NAVIER-STOKES COMPUTATION OF TRANSONIC VORTICES OVER A
ROUND LEADING EDGE DELTA WING

By

BERNHARD MULLER

ARTHUR RIZZI

IMA Preprint Series # 312

May 1987

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
UNIVERSITY OF MINNESOTA
514 Vincent Hall
206 Church Street S.E.
Minneapolis, Minnesota 55455

#	Author(s)	Title	Recent IMA Preprints (continued)	#	Author(s)	Title
203	G. Buttazzo, G. Dal Maso and U. Mosco,	A Derivation Theorem for Capacities with Respect to a Radon Measure	241	Harry Kesten,	Scaling Relations for 2D-Percolation	
204	S. Cowin, M. Mehraabadi,	On the Identification of Material Symmetry for Anisotropic Elastic Materials	242	A. Leizarowitz,	Infinite Horizon Optimization for Markov Process with Finite States Spaces	
205	R.W.R. Darling,	Constructing Nonhomeomorphic Stochastic Flows.	243	Louis H.-Y. Chen,	The Rate of Convergence in A Central Limit Theorem for Dependent Random Variables with Arbitrary Index Set	
206	M. Chipot,	On the Reynolds Lubrication Equation	244	G. Kalianpur,	Stochastic Differential Equations in Duals of Nuclear Spaces with some Applications	
207	R.V. Kohn and G.W. Milton,	On Bounding the Effective Conductivity of Anisotropic Composites	245	Tzuu-Shuh Chang, Yunshong Chow and Yuh-Jia Lee,	Evaluation of Certain Functional Integrals	
208	I.-J. Bakelman,	Notes Concerning the Torsion of Hardening Rods and Its N-Dimensional Generalizations	246	L. Karp and M. Pinsky,	The First Eigenvalue of a Small Geodesic Ball in a Riemannian Manifold	
209	I.-J. Bakelman,	The Boundary Value Problems for Non-Linear Elliptic Equations	247	Chi-Sing Man,	Towards An Acoustoelastic Theory for Measurement of Residual Stress	
210	Guanglu Gong & Mingping Qian,	On the Large Deviation Functions of Markov Chains	248	Andreas Stoll,	Invariance Principles for Brownian Intersection Local Time and Polymer Measures	
211	Arle Leizarowitz,	Control Problems with Random and Progressively Known Targets	249	R.W.R. Darling,	Rate of Growth of the Coalescent Set in a Coalescing Stochastic Flow	
212	R.W.R. Darling,	Ergodicity of a Measure-Valued Markov Chain Induced by Random Transformations	250	R. Cohen, R. Hardt, D. Kinderlehrer, S.-Y. Lin, M. LuskIn,	Minimum Energy for Liquid Crystals: Computational Results	
213	G. Gong, M. Qian & Zhongxin Zhao,	Killed Diffusions and Its Conditioning	251	Suzanne M. Lenhart,	Viscosity Solutions for Weakly Coupled Systems of First Order PDEs	
214	Arle Leizarowitz,	Controlling Diffusion Processes on Infinite Horizon with the Overaking Criterion	252	M. Cranston, E. Fabes, Z. Zhao,	Condition Gauge and Potential Theory for the Schrödinger Operator	
215	Millard Beatty,	The Poisson Function of Finite Elasticity	253	H. Brezis, J.M. Coron, E.H. Lieb,	Harmonic Maps with Defects	
216	David Terman,	Traveling Wave Solutions Arising From a Combustion Model	254	A. Carverhill,	Flows of Stochastic Dynamical Systems: Nontriviality of the Lyapunov Spectrum	
217	Yuh-Jia Lee,	Sharp Inequalities and Regularity of Heat Semi-Group on Infinite Dimensional Spaces	255	A. Carverhill,	Conditioning A 'Lifted' Stochastic System in a Product Case	
218	D. Stroock,	Lecture Notes	256	R.-J. Williams,	Local Time and Excursions of Reflected Brownian Motion	
219	Claudio Canuto,	Spectral Methods and Maximum Principle	257	H. Föllmer, S. Orey,	Large Deviations for the Empirical Field of a Gibbs Measure	
220	Thomas O'Brien,	A Two Parameter Family of Pension Contribution Functions and Stochastic Optimization	258	A. Leizarowitz,	Characterization of Optimal Trajectories On an Infinite Horizon	
221	Takeyuki Hida,	Analysis of Brownian Functionals	259	Y. Giga, T. Miyakawa, H. Osada,	Two Dimensional Navier Stokes Flow with Measures As Initial Vorticity	
222	Leonid Hurwicz,	On Informational Decentralization and Efficiency of Resource Allocation Mechanisms	260	M. Chipot, V. Oikter,	Sur Une Propriete Des Fonctions Propres De L'Operateur De Laplace Beltrami	
223	E.B. Fabes and D.W. Stroock,	A New Proof of Moser's Parabolic Harnack Inequality via the Old Ideas of Nash	261	V. Perez-Abreu,	Decompositions of Semimartingales On Duals of Countably Nuclear Spaces	
224	Minoru Murata,	Structure of Positive Solution to $(-\Delta+V)u = 0$ in R^n	262	J.M. Ball,	Does Rank-One Convexity Imply Quasiconvexity	
225	Paul Dupuis,	Large Deviations Analysis of Reflected Diffusions and Constrained Stochastic Approximation Algorithms in Convex Sets	263	B. Cockburn,	The Quasi-Monotone Schemes for Scalar Conservation Laws. Part I	
226	F. Bernis,	Existence Results for Doubly Nonlinear Higher Order Parabolic Equations on Unbounded Domains.	264	K.A. Perlick-Spector,	On Radially Symmetric Simple Waves in Elasticity	
227	S. Orey and S. Peilkan,	Large Deviations Principles for Stationary Processes of Minimal Surfaces.	265	P.N. Shivakumar, Chi-Sing Man, Simon W. Rabkin,	Modelling of the Heart and Pericardium at End-Diastole	
228	R. Gulliver and S. Hildebrandt,	Boundary Configurations Spanning Continua of Minimal Surfaces.	266	Jose-Luis Menaldi,	Probabilistic View of Estimates for Finite Difference Methods	
229	J. Baxter, G. Dal Maso & U. Mosco,	Stopping Times and T-Convergence.	267	Robert Hardt, Harold Rosenberg,	Open Book Structures and Unicity of Minimal Submanifolds	
230	Julio Bouilliet,	Self-Similar Solutions, Having Jumps and Intervals of Constancy of a Diffusion-heat Conduction Equation	268	Bernardo Cockburn,	The Quasi-Monotone Schemes for Scalar Conservation Laws	
231	R. Hardt, D. Kinderlehrer & F.-H. Lin,	A Remark About the Stability of Smooth Equilibrium Configurations of Static Liquid Crystal	269	H.R. Jauslin, W. Zimmermann, Jr.,	Dynamics of a Model for an Ac Josephson Effect in Superfluid 4He	
232	M. Chipot and M. LuskIn,	The Compressible Reynolds Lubrication Equation	270	A.K. Kapila,	Introductory Lecture on Reacting Flows	
233	J.H. Maddocks,	A Model for Disclinations in Nematic Liquid Crystal	271	J.C. Taylor,	Do Minimal Solutions of Heat Equations Characterize Diffusions?	
234	C. Fofas, G.R. Sell and R. Temam,	Inertial Manifolds for Nonlinear Evolutionary Equations	272	J.C. Taylor,	The Minimal Eigenfunctions Characterize the Ornstein-Uhlenbeck Process	
235	P.-L. Chow,	Expectation Functionals Associated with Some Stochastic Evolution	273	Chi-Sing Man, Quan-Xin Sun,	On the Significance of Normal Stress Effects in the Flow of Glaciers	
236	Giuseppe Buttazzo,	Reinforcement by a Thin Layer with Oscillating Thickness	274	Omar Hijab,	On Partially Observed Control of Markov Processes	
237	W.H. Fleming, S.-J. Shou and H.M. Soner,	On Existence of the Dominant Eigenfunction and its Application to the Large Deviation Properties of an Ergodic Markov Process	275	Lawrence Gray,	The Behavior of Processes with Statistical Mechanical Properties	
238	R. Jensen and P.-E. Souganidis,	A Regularity Result for Viscosity Solutions of Hamilton-Jacobi Equations in one Space Dimension	276	R. Hardt, D. Kinderlehrer, M. LuskIn,	Remarks About the Mathematical Theory of Liquid Crystals	
239	B. Boczar-Karakli, J.-L. Bona and D.-L. Cohen,	Interaction of Shallow-Water Waves and Bottom Topography				
240	F. Colonius and W. Klemann,	Infinite Time Optimal Control and Periodicity Dirichlet Problems				

