, b]$ with $0 < a < b < \infty$. Define $g_n = r^2 H'_n$ and $g = r^2 H'$. Then $g'_n \rightarrow g'$ uniformly on $[a, b]$. It can be seen that $\{g_n\}$ is a bounded sequence in $W^{1,2}(a, b)$, hence g_n converges uniformly to g on $[a, b]$. This proves $g_n \rightarrow g$ in $C^1[a, b]$ which implies $H_n \rightarrow H$ in $C^2[a, b]$. Taking $n \rightarrow \infty$ in Eq.(10) we conclude with Eq.(12).

LEMMA 3. $\eta = 0$.

Proof. It follows from the simple inequality

$$|K_n(r) - \frac{2}{3}| \leq \frac{1}{2} (r A(H_n, K_n))^{\frac{1}{2}}$$

that $\lim_{r \rightarrow 0} K(r) = \frac{2}{3}$. In particular

$$(13) \quad \lim_{r \rightarrow 0} \cos^2 \alpha(K(r)) = \frac{1}{4}.$$

Suppose $\eta > 0$. From Eq.(12) and (13), we see that $(r^2 H')' > 0$ if $r > 0$ is sufficiently small. Thus $\lim_{r \rightarrow 0} r^2 H'$ exists since $H' \geq 0$. We claim $\lim_{r \rightarrow 0} r^2 H' = 0$. Otherwise one may find suitable $\varepsilon_0, \delta > 0$, so that $r^2 H' \geq \varepsilon_0$ for $r \in (0, \delta)$. This contradicts the fact that $r H' \in L^2(0, \infty)$.

The above observation, Eq.(12), L'Hospital's rule, and (13) lead to the limit

$$\begin{aligned} \lim_{r \rightarrow 0} r H'(r) &= \lim_{r \rightarrow 0} \frac{r^2 H'(r)}{r} \\ &= \lim_{r \rightarrow 0} (r^2 H'(r))' = 48 \lim_{r \rightarrow 0} \cos^2 \alpha(K(r)) = 12\eta > 0. \end{aligned}$$

Therefore a small $\delta > 0$ can be chosen to make

$$rH'(r) > \eta, \quad r \in (0, \delta).$$

By virtue of the above inequality, one has

$$H(r_2) - H(r_1) = \int_{r_1}^{r_2} \frac{1}{r} rH'(r) dr > \eta \log \left(\frac{r_2}{r_1} \right), \quad 0 < r_1 < r_2 < \delta,$$

which obviously violates the fact that $H(r) \rightarrow \eta \in [0, 1]$ as $r \rightarrow 0$ established earlier.

Lemma 3 is proved.

LEMMA 4. $K(r)$ is a nonincreasing function.

Proof. Suppose there are $0 < a < b$ so that $K(a) < K(b)$. Let $r_0 \in (0, b)$ satisfy

$$r_0 = \sup\{r' \in (0, b) \mid K(r') = \inf_{r \in (0, b)} K(r)\}.$$

The existence of such a point is ensured by the comparison $K(a) < \frac{2}{3} = \lim_{r \rightarrow 0} K(r)$. Thus $K(r) > K(r_0)$ for all $r \in (r_0, b)$.

On the other hand, the boundary behavior $\lim_{r \rightarrow \infty} H(r) = 1$ follows from the easily derived inequality

$$|H_n(r) - 1| \leq \int_r^\infty |H'_n(s)| ds \leq \left(\frac{3}{r} A(H_n, K_n) \right)^{\frac{1}{2}}.$$

Hence, from the finite action condition one may find a sequence $\{r_j\}$, $r_j \rightarrow \infty$ as $j \rightarrow \infty$, so that $K(r_j) \rightarrow 0$. Therefore $S \equiv K^{-1}(K(r_0)) \cap (r_0, \infty) \neq \emptyset$ since $K(r_0) > 0$ (see the reason given shortly). Set

$$r_1 = \inf\{r \in S\}.$$

One has $r_1 > b$ and

$$K(r) > K(r_0), \quad r \in (r_0, r_1).$$

Construct a modified function $\tilde{K}(r)$ as follows:

$$\tilde{K}(r) = \begin{cases} K(r_0), & r \in (r_0, r_1), \\ K(r), & r \notin (r_0, r_1). \end{cases}$$

Then the inequality $A(H, \tilde{K}) < A(H, K)$ holds. To see this, we must show that

$$(14) \quad \tilde{I} \equiv \int_{r_0}^{r_1} dr \left\{ \frac{r^2}{3} H'^2 + \frac{3}{r^2} (1 - 4 \cos^2 \alpha(\tilde{K}))^4 + 16 H^2 \cos^2 \alpha(\tilde{K}) \right\}$$

$$< \int_{r_0}^{r_1} dr \left\{ \frac{r^2}{3} H'^2 + 4K'^2 + \frac{3}{r^2} (1 - 4 \cos^2 \alpha(K))^4 + 16H^2 \cos^2 \alpha(K) \right\} \equiv I.$$