#	Author(s)	Title	#	Author(s)	Title
277	Cockburn, B.	The Quasi-Monotone Schemes for Scalar Conservation Laws Part III	312	Bernhard Müller, Arthur Rizzi	Navier-Stokes Computation of Transonic Vortices Over a Round Leading Edge Delta Wing
278	M. Chipot, T. Sideris	On the Abelian Higgs Model	313	J. Thomas Beale	On the Accuracy of Vortex Methods at Large Times
279	C. Foias, B. Nicolaenko, G.R.Sell, R. Temam	Inertial Manifolds for the Kuramoto-Sivashinsky Equation and an Estimate of their Lowest Dimension	314	P. Le Tallec, A. Lotfi	Decomposition Methods for Adherence Problems in Finite Elasticity
280	R. Duran	On the Approximation of Miscible Displacement in Porous Media by a Method of Characteristics Combined with a Mixed Method	315	Jim Douglas, Juan E. Santos	Approximation of Waves in Composite Media
281	M. Axleng, Zheng Bo	The Convergence for Nodal Expansion Method	316	Tod Arbogast	The Double Porosity Model for Single Phase Flow in Naturally Fractured Reservoirs
282	V. Tverzsky	Dispersive Bulk Parameters for Coherent Propagation in Correlated Random Distributions	317	Tod Arbogast, Jim Douglas, Jr., Juan E. Santos	Two-Phase Immiscible Flow in Naturally Fractured Reservoirs
283	W.Th.F. den Hollander	Mixing Properties for Random Walk in Random Scenery	318	Jim Douglas, Jr., Yuan Yirang	Numerical Simulation of Immiscible Flow in Porous Media Based on Combining the Method of Characteristics with Finite Element Procedures
284	H.R. Jauslin	Nondifferentiable Potentials for Nonequilibrium Steady States	319	R. Duran, R.H. Nochetto, J. Wang	Sharp Maximum Norm Error Estimates for Finite Element Approximations of the Stokes Problem in 2-D
285	K. Meyer, G.R.Sell	Homoclinic Orbits and Bernoulli Bundles in Almost Periodic Systems	320	Andreas Greven	A Phase Transition for a System of Branching Random Walks in a Random Environment
286	J. Douglas, Jr., Y. Yuan	Finite Difference Methods for the Transient Behavior of a Semiconductor Device	321	J.M. Harrison, R.J. Williams	Brownian Models of Open Queueing Networks with Homogeneous Customer Populations
287	L.J. Kaftel, Yan Ningning	The Extrapolation for Boundary Finite Elements	322	Ana Bela Cruzeiro	Solutions Et Mesures Invariantes Pour Des Equations d'Evolution Stochastiques Du Type Navier-Stokes
288	R. Durrett, R.H. Schonmann	Stochastic Growth Models	323	Salah-Eddin A. Mohammed	The Lyapunov Spectrum and Stable Manifolds for Stochastic Linear Delay Equations
289	David Kinderlehrer	Remarks about Equilibrium Configurations of Crystals	324	Bao Gia Nguyen	Typical Cluster Size for 2-DIM Percolation Processes (Revised)
290	D.G. Aronson, J.L. Vazquez	Eventual C^∞ -Regularity and Concavity for Flows in One-Dimensional Porous Media	325	Robert Hardt	David Kinderlehrer, Fanghua Lin, Stable Defects of Minimizers of Constrained Variational Principles
291	L.R. Scott, J.M. Boyle, B. Bagheri	Distributed Data Structures for Scientific Computation	326	Michel Chipot, David Kinderlehrer	Equilibrium Configurations of Crystals
292	J. Douglas, Jr., P.J.Paes Leme, T. Arbogast, T. Schmitt	Simulation of Flow in Naturally Fractured Petroleum Reservoirs			
293	D.G. Aronson, L.A. Caffarelli	Optimal Regularity for One-Dimensional Porous Medium Flow			
294	Meim Berezis	Liquid Crystals and Energy Estimates for S^2 -Valued Maps			
295	Todd Arbogast	Analysis of the Simulation of Single Phase Flow Through a Naturally Fractured Reservoir			
296	He Yimin, Li Kaftel	The Coupling Method of Finite Elements and Boundary Elements for Radiation Problems			
297	T. Cazenave, A. Haraux, L. Vazquez, F.B. Weissler	Nonlinear Effects in the Wave Equation with a Cubic Restoring Force			
298	M. Chipot, F.B. Weissler	Some Blow Up Results for a Nonlinear Parabolic Parabolic Equation with a Gradient Term			
299	Li Kaftel	Perturbation Solutions of Simple and Double Bifurcation Problems for Navier-Stokes Equations			
300	Chen Zhongxin, Li Kaftel	The Convergence on the Multigrid Algorithm for Navier-Stokes Equations			
301	A. Gerasi, G. Mappo	Martingale Approach for Modeling DNA Synthesis			
302	D.M. Arnold, L. Ridgway Scott, Michael Vogelius	Regular Inversion of the Divergence Operator with Dirichlet Boundary Conditions on a Polygon			
303	Duran, Ricardo G.	Error Analysis in L^p , $1 \leq p \leq \infty$, for Mixed Finite Element Methods for Linear and Quasi-Linear Elliptic Problems			
304	Nochetto, Ricardo H., Verdi, Claudio	An Efficient Linear Scheme to Approximate Parabolic Free Boundary Problems: Error Estimates and Implementation			
305	K.A. Perlick-Spector, Scott J. Spector	Nonuniqueness for a Hyperbolic System: Cavitation in Nonlinear Elastodynamics			
306	E.G. Kalnins, Willard Miller, Jr.	q-Series and Orthogonal Polynomials Associated with Barnes' First Lemma			
307	D.M. Arnold, Richard S. Falk	A Uniformly Accurate Finite Element Method for the Mindlin-Reissner Plate			
308	Chi-Wang Shu	TVD Properties of a Class of Modified Eno Schemes for Scalar Conservation Laws			
309	Edmund Dikow, Ulrich Hornung	A Random Boundary Value Problem Modeling Spatial Variability in Porous Media Flow			
310	Jack K. Hale	Compact Attractors and Singular Perturbations			
311	Boerger, A. Cockburn, B.	The TVD-Projection Method for Solving Implicit Numerical Schemes for Scalar Conservation Laws: A Numerical Study of a Simple Case			

NAVIER-STOKES COMPUTATION OF TRANSONIC VORTICES OVER A ROUND LEADING EDGE DELTA WING

by

Bernhard Müller and Arthur Rizzi
FFA, The Aeronautical Research Institute of Sweden,
S-161 11 BROMMA, Sweden

Abstract

A 3D Navier-Stokes solver has been developed to simulate laminar compressible flow over quadrilateral wings. The finite volume technique is employed for spatial discretization with a novel variant for the viscous fluxes. An explicit three-stage Runge-Kutta scheme is used for time integration taking local time steps according to the linear stability condition derived for application to the Navier-Stokes equations. The code is applied to compute primary and secondary separation vortices at transonic speeds over a 65° swept delta wing with round leading edges and cropped tips. The results are compared with experimental data and Euler solutions, and Reynolds number effects are investigated.

I. Introduction

Even at moderate angles of attack, the flow over a swept delta wing is characterized by a pair of symmetric primary vortices separating on the leeward side from the leading edges and symmetric counter-rotating secondary vortices separating from the leeward side inboard of the leading edges [1].

Since the augmented lift induced by the leeside vortices is of great aerodynamic usefulness, a number of experimental and theoretical investigations have been performed (see [2] for a recent overview). Panel and Euler methods have frequently been used, while recently, Navier-Stokes methods have been employed.

In the present paper, laminar transonic flow over the 65° round leading edge delta wing proposed for the International Vortex Flow Experiment on Euler Code Validation (subject of the International Vortex Flow Symposium held in Stockholm on October 1-3, 1986), is simulated at $M_\infty=0.85$, $\alpha=10^\circ$, $Re_{\infty, c_R} = 2.38 \times 10^6$ and $Re_{\infty, SPAN}=10^m$, $m=5,6,7$. The compressible Navier-Stokes equations (Section II) are discretized using the finite-volume technique in space involving 25 cells to form the viscous flux difference opposed to the conventional 19 cells. Nonlinear second-order and linear fourth-order damping terms are added to the physical difference operator. The resulting system of ordinary differential equations is solved by an explicit three-stage Runge-Kutta scheme in time. A linear stability condition is derived for a scalar model equation to determine the local time steps and the allowable damping coefficients (Section III). The 65° swept delta wing has 15% taper, and is defined by one section with 0.7% nose radius and with 5%

thickness at 40% local chord. The fine O-O type mesh has 129, 49 and 65 grid points in the chordwise, near normal and spanwise directions, respectively (Section IV). The delta wing surface is assumed to be adiabatic. Locally one-dimensional characteristic boundary conditions are used in the farfield. (Section V). Results are presented for the above mentioned cases and compared with experiment and Euler solution (Section VI).

II. Governing Equations

Fluid motion is governed by the conservation laws for mass, momentum and energy. The investigated fluid here is a perfect gas obeying Newton's and Fourier's laws. External forces and heat sources are not taken into account. Considering an arbitrary stationary cell V with the boundary ∂V and the outer normal unit vector $\underline{n}$ in an absolute frame of reference, the Navier-Stokes equations read [3].

$$\int_V \frac{\partial Q}{\partial t} dV + \int_{\partial V} \underline{H} \cdot \underline{n} dA = 0 \quad , \quad (1)$$

where

$$\underline{Q} = \begin{pmatrix} \rho \\ \rho \underline{u} \\ e \end{pmatrix} \quad \underline{H} = \begin{pmatrix} \rho \underline{u} \\ \rho \underline{u} \underline{u} + p \underline{I} - \underline{\tau} \\ (e+p) \underline{u} - \underline{\tau} \cdot \underline{u} + \underline{q} \end{pmatrix} .$$

$\underline{Q}$ is the vector of the conservative variables, i.e. density, momentum density and total energy density. $\underline{H}$ represents the flux tensor, $\underline{I}$ the unit tensor. Pressure p and temperature T are related to the conservative variables by the equations of state for perfect gas with the ratio of the constant specific heats $\gamma=1.4$. The stress tensor is given by Newton's law:

$$\underline{\tau} = \mu [\text{grad } \underline{u} + (\text{grad } \underline{u})^T] + \lambda \text{div } \underline{u} \underline{I} \quad (2)$$

Fourier's law states for the heat flux:

$$\underline{q} = -\kappa \text{grad } T \quad (3)$$

The viscosity coefficients μ and λ are related by Stokes's hypothesis. The Prandtl number is assumed to be constant, namely $Pr=0.72$, thus giving a simple relationship between the thermal conductivity coefficient κ and the viscosity coefficient μ . The dependence of μ on the temperature T is obtained from the Sutherland law with the Sutherland constant $S/T_\infty = 0.375$.