First we observe that, if $K(r) < \frac{2}{3}$ hold for some $r > 0$, then, at this point, the K -component of the variational equation $\delta A = 0$ of the functional (5) is valid:

$$(15) \quad 4K''(r) = \left(-\frac{24}{r^2} (1 - 4 \cos^2 \alpha(K))^3 + 8H^2(r) \right) \left(1 - \left(\frac{3}{2} K \right)^2 \right)^{-\frac{1}{2}} \cos \alpha(K) \sin \alpha(K).$$

Applying Eq.(15) to the point $r = r_0$ we see that $K(r_0) > 0$. In fact, if $K(r_0) = 0$, then since $K'(r_0) = 0$ because $K(r)$ attains its local minimum $K(r_0)$ at $r = r_0$, we would conclude that $K(r) \equiv 0$ by virtue of the uniqueness theorem of the initial value problems of ordinary differential equations, which contradicts the boundary behavior $\lim_{r \rightarrow 0} K(r) = \frac{2}{3}$.

Consequently, using $K''(r_0) \geq 0$, we have the following useful inequality

$$(16) \quad H^2(r_0) - \frac{3}{r_0^2} (1 - 4 \cos^2 \alpha(K(r_0)))^3 \geq 0.$$

because $\cos \alpha(K(r_0)) \sin \alpha(K(r_0)) > 0$.

Next, we consider the difference

$$\begin{aligned} I - \tilde{I} &= \int_{r_0}^{r_1} dr \left\{ 4K'^2 + \frac{3}{r^2} [(1 - 4 \cos^2 \alpha(K))^4 - (1 - 4 \cos^2 \alpha(K(r_0)))^4] \right. \\ &\quad \left. + 16H^2 [\cos^2 \alpha(K) - \cos^2 \alpha(K(r_0))] \right\} \\ &> \int_{r_0}^{r_1} dr (\cos^2 \alpha(K) - \cos^2 \alpha(K(r_0))) \times \\ &\quad \times \left\{ 16H^2 - \frac{12}{r^2} ([1 - 4 \cos^2 \alpha(K)] + [1 - 4 \cos^2 \alpha(K(r_0))]) \times \right. \\ &\quad \left. \times ([1 - 4 \cos^2 \alpha(K)]^2 + [1 - 4 \cos^2 \alpha(K(r_0))]^2) \right\} \\ &\equiv \int_{r_0}^{r_1} dr \{ (\cos^2 \alpha(K) - \cos^2 \alpha(K(r_0))) \sigma(r) \}. \end{aligned}$$

From $\alpha(K(r)) < \alpha(K(r_0))$, $r \in (r_0, r_1)$, it follows that $\cos^2 \alpha(K(r)) > \cos^2 \alpha(K(r_0))$. In particular, $1 - 4 \cos^2 \alpha(K) < 1 - 4 \cos^2 \alpha(K(r_0))$, $r \in (r_0, r_1)$.

We know that $H(r)$ is nondecreasing, so $H(r_0) \leq H(r)$, $r \in (r_0, r_1)$.

Combining the above results and (16) we have

$$\sigma(r) \geq 16H^2(r_0) - \frac{48}{r_0^2} (1 - 4 \cos^2 \alpha(K(r_0)))^3 \geq 0.$$

This proves the desired comparison $I - \tilde{I} > 0$.

If $K(r) \rightarrow 0$ as $r \rightarrow \infty$, then $(H, \tilde{K}) \in \mathcal{X}$. But $A(H, K) > A(H, \tilde{K})$, this violates our earlier conclusion that $A(H, K) \leq m$.

Since $H(r) \rightarrow 1$ as $r \rightarrow \infty$, so there is an $R_0 > 0$ such that $H^2(r) - \frac{3}{r^2}(1 - 4\cos^2 \alpha(r))^3 > 0$ for $r > R_0$. Suppose $K(r)$ attains its local minimum at $r = r^* \in (R_0, \infty)$. Then $0 < K(r^*) < \frac{2}{3}$. Evaluating $K''(r^*)$ at the point $r = r^*$ by using Eq.(15) we have $K''(r^*) > 0$. Hence the set of local minimum points of $K(r)$ in (R_0, ∞) must be isolated. Suppose $K(r) \nrightarrow 0$ (as $r \rightarrow \infty$). Then local minimum points of $K(r)$ in (R_0, ∞) are countably many, say $\{r_j\}$.

Set $r'_1 = r_1$. Let $r'_j \in \{r_j\}$ be such that

$$r'_j = \inf\{r \in \{r_j\} \mid K(r) < K(r'_{j-1})\}, \quad j = 2, 3, \dots.$$

Because $K(r_j) \rightarrow 0$ as $j \rightarrow \infty$, so $\{r'_j\}$ is countable and satisfies

$$r'_j < r'_{j+1}, \quad K(r'_j) > K(r'_{j+1}), \quad j = 1, 2, \dots, \quad r'_j \rightarrow \infty \text{ as } j \rightarrow \infty.$$

Since $K'(r'_j) = 0$ and $K''(r'_j) > 0$, there is an interval $(r'_j, r'_j + \delta_j)$, in which, $K(r) > K(r'_j)$. Thus $S_j = K^{-1}(K(r'_j)) \cap (r'_j, r'_{j+1}) \neq \emptyset$. Define

$$r''_j = \sup\{r \in S_j\}.$$

We have

$$K(r) > K(r'_j), \quad r \in (r'_j, r''_j).$$

Introduce a new function $\tilde{K}(r)$:

$$\tilde{K}(r) = \begin{cases} K(r'_j), & r \in (r'_j, r''_j), \quad j = 1, 2, \dots, \\ K(r), & r \notin \cup_{j=1}^{\infty} (r'_j, r''_j). \end{cases}$$

By the same method as before, we can verify the inequality $A(H, \tilde{K}) < A(H, K)$.