A spatial vector is expressed in Cartesian coordinates x, y and z , i.e. chordwise from apex, spanwise from plane of bilateral symmetry and normal from wing plane, respectively. A velocity vector $\underline{u}$ is represented by its components u, v and w in the corresponding coordinate directions.

III. Numerical Method

Spatial Discretization

The Navier-Stokes equations (1) are discretized in hexahedrons (Fig. 1) using the finite-volume technique [3]. Since the conservative variables are assumed to be defined by their cell averages, the volume integral in (1) over a cell P is expressed by:

$$\int_{V_P} \frac{\partial Q}{\partial t} dV = \frac{\partial Q_P}{\partial t} \int_{V_P} dV \quad (4)$$

The surface integral in (1) over the boundary of cell P is approximated by assuming the mean-value of the flux tensor on each side to be equal to the arithmetic average of the flux tensor in the adjacent cells:

$$\int_{\partial V_P} \underline{H} \cdot \underline{n} dA \cong \sum_{k=1}^6 \underline{H}_{Pk} \cdot \int_{\partial V_{Pk}} \underline{n} dA \quad (5)$$

where

$$\underline{H}_{Pk} = \frac{1}{2} (\underline{H}_P + \underline{H}_k) .$$

∂V_{Pk} denotes the common part of the boundaries of P and its neighbouring cell k. The cell volume in (4) is computed as the sum of five tetrahedra [4]. The surface normal in (5) is determined as the sum of the surface normals of two triangles [5].

With the conservative variables given, all terms of the flux tensor are readily available in cell P, except for the gradients of the velocity components and temperature as well as $\text{div } \underline{u}$. Following the definition of the conservative variables as cell averages, the gradients in cell P are defined by:

$$\text{grad } \phi_P = \int_{V_P} \text{grad } \phi dV / \int_{V_P} dV \quad (6)$$

where $\phi = u, v, w, \text{ or } T$.

Using the gradient theorem, the volume integral in (6) can be expressed by a surface integral, which is approximated similarly to (5) [6]:

$$\text{grad } \phi_P = \int_{V_P} \phi \underline{n} dA / \int_{V_P} dV \cong \sum_{k=1}^6 \phi_{Pk} \int_{\partial V_{Pk}} \underline{n} dA / \int_{V_P} dV \quad (7)$$

where

$$\phi_{Pk} = \frac{1}{2} (\phi_P + \phi_k) .$$

$\text{div } \underline{u}_P$ is evaluated similarly to $\text{grad } \phi_P$.

On a Cartesian equidistant grid, the present finite-volume approximation is equivalent to a second-order central difference discretization involving 25 points opposed to the conventional 19 ones.

Numerical Damping

The spatial discretization constitutes the physical difference operator $\underline{F}_{PH}$ defined by the right hand side of (5) divided by the cell volume. These central differences do not damp unphysical oscillations caused by flow discontinuities and waves with short wavelengths. The numerical damping terms, which are therefore added to $\underline{F}_{PH}(\underline{Q})$, comprise nonlinear second-order differences sensed by the discretized second derivative of the pressure, and linear fourth-order differences of the conservative variables [4]:

$$\begin{aligned} \underline{F}_N(\underline{Q}) = & (\text{CFL}/\Delta t) \{ \chi (\delta_I [s_I(p) \delta_I] + \delta_J [s_J(p) \delta_J] + \\ & + \delta_K [s_K(p) \delta_K]) - \Lambda (\delta_I^4 + \delta_J^4 + \delta_K^4) \} \underline{Q} \end{aligned} \quad (8)$$

with CFL the maximum CFL number used (cf. chapter on stability below) and Δt the time step. The constants χ and Λ used lie in the ranges: $0 \leq \chi \leq 0.005$ and $0.005 \leq \Lambda \leq 0.02$ (cf. chapter on stability below). The sensors s_I , s_J and s_K are of similar form, e.g. s_I for a cell indexed by I, J, K :

$$s_I(p_{I \pm 1/2, J, K}) = \mu_I |\delta_I^2 p_{I \pm 1/2, J, K}| / \max_{I', J', K'} |\delta_I^2 p_{I', J', K'}| \quad (9)$$

The classical finite-difference operators are defined by:

$$\delta_I a_{I, J, K} = a_{I+1/2, J, K} - a_{I-1/2, J, K} \quad (10)$$

$$\mu_I a_{I, J, K} = (a_{I+1/2, J, K} + a_{I-1/2, J, K})/2$$

and similarly for J and K .

The numerical damping operator $\underline{F}_N$ is modified near wing surface and farfield to ensure its dissipative property also there [7]. Using the periodic and symmetry boundary conditions (cf. Section V), $\underline{F}_N$ can be determined near these boundaries without modifying (8) or (9).

Time Integration

Thus the semi-discrete approximation of the Navier-Stokes equations can be written as:

$$\frac{d\underline{Q}}{dt} = \underline{F}(\underline{Q}) \quad (11)$$

where

$$\underline{F} = \underline{F}_{PH} + \underline{F}_N .$$

Equation (11) represents a large system of first-order ordinary differential equations. It is solved for the steady state by the second-order explicit three-stage Runge-Kutta scheme [8]:

$$\begin{aligned}
 \underline{Q}^{(0)} &= \underline{Q}^n \\
 \underline{Q}^{(1)} &= \underline{Q}^{(0)} + \Delta t \underline{F}(\underline{Q}^{(0)}) \\
 \underline{Q}^{(2)} &= \underline{Q}^{(0)} + \Delta t [(1-\theta)\underline{F}(\underline{Q}^{(0)}) + \theta \underline{F}(\underline{Q}^{(1)})] \\
 \underline{Q}^{(3)} &= \underline{Q}^{(0)} + \Delta t [(1-\theta)\underline{F}(\underline{Q}^{(0)}) + \theta \underline{F}(\underline{Q}^{(2)})] \\
 \underline{Q}^{n+1} &= \underline{Q}^{(3)}
 \end{aligned} \tag{12}$$

with $\theta=1/2$. n denotes the time level.

Stability

The stability of explicit Runge-Kutta schemes applied to the semi-discretization (11) of the Navier-Stokes equations (1) is studied for the scalar model equation:

$$\begin{aligned}
 \frac{\partial a}{\partial t} + \lambda_{\xi} \frac{\partial a}{\partial \xi} + \lambda_{\eta} \frac{\partial a}{\partial \eta} + \lambda_{\zeta} \frac{\partial a}{\partial \zeta} &= \nu_{\xi} \frac{\partial^2 a}{\partial \xi^2} + \nu_{\eta} \frac{\partial^2 a}{\partial \eta^2} + \\
 + \nu_{\zeta} \frac{\partial^2 a}{\partial \zeta^2} + \nu_{\xi\eta} \frac{\partial^2 a}{\partial \xi \partial \eta} + \nu_{\xi\zeta} \frac{\partial^2 a}{\partial \xi \partial \zeta} + \\
 + \nu_{\eta\zeta} \frac{\partial^2 a}{\partial \eta \partial \zeta} + \varepsilon_{\xi} \frac{\partial^4 a}{\partial \xi^4} + \varepsilon_{\eta} \frac{\partial^4 a}{\partial \eta^4} + \varepsilon_{\zeta} \frac{\partial^4 a}{\partial \zeta^4}
 \end{aligned} \tag{13}$$

where

$$\begin{aligned}
 \lambda_h &= |\underline{u} \cdot \text{grad } h| + c |\text{grad } h| \\
 \nu_h &= \nu |\text{grad } h \cdot \text{grad } h| + \Delta h^2 \chi_h \text{ CFL} / \Delta t \\
 \nu_{hg} &= 2\nu |\text{grad } h \cdot \text{grad } g| \\
 \varepsilon_h &= -\Delta h^4 \Lambda \text{ CFL} / \Delta t
 \end{aligned}$$

with c the speed of sound, and

$$\nu = \max \{ \mu, \lambda + 2\mu, \mu\gamma / \text{Pr} \} / \rho, \quad \chi_h = \chi s_h \text{ and } h, g \in \{ \xi, \eta, \zeta \}$$

Equation (13) is derived from the differential form of the Navier-Stokes equations in transformed coordinates ξ, η, ζ [3] by linearization and from the differential expressions of the damping terms in (8). λ_h is chosen equal to the maximum modulus of the eigenvalues belonging to the Jacobian matrix of the flux in h -direction. ν_h and ν_{hg} are determined by the maximum moduli of the coefficients of the second derivatives $\partial^2 / \partial h^2$ and $\partial^2 / \partial h \partial g$, respectively, in the linearized momentum and energy equations, including the differential expressions of the linearized second-order damping terms. ε_h represents the coefficient of the fourth-order damping term in h -direction. Thus, the scalar ansatz (13) models the full Navier-Stokes equations including the numerical damping terms, whereas inviscid, diffusion and mixed-derivative vector parts of the linearized Navier-Stokes equations are considered separately by [9], [10], [11], to determine the respective time steps of time splitting schemes.

The first and second spatial derivatives in (13) are discretized by second-order central finite-differences corresponding to the finite-volume approximation (11):

$$\frac{\partial a}{\partial h} \Big|_{I,J,K} = \frac{1}{\Delta h} \mu_h \delta_h a_{I,J,K} + O(\Delta h^2) \quad (14a)$$

$$\frac{\partial^2 a}{\partial h^2} \Big|_{I,J,K} = \frac{1}{\Delta h^2} (\mu_h \delta_h)^2 a_{I,J,K} + O(\Delta h^2) \quad (14b)$$

$$\frac{\partial^2 a}{\partial h \partial g} \Big|_{I,J,K} = \frac{1}{\Delta h \Delta g} \mu_h \delta_h \mu_g \delta_g a_{I,J,K} + O(\Delta h^2) \quad (14c)$$

The finite-difference operators in (14) are defined by (10) with ξ corresponding to I , and similarly η to J and ζ to K . The fourth derivatives in (13) are discretized according to (8).

Note that $a_{I\pm 1,J,K}$, $a_{I,J\pm 1,K}$ and $a_{I,J,K\pm 1}$ do not appear in (14b). The second-order truncation error in (14b) is four times greater than the one, if the compact differencing δ_h^2 instead of $(\mu_h \delta_h)^2$ were used.

The linear stability of an explicit Runge-Kutta scheme is investigated for the application to the model equation (13) with frozen coefficients and spatially discretized by (14). The analysis shows that the shortest resolvable waves are undamped by (14b) in contrast to the compact differencing δ_h^2 . The stability condition reads:

$$\begin{aligned} |RK| \Big[& \frac{\tilde{v}_\xi}{\Delta \xi^2} + \frac{\tilde{v}_\eta}{\Delta \eta^2} + \frac{\tilde{v}_\zeta}{\Delta \zeta^2} + \frac{v_{\xi\eta}}{\Delta \xi \Delta \eta} + \frac{v_{\xi\zeta}}{\Delta \xi \Delta \zeta} + \frac{v_{\eta\zeta}}{\Delta \eta \Delta \zeta} - \\ & - 16 \left(\frac{\varepsilon_\xi}{\Delta \xi^4} + \frac{\varepsilon_\eta}{\Delta \eta^4} + \frac{\varepsilon_\zeta}{\Delta \zeta^4} \right) \Big]^{-1} \end{aligned} \quad (15)$$

where

$\tilde{v}_h = \alpha v \text{ grad } h \cdot \text{grad } h + 4\Delta h^2 \chi_h \text{ CFL}/\Delta t$
with $\alpha = 1$ for (14b), but $\alpha = 4$ for the compact differencing δ_h^2 .

The negative and positive stability bounds RK and CFL, resp., are chosen such that all complex numbers z with $RK \leq \text{Re}(z) \leq 0$ and $|\text{Im}(z)| \leq \text{CFL}$ lie inside the stability region of the Runge-Kutta method. For the three-stage Runge-Kutta method (12) the following choice is taken: $\text{CFL}=1.5, RK=-1$ (Fig. 2).

The viscous contributions in (15) (denoted here by S) involving coefficient v and the second- and fourth-order damping coefficients in (15) (here referred to as D_2 and D_4) share the negative real axis of the stability region. Their distribution may be prescribed by

$$S \Delta t \leq \beta_1 |RK| \quad (16a)$$

$$D_2 \Delta t \leq \beta_2 |RK| \quad (16b)$$

$$D_4 \Delta t \leq \beta_3 |RK| \quad (16c)$$

where $0 \leq \beta_\lambda$ and $\sum_{\lambda=1}^3 \beta_\lambda = 1$.

Equations (16b) and (16c) yield the following conditions for the damping coefficients:

$$\chi \leq (\beta_2/4d) |RK|/CFL \quad (17a)$$

$$\Lambda \leq (\beta_3/16d) |RK|/CFL \quad (17b)$$

with d the dimension, i.e. $d=3$ for 3D.