$\tilde{K}$ has the property that $\tilde{K}(r)$ is nonincreasing in $[r'_1, \infty)$. Indeed, we need only to prove the monotonicity of $\tilde{K}$ in each $[r''_j, r'_{j+1}]$, $j = 1, 2, \dots$.

We first claim that $K'(r''_j) < 0$. Suppose otherwise $K'(r''_j) \geq 0$. Then $K''(r''_j) > 0$ implies $K'(r) > 0$ for $r \in \text{some } (r''_j, r''_j + \delta)$. Hence there is a point $r_j^* \in (r''_j, r'_{j+1})$ so that $K(r_j^*) = K(r'_j)$ which contradicts the definition of r''_j .

From the inequality $K'(r''_j) < 0$, we may find a $\delta > 0$ to make $K(r) < K(r''_j)$, $r \in (r''_j, r''_j + \delta)$.

There must be a point $r' > r'_{j+1}$ so that $K(r') = \frac{2}{3}$. If this is not true, then $K(r)$ satisfies Eq.(15) for all $r \geq r'_{j+1}$. Using (15) we obtain $K''(r) > 0$, $r > r'_{j+1}$. But

$K'(r'_{j+1}) = 0$, so $K'(r) > 0$ for all $r > r'_{j+1}$. In particular $K(r) > K(r'_{j+1}) > 0, r \in (r'_{j+1}, \infty)$. This is false.

Put $S'_j = K^{-1}(K(r'_j)) \cap (r''_j, r')$. Obviously, $S'_j \neq \emptyset$. Set

$$r'' = \inf\{r \in S'_j\}.$$

There holds

$$K(r) < K(r''_j), \quad r \in (r''_j, r'').$$

Using Eq.(15) in (r''_j, r'') , we have $K''(r) > 0, r \in (r''_j, r'')$. Therefore, K' has exactly one zero, say $\bar{r}$, in (r''_j, r'') . At $r = \bar{r}$, $K(r)$ attains its local minimum and $K(r'_j) = K(r''_j) > K(\bar{r})$. By the definition of the sequence $\{r'_j\}$, we get immediately $\bar{r} = r'_{j+1}$. This proves $K'(r) < 0$ for $r \in (r''_j, r'_{j+1})$ and the global monotonicity of $\tilde{K}(r)$ follows.

The above result leads to the conclusion that $\tilde{K}(r) \rightarrow 0$ as $r \rightarrow \infty$. Hence $(H, \tilde{K}) \in \mathcal{X}$. But $A(H, \tilde{K}) < A(H, K) \leq m$, which is a contradiction.

Lemma 4 is proved.

We have shown that $K(r)$ is nonincreasing over $(0, \infty)$. Thus $K(r) \rightarrow 0$ as $r \rightarrow \infty$ and $K^{-1}(\frac{2}{3}) = [0, a]$ for some $a \geq 0$. If $a = 0$, the desired inequality (8), namely, $0 < K(r) < \frac{2}{3}, r \in (0, \infty)$, will follow.

LEMMA 5. Let $K^{-1}(\frac{2}{3}) = [0, a]$. Then $a = 0$.

Proof. Suppose otherwise that $a > 0$. Since $K(r) < \frac{2}{3}, r \in (a, \infty)$, so $K(r)$ satisfies Eq.(15) over (a, ∞) . Obviously, $H(a) > 0$. Indeed, if $H(a) = 0$, then the monotonicity of $H(r)$ implies $H(r) \equiv 0, r \in [0, a]$. So $H'(a) = 0$. From the uniqueness theorem of the initial value problems of ordinary differential equations, we get $H(r) \equiv 0, r \in (0, \infty)$, which is false.

Hence, $H(a) > 0$, Eq.(15) and the limit $K(r) \rightarrow \frac{2}{3}, r \rightarrow a^+$ enable us to find a $\delta > 0$, so that $K''(r) > 0, r \in (a, a + \delta)$. In particular, $K'(r)$ is increasing in $(a, a + \delta)$. Consequently, as $r \rightarrow a^+$, $K'(r) \rightarrow$ a nonpositive finite number or minus infinity because $K'(r) \leq 0$.

Let us first exclude the possibility $\lim_{r \rightarrow a^+} K'(r) = 0$.

In fact, if this is true, then $K'(r) > 0, r \in (a, a + \delta)$, which violates the conclusion $K'(r) \leq 0$ proved earlier.