Using then (16a) in (15), the stability condition is obtained:

$$\Delta t \leq \min \left\{ CFL \left[\frac{|\lambda_\xi|}{\Delta \xi} + \frac{|\lambda_\eta|}{\Delta \eta} + \frac{|\lambda_\zeta|}{\Delta \zeta} \right]^{-1}, \right. \\ \left. \beta_1 |RK| \left[\frac{v_{\xi\xi}}{\Delta \xi^2} + \frac{v_{\eta\eta}}{\Delta \eta^2} + \frac{v_{\zeta\zeta}}{\Delta \zeta^2} + \frac{v_{\xi\eta}}{\Delta \xi \Delta \eta} + \frac{v_{\xi\zeta}}{\Delta \xi \Delta \zeta} + \frac{v_{\eta\zeta}}{\Delta \eta \Delta \zeta} \right]^{-1} \right\} \quad (18)$$

where

$$v_{hh} = \alpha \, v \, \text{grad } h \cdot \text{grad } h$$

with $\alpha=1$ for (14b), but $\alpha=4$ for the compact differencing δ_h^2 .

In order to apply the stability condition (18) to the finite-volume discretization, the metric terms are related to geometrical quantities [12]:

cell volume

$$\Delta \xi \Delta \eta \Delta \zeta \, J^{-1} = v = \int_V dV \quad (19a)$$

surface normal in G-direction:

$$\Delta \xi \Delta \eta \Delta \zeta \, J^{-1} \, \text{grad } g / \Delta g = \underline{S}_G = \int_{\partial V_G} \underline{n} \, dA \quad (19b)$$

where $g=\xi, \eta, \zeta$ corresponds to $G=I, J, K$, and ∂V_G denotes a cell boundary with constant G . J represents the Jacobian determinant of the transformation $(\xi, \eta, \zeta)^T (x, y, z)$.

Thus, the stability condition (18) with (17a) and (17b) for an explicit Runge-Kutta method like (12) applied to the finite-volume discretization of the Navier-Stokes equations (11) reads:

$$\Delta t \leq \min \{ CFL \vee [|\underline{u} \cdot \underline{S}_I| + c |\underline{S}_I| + |\underline{u} \cdot \underline{S}_J| + c |\underline{S}_J| + |\underline{u} \cdot \underline{S}_K| + c |\underline{S}_K|]^{-1}, \beta_1 |\text{RK}| \vee^2 [\vee \{ \alpha (\underline{S}_I \cdot \underline{S}_I + \underline{S}_J \cdot \underline{S}_J + \underline{S}_K \cdot \underline{S}_K) + 2 (|\underline{S}_I \cdot \underline{S}_J| + |\underline{S}_I \cdot \underline{S}_K| + |\underline{S}_J \cdot \underline{S}_K|) \}] \} \quad (20)$$

with $\alpha=1$.

If the compact differencing δ_h^2 were used instead of (14b), the stability condition would be more restrictive, since $\underline{S}_I \cdot \underline{S}_I + \underline{S}_J \cdot \underline{S}_J + \underline{S}_K \cdot \underline{S}_K$ would have to be multiplied by $\alpha=4$.

$\beta_1=1/2$, $\beta_2=1/30$ and $\beta_3=1/2$ were used in (17) and (20) for the present calculation with $CFL=0.5$ and $RK=-1$. Although $\sum_{\ell=1}^3 \beta_\ell > 1$, stability was maintained indicating that the stability bound RK could have been chosen lower than -1 for the Runge-Kutta method (12). For the $65 \times 25 \times 17$ medium mesh, even $CFL=1.5$, $RK=-1$, $\beta_1=1/2$, $\beta_2=1/10$, and $\beta_3=3/2$ were used. Computational work is saved by computing the local time steps according to (20) only at those time levels which are powers of 2.

IV. Mesh

The round leading edge delta wing proposed for the International Vortex Flow Experiment on Euler Code Validation has 65° sweep and 15% taper. It is defined by one section in terms of X- and Z-coordinates, i.e. chordwise from leading ledge and normal from wing plane, respectively, non-dimensionalized by local chord:

$$Z = \begin{cases} \pm [0.1183/X - 0.2101X + 0.3501X^2 - 0.3406X] & \text{for } 0 \leq X \leq 0.4 \\ \text{NACA 64A005} & \text{for } 0.4 \leq X \leq 1 \end{cases} \quad (21)$$

The nose radius is 0.7%, the maximum thickness at 40% is 5% local chord, and the trailing edge is sharp.

The O-O mesh topology introduces periodic, symmetry, wing and farfield boundaries to determine bilaterally symmetric flow over a quadrilateral wing. The symmetry boundary lies in the $y=0$ plane. A hemisphere with a radius of 3 root chord lengths from root midchord is chosen as farfield boundary. The periodic boundaries extend from the trailing edge and tip of the wing in the positive x- and y-directions, respectively.

The O-O meshes are generated by the transfinite interpolation method [13]. The fine mesh consists of 129, 49 and 65 grid points in the chordwise I-, near normal J- and spanwise K-directions, respectively, i.e. 410 865 grid points in total. On the wing, the mesh points are clustered near the leading edge and tip and to a less extent also near the trailing edge and symmetry boundary with larger spacings in the mid sections (Fig. 3). On the hemisphere in the farfield, the grid points are distributed equally (Figs. 4 and 5), except for a clustering near the two points lying on the two parabolic singular

lines extending from the tip leading and trailing edges [13]. The mesh is nearly orthogonal at the wing contour, except for the points near the trailing edge and tip. The grid points between wing contours and farfield are clustered near the wing to resolve the boundary layer for the respective Reynolds number (Figs. 4 and 5 for $Re_{\infty, C_R} = 2.38 \times 10^6$). For $Re_{\infty, SPAN} = 10^m$, $m=5, 6, 7$, $65 \times 25 \times 33$ medium grids have been used, each with a different clustering in the near normal direction.

V. Initial and Boundary conditions

For $M_{\infty} = 0.85$, $\alpha = 10^\circ$, $Re_{\infty, SPAN} = 10^5$, the calculation is started from freestream on the coarse mesh obtained by cancelling every other point of the medium 53×625 grid points mesh. Then the result is interpolated on the medium mesh, and 500 time steps are taken to obtain the result on the medium mesh. This result is used as initial condition for the next higher Reynolds number, etc.

For $M_{\infty} = 0.85$, $\alpha = 10^\circ$, $Re_{\infty, C_R} = 2.38 \times 10^6$, the calculation is started from freestream on a coarse mesh using a large second-order damping coefficient ($\chi = 0.1$), which is subsequently reduced. The converged result is interpolated on the medium mesh and so on to the fine mesh.