Thus, one may find $\eta, \delta > 0$, such that

$$K'(r) < -\eta, \quad r \in (a, a + \delta).$$

As a consequence,

$$\begin{aligned} 1 - \left(\frac{3}{2}K(r)\right)^2 &= -\frac{9}{2}K(\xi)K'(\xi)(r - a) \quad (a < \xi < r < a + \delta) \\ &> \eta(r - a), \quad r \in (a, a + \delta), \end{aligned}$$

where $\delta > 0$ has been chosen small enough to make $K(a + \delta) > \frac{2}{9}$. So, using Eq.(15), we have the estimate

$$0 < K''(r) < 8[\eta(r - a)]^{-\frac{1}{2}}, \quad r \in (a, a + \delta),$$

which implies

$$\begin{aligned} |K'(r_1) - K'(r_2)| &\leq 8\eta^{-\frac{1}{2}} \left| \int_{r_2}^{r_1} (r - a)^{-\frac{1}{2}} dr \right|, \quad r_1, r_2 \in (a, a + \delta) \\ &\rightarrow 0, \quad \text{as } r_1, r_2 \rightarrow a^+. \end{aligned}$$

Therefore $\lim_{r \rightarrow a^+} K'(r)$ exists and is a finite number, say $K'(a^+)$.

At this stage, we need to use the fact $\lim_{r \rightarrow \infty} K'(r) = 0$ which follows readily from the monotonicity of $K'(r)$ at large values of r (see Eq.(15)) and the finite action condition.

Define a translation $K_\tau(r) = K(r + \tau)$ where $\tau < a$ and it is understood that $K(r) = K(0) = \frac{2}{3}$ for $r < 0$. We have $(H, K_\tau) \in \mathcal{X}$.

Finally, the inequality $A(H, K_\tau) \geq A(H, K_0)$, the translation invariance

$$\int_0^\infty K'^2(r) dr = \int_0^\infty K_\tau'^2(r) dr,$$

the finite action condition, and the behavior $\lim_{r \rightarrow a^+} K'(r) = K'(a^+) \neq 0$ enable us to carry through the following calculation:

$$\begin{aligned} 0 &= \frac{d}{d\tau} A(H, K_\tau) \Big|_{\tau=0} = \int_a^\infty dr \left\{ \left(-\frac{48}{r^2} (1 - 4 \cos^2 \alpha(K))^3 + 16H^2 \right) \times \right. \\ &\quad \left. \times \left(1 - \left(\frac{3}{2} K \right)^2 \right)^{-\frac{1}{2}} \cos \alpha(K) \sin \alpha(K) K' \right\} \\ &= \int_a^\infty 8K' K'' dr = -4K'^2(a^+). \end{aligned}$$

This is again a contradiction. The proof of the lemma is complete.

Now that (8) and Eqs.(12) and (15) hold for all $r > 0$, it is not hard to observe that $H(r)$ ($K(r)$) is strictly increasing (decreasing).

Indeed, if there are r_1, r_2 , so that $H(r_1) = H(r_2)$, $r_1 < r_2$, then $H(r) \equiv H(r_1)$, $r \in (r_1, r_2)$ because $H(r)$ is nondecreasing. Substituting this result into Eq.(12) and using the uniqueness theorem for the initial value problems of ordinary differential equations we reach $H(r) \equiv 0$, $r > 0$. This is false. Similarly, $K(r_1) = K(r_2)$ implies $K(r) \equiv K(r_1)$, $r \in (r_1, r_2)$. Inserting this into Eq.(15) we get $H(r) = \frac{C}{r}$, $r \in (r_1, r_2)$, where $C > 0$ is a constant, which contradicts the fact that $H(r)$ is an increasing function.

It will be interesting to notice that the fields H and K play rather uneven roles in the model: H and K characterize respectively long and short range interaction. More

precisely, it can be verified that $1 - H(r) = O(\frac{1}{r})$ while $K(r) = O(e^{-\theta r})$, $\forall \theta \in (0, 1)$, as $r \rightarrow \infty$. We proceed as follows.

Since $H(r) \rightarrow 1$, $K(r) \rightarrow 0$ as $r \rightarrow \infty$, from Eq.(15), there holds $K''(r) > 0$ for large r . Thus $K'(r) \neq 0$ if r is sufficiently large which enables us to find by L'Hospital's rule,

$$(17) \quad \lim_{r \rightarrow \infty} \frac{\cos \alpha(K)}{K} = \frac{1}{2}.$$

As a consequence, $\forall \theta \in (0, 1)$, $\exists r_0 > 0$, such that

$$K''(r) = \frac{1}{4} \text{the right-hand-side of (15)} \geq \theta^2 K(r), \quad r > r_0.$$

Therefore, it follows that,

$$(18) \quad K(r) \leq C_0(\theta) e^{-\theta r}, \quad r > 0,$$

where $C_0(\theta) > 0$ is a suitable constant.

The decay rate of $|K'(r)|$ may similarly be obtained. In fact, a differentiation of Eq.(15) yields

$$(19) \quad \begin{aligned} K'''(r) &= \left(-\frac{3}{r^2} (1 - 4 \cos^2 \alpha(K))^3 + H^2 \right) \left(1 - \left(\frac{3}{2} K \right)^2 \right) \sin^2 \alpha(K) K' \\ &\quad + f(H, H', K, K', r) \cos \alpha(K) \\ &\leq (\theta + \varepsilon)^2 K'(r) + C_1 e^{-\theta r}, \quad r > \text{some } r_0(\varepsilon), \end{aligned}$$

by virtue of (17), (18), the boundedness of H' (see below) and K' in any closed region away from $r = 0$. Here we have made $\varepsilon > 0$ small enough to achieve $\theta + \varepsilon < 1$.