At periodic boundaries, grid points on upper and lower surfaces are mapped on each other. For the conservative variables to be uniquely defined, the periodic boundary condition requires:

$$\underline{Q}_{IMAX, J, K} = \underline{Q}_{1, J, K}; \quad \underline{Q}_{I, J, KMAX} = \underline{Q}_{IMAX-I, J, KMAX-1} \quad (22)$$

At the symmetric boundary, ρ, u, w , and e are even functions with respect to y , and v is an odd one:

$$(\rho, u, v, w, e)^T(x, y, z) = (\rho, u, -v, w, e)^T(x, -y) \quad (23)$$

The no-slip condition holds on the wing, which is assumed to be adiabatic. The pressure is obtained by neglecting the viscous terms in the wall normal momentum equation:

$$\underline{u}_w = 0 \quad ; \quad \left. \frac{\partial T}{\partial n} \right|_w = 0 \quad ; \quad \left. \frac{\partial p}{\partial n} \right|_w = 0 \quad (24)$$

The pressure and the stress tensor at the wing interface of the first cell above the wing are approximated by their values in that cell.

The boundary conditions at the farfield boundary are based on the theory of characteristics for locally one-dimensional inviscid flow [14]. For supersonic inflow or outflow, the locally one-dimensional Riemann invariants, entropy and tangential velocity component, i.e. (with the speed of sound c):

$$\begin{aligned}
 R_1 &= \underline{u} \cdot \underline{n} - \frac{2}{\gamma-1} c & ; & \quad R_2 = \underline{u} \cdot \underline{n} + \frac{2}{\gamma-1} c \\
 R_3 &= \ln \left(\frac{p}{\rho^\gamma} \right) & ; & \quad R_4 = \underline{u} - (\underline{u} \cdot \underline{n}) \underline{n}
 \end{aligned}
 \tag{25a}$$

are given from outside or inside the region of interest, respectively:

$$R_m = R_{m_\infty} \quad \text{or} \quad \frac{\partial R_m}{\partial n} = 0
 \tag{25b}$$

For subsonic inflow, R_1, R_3 , and R_4 are given from outside and R_2 from inside. R_2, R_3 , and R_4 are determined from inside and R_1 from outside for subsonic outflow.

The conditions (25b) are used to determine R_m in a fictitious cell outside the domain of integration either by free-stream or by R_m in the cell next to the farfield.

VI. Results

A computer program has been developed based on the Runge-Kutta finite-volume method for the three-dimensional Navier-Stokes equations. The code is implemented on the CYBER 205 (2 pipes and 8 million words of memory) of the SCIENTIFIC INFORMATION SERVICE in Kansas City, MO. The fine mesh computations were performed with the CYBER 205 of the Minnesota Supercomputer Center located at the University of Minnesota. The vector coding concept of [15] is employed to fully vectorize the computer program. Using half precision and a fine $129 \times 49 \times 65$ mesh, the latest version takes 23×10^{-6} CPU seconds per time step and per grid point.

Laminar flow over the 65° swept round leading edge cropped delta wing described in Section IV, has been simulated at $M_\infty = 0.85$, $\alpha = 10^\circ$ and $Re_\infty, c_R = 2.38 \times 10^6$ (based on root chord c_R).

The convergence history (Fig. 6a) shows a rapid decrease of the density change in the coarse and medium mesh. The "spikes" are due to the normalization $\|\rho^n - \rho^{n-1}\|_2 / \|\rho^1 - \rho^0\|_2$ at restarts and due to interpolations from coarser to finer grids. The convergence in the fine mesh is poor, because the CFL number had to be decreased from 1.5 and 1 to 0.5 to allow a larger second-order damping coefficient χ (cf. (17a)) in the first cycles on the fine mesh. Decreasing χ had a negative effect on convergence, but apparently no effect on lift and drag, except for the coarse mesh (Fig. 6b). On each mesh level, C_L and C_D become constant and the density change is deemed low enough to assume steady flow results.

The comparison of the lift, drag and pitching moment (around $x/c_R = 0.57$) coefficients for the different mesh levels (Table 1) shows that the coarse $33 \times 13 \times 17$ mesh solution underpredicts C_L

and C_D considerably. The medium mesh C_L and the fine grid C_D happen to be closest to the experiment [16] measured at the higher Reynolds number $Re_\infty, c_R = 9 \times 10^6$.

The pressure coefficients at the stations $x/c_R = 0.3, 0.6, 0.9, 0.95$ and $2y/b = 0.55$ (b denotes span, cf. Fig. 7) are compared with experimental data at $Re_\infty, c_R = 2.38 \times 10^6$ [17] and Euler solutions [18] with 1 M points. $s(x)$ denotes the local semi-span, $l(y)$ the local chord, and x_{LE} the local leading edge x-component. The location of the primary vortex core is predicted in good agreement with the experiment, whereas the pressure minimum in the Euler solution lies too close to the leading edge (Fig. 8). The pressure level under the primary vortex core is predicted right with the fine mesh, while the medium mesh overpredicts the pressure, probably because of poor vortex resolution (e.g. at $2y/b = 0.55$ in Fig. 8) [19].

Between the primary and secondary separation cores, at $x/c_R = 0.3, 0.6$ and 0.8 , the pressure is overpredicted with the fine grid as well, and the pressure under the secondary vortex core is lower than in the experiment. At x/c_R , a pressure rise can be seen close to the leading edge. That pressure increase is neither apparent in the medium mesh solution [19], nor in the experiment. Even with finer meshes, however, complete agreement cannot be expected from a laminar computation, because the flow was observed to be transitional in the experiment. Moreover, the wind tunnel model was a wing-body combination on the windward side.

Figure 9 shows pressure coefficient, Mach number and total pressure $(1 - p_0/p_{0\infty})$ contours as well as velocity vectors at $x/c_R = 0.3, 0.6, 0.8$ and 0.95 . The growth of the primary vortex can be well identified. However, the secondary vortex is hardly visible, even in the enlargement of the velocity vectors near leading edge and tip, respectively. It is noted that the velocity vectors in Figs. 9 and 10 are not given at mesh points, but are interpolated to a Cartesian mesh. The structure of the primary vortex can be seen in a chordwise cut in Fig. 10.

In Fig. 11a, the secondary separation line at $y/s(x) \sim 0.74$ is indicated by the line towards which the skin friction lines converge. The skin friction lines emanating from points a short distance away from the tip and trailing edge turn in upstream direction and apparently terminate in a nodal point of separation on the secondary separation line.