Using the comparison function $\zeta(r) = C e^{-\theta r}$ ($C > 0$) and (19), we have

$$(20) \quad (\zeta + K')'' \leq (\theta + \varepsilon)^2 (\zeta + K') - [(\theta + \varepsilon)^2 - \theta^2] C - C_1 e^{-\theta r}, \quad r > r_0(\varepsilon).$$

Now choose $C > 0$ large so that $(\zeta(r) + K'(r))|_{r=r_0} \geq 0$ and $[(\theta + \varepsilon)^2 - \theta^2] C - C_1 \geq 0$. Applying these and the behavior $\zeta(r) + K'(r) \rightarrow 0$ ($r \rightarrow \infty$) we reach

$$|K'(r)| \leq \zeta(r), \quad r > r_0,$$

i.e., $|K'(r)|$ decays as fast as $K(r)$.

From (17), (18), and $\lim_{r \rightarrow 0} r^2 H'(r) = 0$, we obtain by integrating Eq.(12) that

$$(21) \quad 0 \leq H'(r) \leq \frac{C_1}{r^2}, \quad 0 < 1 - H(r) \leq \frac{C}{r}, \quad r > 0,$$

where C, C_1 are some positive constants.

Let us show the sharpness of the above estimates, namely, $H(r)$ cannot approach $H(\infty) = 1$ with the rate $O(r^{-\beta})$ for $\beta > 1$. To see this, we introduce a new comparison function $\zeta(r) = Cr^{-\beta}$ where $C > 0$ is a constant to be determined. Then,

$$(22) \quad \begin{aligned} (r^2[(1-H) - \zeta]')' &\leq 48 \cos^2 \alpha(K)(1-H) - \beta(\beta-1)\zeta \\ &\leq 48 \cos^2 \alpha(K)([1-H] - \zeta), \quad r > r_0, \end{aligned}$$

where $r_0 > 0$ is so large that $48 \cos^2 \alpha(K(r)) \leq \beta(\beta-1), r > r_0$.

Choose $C > 0$ in the expression of $\zeta(r)$ sufficiently small to make $([1-H] - \zeta)|_{r=r_0} \geq 0$. Hence we may use the maximum principle in (22) to conclude that $1-H(r) \geq \zeta(r) = Cr^{-\beta}$ for $r > r_0$. Therefore $1-H(r)$ does not decay faster than $r^{-\beta}$ for $\beta > 1$.

In summary, we have proved the existence of a finite action solution of the seven-dimensional generalized YMH equations derived through a dimension reduction from Tchrakian's eight-dimensional $SO(8)$ YM theory under the spherically symmetric ansatz so that the radial functions $h(r), k(r)$ are given by

$$(23) \quad h(r) = \frac{H(r)}{1 - 4 \cos^2 \alpha(K(r))}, \quad k(r) = 2 \cos \alpha(K(r)), \quad r > 0,$$

where $H(r)$ ($K(r)$) is strictly increasing (decreasing) over $r > 0$ and H and K satisfy the boundary condition (7), $0 < H(r) < 1, 0 < K(r) < \frac{2}{3}$ for $r > 0$, and $1-H(r) = O(\frac{1}{r}), K(r) = O(e^{-\theta r}), \forall \theta \in (0, 1)$, as $r \rightarrow \infty$. This solution is topologically non-trivial.

To conclude, we remark that our solution here cannot be self-dual which answers the question whether or not the variational solution above may be promoted to a self-dual solution, also raised in [1].

In fact, the generalized Bogomol'nyi equations derived from the self-duality condition are [1,4]:

$$(24) \quad \begin{cases} h' = \frac{3}{r^2}(k^2 - 1) - \frac{2h^2k^2}{k^2 - 1}, \\ k' = hk, \end{cases} \quad r > 0.$$

If (23) gave rise to a solution of Eqs.(24), then one would have $k'(r) > 0, r \in (0, \infty)$, since $h(r), k(r) > 0$. On the other hand, from (23) we know $k(r)$ is a decreasing function. This is a contradiction.

REFERENCES

- [1] BURZLAFF, J., O'SÉ, D. AND TCHRAKIAN, D. H., Lett. Math. Phys. **13** (1987), 121.
- [2] TCHRAKIAN, D. H., J. Math. Phys. **21** (1980), 166.
- [3] GROSSMAN, B., KEPHART, T. W. AND STASHEFF, J. D., Commun. Math. Phys. **96** (1984), 431.
- [4] TCHRAKIAN, D. H., Phys. Lett. B **150** (1985), 360.
- [5] BELAVIN, A. A., POLYAKOV, A. M., SCHWARTZ, A. S. AND TYUPKIN, YU. S., Phys. Lett. B **59** (1975), 85.
- [6] ACTOR, A., Rev. Modern Phys. **51** (1979), 461.
- [7] WITTEN, E., Phys. Rev. Lett. **38** (1977), 121.
- [8] CHAKRABARTI, A., SHERRY, T. N. AND TCHRAKIAN, D. H., Phys. Lett. B **162** (1985), 340.
- [9] JULIA, B. AND ZEE, A., Phys. Rev. D **11** (1975), 2227.
- [10] TYUPKIN, YU. S., FATEEV, V. A. AND SHVARTS, A. S., Theoret. Math. Phys. **26** (1976), 270.

#	Author/s	Title
495	Lucas Hsu, Niky Kamran and Peter J. Olver,	Equivalence of Higher Order Lagrangians II. The Cartan Form for Particle Lagrangians
496	D.J. Kaup and Peter J. Olver,	Quantization of BiHamiltonian Systems
497	Metin Arik, Fahrünisa Neyzi, Yavuz Nutku, Peter J. Olver and John M. Verosky	Multi-Hamiltonian Structure of the Born-Infeld Equation
498	David H. Wagner,	Detonation Waves and Deflagration Waves in the One Dimensional ZND Model for High Mach Number Combustion
499	Jerrold R. Griggs and Daniel J. Kleitman,	Minimum Cutsets for an Element of a Boolean Lattice
500	Dieter Jungnickel,	On Affine Difference Sets
501	Pierre Leroux,	Reduced Matrices and q-log Concavity Properties of q-Stirling Numbers
502	A. Narain and Y. Kizilyalli,	The Flow of Pure Vapor Undergoing Film Condensation Between Parallel Plates
503	Donald A. French,	On the Convergence of Finite Element Approximations of a Relaxed Variational Problem
504	Yisong Yang,	Computation, Dimensionality, and Zero Dissipation Limit of the Ginzburg-Landau Wave Equation
505	Jürgen Sprekels,	One-Dimensional Thermomechanical Phase Transitions with Non-Convex Potentials of Ginzburg-Landau Type
506	Yisong Yang,	A Note On Nonabelian Vortices
507	Yisong Yang,	On the Abelian Higgs Models with Sources
508	Chjan. C. Lim,	Existence of Kam Tori in the Phase Space of Vortex Systems
509	John Weiss,	Bäcklund Transformations and the Painlevé Property
510	Pu Fu-cho and D.H. Sattinger,	The Yang-Baxter Equation for Integrable Systems
511	E. Bruce Pitman and David G. Schaeffer,	Instability and Ill-Posedness in Granular Flow
512	Brian A. Coomes,	Polynomial Flows on $\mathbb{C}^n$
513	Bernardo Cockburn, Suchung Hou and Chi-Wang Shu,	The Runge-Kutta Local Projection Discontinuous Galerkin Finite Element Method for Conservation Laws IV: The Multidimensional Case
514	Peter J. Olver,	Invariant Theory, Equivalence Problems, and the Calculus of Variations
515	Daniel D. Joseph and Thomas S. Lundgren with an appendix by R. Jackson and D.A. Saville,	Ensemble Averaged and Mixture Theory Equations
516	P. Singh, Ph. Caussignac, A. Fortes, D.D. Joseph and T. Lundgren,	Stability of Periodic Arrays of Cylinders Across the Stream by Direct Simulation
517	Daniel D. Joseph,	Generalization of the Foscolo-Gibilario Analysis of Dynamic Waves
518	A. Narain and D.D. Joseph,	Note on the Balance of Energy at a Phase Change Interface
519	Daniel D. Joseph,	Remarks on inertial radii, persistent normal stresses, secondary motions, and non-elastic extensional viscosities
520	D. D. Joseph,	Mathematical Problems Associated with the Elasticity of Liquids
521	Henry C. Simpson and Scott J. Spector,	Some Necessary Conditions at an Internal Boundary for Minimizers in Finite Elasticity
522	Peter Gritzmann and Victor Klee,	On the 0-1 Maximization of Positive Definite Quadratic Forms
523	Fu-Cho Pu and D.H. Sattinger,	The Yang-Baxter Equations and Differential Identities
524	Avner Friedman and Fernando Reitich,	A Hyperbolic Inverse Problem Arising in the Evolution of Combustion Aerosol
525	E.G. Kalnins, Raphael D. Levine and Willard Miller, Jr.,	Conformal Symmetries and Generalized Recurrences for Heat and Schrödinger Equations in One Spatial Dimension
526	Wang Jinghua and Gerald Warnecke,	On Entropy Consistency of Large Time Step Godunov and Glimm Schemes
527	C. Guillopé and J.C. Saut,	Existence Results for the Flow of Viscoelastic Fluids with a Differential Constitutive Law
528	H.L. Bodlaender, P. Gritzmann, V. Klee and J. Van Leeuwen	Computational Complexity of Norm-Maximization
529	Li Ta-tsien (Li Da-qian) and Yu Xin,	Life-Span of Classical Solutions to Fully Nonlinear Wave Equations
530	Jong-Shenq Guo,	A Variational Inequality Associated with a Lubrication Problem
531	Jong-Shenq Guo,	On the Semilinear Elliptic Equation $\Delta w - \frac{1}{2}y \cdot \nabla w + \lambda w - w^{-\beta} = 0$ in R^n
532	Andrew E. Yagle,	Inversion of the Bloch transform in magnetic resonance imaging using asymmetric two-component inverse scattering

#	Author/s	Title
495	Lucas Hsu, Niky Kamran and Peter J. Olver,	Equivalence of Higher Order Lagrangians II. The Cartan Form for Particle Lagrangians
496	D.J. Kaup and Peter J. Olver,	Quantization of BiHamiltonian Systems
497	Metin Arik, Fahrünisa Neyzi, Yavuz Nutku, Peter J. Olver and John M. Verosky	Multi-Hamiltonian Structure of the Born-Infeld Equation
498	David H. Wagner,	Detonation Waves and Deflagration Waves in the One Dimensional ZND Model for High Mach Number Combustion
499	Jerrold R. Griggs and Daniel J. Kleitman,	Minimum Cutsets for an Element of a Boolean Lattice
500	Dieter Jungnickel,	On Affine Difference Sets
501	Pierre Leroux,	Reduced Matrices and q-log Concavity Properties of q-Stirling Numbers
502	A. Narain and Y. Kizilyalli,	The Flow of Pure Vapor Undergoing Film Condensation Between Parallel Plates
503	Donald A. French,	On the Convergence of Finite Element Approximations of a Relaxed Variational Problem
504	Yisong Yang,	Computation, Dimensionality, and Zero Dissipation Limit of the Ginzburg-Landau Wave Equation
505	Jürgen Sprekels,	One-Dimensional Thermomechanical Phase Transitions with Non-Convex Potentials of Ginzburg-Landau Type
506	Yisong Yang,	A Note On Nonabelian Vortices
507	Yisong Yang,	On the Abelian Higgs Models with Sources
508	Chjan. C. Lim,	Existence of Kam Tori in the Phase Space of Vortex Systems
509	John Weiss,	Bäcklund Transformations and the Painlevé Property
510	Pu Fu-cho and D.H. Sattinger,	The Yang-Baxter Equation for Integrable Systems
511	E. Bruce Pitman and David G. Schaeffer,	Instability and Ill-Posedness in Granular Flow
512	Brian A. Coomes,	Polynomial Flows on $\mathbb{C}^{n*}$
513	Bernardo Cockburn, Suchung Hou and Chi-Wang Shu,	The Runge-Kutta Local Projection Discontinuous Galerkin Finite Element Method for Conservation Laws IV: The Multidimensional Case
514	Peter J. Olver,	Invariant Theory, Equivalence Problems, and the Calculus of Variations
515	Daniel D. Joseph and Thomas S. Lundgren with an appendix by R. Jackson and D.A. Saville,	Ensemble Averaged and Mixture Theory Equations
516	P. Singh, Ph. Caussignac, A. Fortes, D.D. Joseph and T. Lundgren,	Stability of Periodic Arrays of Cylinders Across the Stream by Direct Simulation
517	Daniel D. Joseph,	Generalization of the Foscolo-Gibilaro Analysis of Dynamic Waves
518	A. Narain and D.D. Joseph,	Note on the Balance of Energy at a Phase Change Interface
519	Daniel D. Joseph,	Remarks on inertial radii, persistent normal stresses, secondary motions , and non-elastic extensional viscosities
520	D. D. Joseph,	Mathematical Problems Associated with the Elasticity of Liquids
521	Henry C. Simpson and Scott J. Spector,	Some Necessary Conditions at an Internal Boundary for Minimizers in Finite Elasticity
522	Peter Gritzmann and Victor Klee,	On the 0-1 Maximization of Positive Definite Quadratic Forms
523	Fu-Cho Pu and D.H. Sattinger,	The Yang-Baxter Equations and Differential Identities
524	Avner Friedman and Fernando Reitich,	A Hyperbolic Inverse Problem Arising in the Evolution of Combustion Aerosol
525	E.G. Kalnins, Raphael D. Levine and Willard Miller, Jr.,	Conformal Symmetries and Generalized Recurrences for Heat and Schrödinger Equations in One Spatial Dimension
526	Wang Jinghua and Gerald Warnecke,	On Entropy Consistency of Large Time Step Godunov and Glimm Schemes
527	C. Guillopé and J.C. Saut,	Existence Results for the Flow of Viscoelastic Fluids with a Differential Constitutive Law
528	H.L. Bodlaender, P. Gritzmann, V. Klee and J. Van Leeuwen	Computational Complexity of Norm-Maximization
529	Li Ta-tsien (Li Da-qian) and Yu Xin,	Life-Span of Classical Solutions to Fully Nonlinear Wave Equations
530	Jong-Shenq Guo,	A Variational Inequality Associated with a Lubrication Problem
531	Jong-Shenq Guo,	On the Semilinear Elliptic Equation $\Delta w - \frac{1}{2}y \cdot \nabla w + \lambda w - w^{-\beta} = 0$ in $\mathbb{R}^n$
532	Andrew E. Yagle,	Inversion of the Bloch transform in magnetic resonance imaging using asymmetric two-component inverse scattering

Recent IMA Preprints (Continued)

#	Author/s	Title
533	Bei Hu , A Fiber Tapering Problem	
534	Peter J. Olver , Canonical Variables for BiHamiltonian Systems	
535	Michael Renardy , A Well-Posed Boundary Value Problem for Supercritical Flow of Viscoelastic Fluids of Maxwell Type	
536	Michael Renardy , Ill-Posedness Resulting from Slip As a Possible Explanation of Melt Fracture	
537	Michael Renardy , Compatibility Conditions at Corners Between Walls and Inflow Boundaries for Fluids of Maxwell Type	
538	Rolf Rees , The Spectrum of Restricted Resolvable Designs with $r = 2$	
539	D. Lewis and J.C. Simo , Nonlinear stability of rotating pseudo-rigid bodies	
540	Robert Hardt and David Kinderlehrer , Variational Principles with Linear Growth	
541	San Yih Lin and Yisong Yang , Computation of Superconductivity in Thin Films	
542	A. Narain , Pressure Driven Flow of Pure Vapor Undergoing Laminar Film Condensation Between Parallel Plates	
543	P.J. Vassiliou , On Local Equivalence for Vector Field Systems	
544	Brian A. Coomes , On Conditions Sufficient for Injectivity of Maps	
545	Yanchun Zhao , A Class of Global Smooth Solutions of the One Dimensional Gas Dynamics System	
546	H. Holden, L. Holden and N.H. Risebro , Some Qualitative Properties of 2×2 Systems of Conservation Laws of Mixed Type	
547	M. Slemrod , Dynamics of Measured Valued Solutions to a Backward-Forward Heat Equation	
548	Avner Friedman and Jürgen Sprekels , Steady States of Austenitic–Martensitic–Domains in the Ginzburg–Landau Theory of Shape Memory Alloys	
549	Avner Friedman and Bei Hu , Degenerate Hamilton–Jacobi–Bellman Equations in a Bounded Domain	
550	E.G. Kalnins, Willard Miller, Jr., and M.V. Tratnik , Families of Orthogonal and Biorthogonal Polynomials on the N-Sphere	
551	Heinrich Freistühler , On Compact Linear Degeneracy	
552	Matthew Witten , Quantifying the Concepts of Rate and Acceleration/Deceleration of Aging	
553	J.P. Albert and J.L. Bona , Total Positivity and the Stability of Internal Waves in Stratified Fluids of Finite Depth	
554	Brian Coomes and Victor Zurkowski , Linearization of Polynomial Flows and Spectra of Derivations	
555	Yuriko Renardy , A Couette-Poiseuille Flow of Two Fluids in a Channel	
556	Michael Renardy , Short wave instabilities resulting from memory slip	
557	Daniel D. Joseph and Michael Renardy , Stokes' first problem for linear viscoelastic fluids with finite memory	
558	Xiaxi Ding , Superlinear Conservation Law with Viscosity	
559	J.L. Ericksen , Liquid Crystals with Variable Degree of Orientation	
560	F. Robert Ore, Jr. and Xinfu Chen , Electro-Optic Modulation in an Arbitrary Cross-Section Waveguide	
561	M.V. Tratnik , Multivariable biorthogonal continuous-discrete Wilson and Racah polynomials	
562	Yisong Yang , Existence of Solutions for a Generalized Yang-Mills Theory	
563	Peter Gritzmann, Laurent Habsieger and Victor Klee , Good and Bad Radii of Convex Polygons	
564	Martin Golubitsky, Martin Krupa and Chjan. C. Lim , Time-Reversibility and Particle Sedimentation	
565	G. Yin , Recent Progress in Parallel Stochastic Approximations	
566	G. Yin , On H-Valued SA: Finite Dimensional Approximations	
567	Chien-Cheng Chang , Accurate Evaluation of the Effect of Diffusion and Conductivity in Certain Equations	
568	Chien-Cheng Chang and Rucy-Ling Chern , The Effect of Viscous Diffusion in Discrete Vortex Dynamics for Slightly Viscous Flows	
569	Li Ta-Tsien (Li Da-qian) and Zhao Yan-Chun , Global Existence of Classical Solutions to the Typical Free Boundary Problem for General Quasilinear Hyperbolic Systems and its Applications	
570	Thierry Cazenave and Fred B. Weissler , The Structure of Solutions to the Pseudo-Conformally Invariant Nonlinear Schrödinger Equation	
571	Marshall Slemrod and Athanasios E. Tzavaras , A Limiting Viscosity Approach for the Riemann Problem in Isentropic Gas Dynamics	
572	Richard D. James and Scott J. Spector , The Formation of Filamentary Voids in Solids	
573	P.J. Vassiliou , On the Geometry of Semi-Linear Hyperbolic Partial Differential Equations in the Plane Integrable by the Method of Darboux	
574	Jerome V. Moloney and Alan C. Newell , Nonlinear Optics	
575	Keti Tenenblat , A Note on Solutions for the Intrinsic Generalized Wave and Sine-Gordon Equations	
576	P. Szmolyan , Heteroclinic Orbits in Singularly Perturbed Differential Equations	
577	Wenxiong Liu , A Parabolic System Arising In Film Development	