The pressure minimum (Fig. 11b) almost coincides with the secondary separation line, around which large gradients of the modulus of vorticity (Fig. 11c) can be seen. Pressure and vorticity are nearly conical up to $x/c_R \sim 0.55$. Their gradients near the leading edge between $x/c_R \sim 0.55$ and ~ 0.75 might be indications for a tertiary separation vortex (cf. $x/c_R = 0.6$ in Fig. 8).

The streamlines (Fig. 12) show two distinct vortices which lift before the trailing edge.

The velocity vectors in grid surfaces (at ~37% local chord on the wing) seen from behind show the flow directions (Fig. 13). On the leeward side, the boundary layer flow is directed in the clockwise direction from the symmetry plane up to the secondary separation line, where it meets the flow going in the opposite direction. The primary and secondary vortices are clearly visible.

In the medium mesh computations, the Reynolds number has been varied (Fig. 14). For $Re_{\infty, SPAN}=10^5$, primary and secondary vortices are well resolved, while only the secondary vortices are well represented for $Re_{\infty, SPAN}=10^6$ and 10^7 . A comparison of the location of the pressure minimum (not shown) and the vortex core for these Reynolds numbers shows that the primary and secondary vortices lie closer to the leading edge and tip for higher Reynolds numbers. This observation is backed by the experimental finding that the primary vortex axis moves closer to the sharp leading edge of a 75° swept delta wing as the Reynolds number increases [20]. However, this effect vanishes for $Re_{\infty, CR} > 10^6$ [20].

VII. Conclusions

A new Navier-Stokes analysis code for laminar compressible flow over quadrilateral wings has been developed. The finite-volume technique is employed with a larger difference molecule than the conventional compact differencing approach for the viscous fluxes. A linear stability condition is derived to determine the local time steps and the allowable damping coefficients of the explicit Runge-Kutta scheme used for time integration. The method has been applied to simulate transonic flow over a 65° swept round leading edge cropped delta wing. The computation with a $129 \times 49 \times 65$ 0-0 mesh shows that the location of and pressure level under the primary vortex are predicted well compared with experimental data for $M_\infty=0.85$, $\alpha=10^\circ$ and $Re_{\infty, CR} = 2.38 \times 10^6$. However, the pressure level between the primary and the secondary vortices is overpredicted. The results obtained at $M_\infty=0.85$, $\alpha=10^\circ$ and $Re_{\infty, SPAN}=10^m$, $m=5,6,7$, indicate that the primary and secondary separation vortices lie closer to the leading edge for higher Reynolds numbers.

Acknowledgements

The authors thank Dr. G. Drougge, FFA, for valuable discussions. The extension of the second author's Euler solver to a Navier-Stokes code for quadrilateral wings was supported by the Swedish Board for Technical Development, STU. The computing time for the medium mesh results was provided by Control Data Corporation, CDC. The fine mesh investigation was supported in part by the Institute of Mathematics and its Applications, IMA, of the University of Minnesota with funds provided by the National Science Foundation, NSF.

VIII. References

- [1] Hummel, D.: "Experimentelle Untersuchung dreidimensionaler laminarer Grenzschichten an einem schlanken Deltaflügel", Zeitschrift für Flugwissenschaften und Weltraumforschung 10 (1986), Heft 3, pp. 133-145.
- [2] Newsome, R.W., and Kandil, O.A.: "Vortical Flow Aerodynamics - Physical Aspects and Numerical Simulation" AIAA Paper 87-0205, Jan. 1987.
- [3] Peyret, R., and Taylor, T.D.: "Computational Methods for Fluid Flows". Springer-Verlag, New York, 1983.
- [4] Rizzi, A., and Eriksson, L.-E.: "Computation of flow around wings based on the Euler equations". J. Fluid Mechanics, Vol. 148, 1984, pp. 45-71.
- [5] Kordulla, W., and Vinokur, M.: "Efficient Computation of Volume in Flow Predictions". AIAA J., Vol. 21, No. 6, June 1983, pp. 917-918.
- [6] Müller, B., and Rizzi, A.: "Runge- Kutta Finite-Volume Simulation of Laminar Transonic Flow over a NACA0012 Airfoil Using the Navier-Stokes Equations". FFA TN 1986-60, 1986.
- [7] Eriksson, L.-E. and Rizzi, A.: "Computer-Aided Analysis of the Convergence to Steady State of Discrete Approximations to the Euler Equations". J. Comp. Physics, Vol. 57, 1985, pp. 90-128.
- [8] Gary, J.: "On Certain Finite-Difference Schemes for Hyperbolic Systems". Math. Comp., Vol. 18, 1964, pp. 1-18.
- [9] MacCormack, R.W., and Baldwin, B.S.: "A Numerical Method for Solving the Navier-Stokes Equations with Application to Shock-Boundary Layer Interactions". AIAA Paper 75-1, Jan 1975.
- [10] Abarbanel, S., and Gottlieb, D.: "Optimal Time Splitting for Two- and Three-Dimensional Navier-Stokes Equations with Mixed Derivatives". J. Comp. Physics, Vol. 41, 1981, pp. 1-33.
- [11] Nordström, J.: "Stability Criteria for a Second Order Accurate, Time-Split Finite Volume Scheme to Solve the Navier-Stokes Equation". FFA TN 1985-08, 1985.
- [12] Müller, B.: "Navier-Stokes Solution for Hypersonic Flow over an Indented Nosedip". AIAA Paper 85-1504, July 1985.
- [13] Eriksson, L.-E.: "Generation of Boundary Conforming Grids Around Wing-Body Configurations Using Transfinite Interpolation". AIAA J., Vol. 20, No. 10, Oct 1982, pp. 1313-1320.

- [14] Jameson, A., and Baker, T.J.: "Solution of the Euler Equations for Complex Configurations". AIAA Paper 83-1929, 1983.
- [15] Rizzi, A.: "Vector Coding the Finite-Volume Procedure for the CYBER 205". In: Lecture Series Notes 1983-04, von Karman Institute, Brussels, 1983. Also: Parallel Computing 2 (1985), pp. 295-312.
- [16] Hirdes, R.H.C.: "US/European Vortex Flow Experiment Tests Report of Wind Tunnel Measurements on the 65° Wing in the NLR High Speed Wind Tunnel HST". NLR TR 85046 L, 1985.
- [17] Hartmann, K.: "US/European Transonic Vortex Flow Experiment - Data Lists of Pressure Measurements". DFVLR-IB 222-86 A 26, 1986.
- [18] Rizzi, A., Drougge, G., and Purcell, C.J.: "Euler Simulation of Shed Vortex Flows over the 65 Degree Delta Wings". International Vortex Flow Symposium, Stockholm, Oct. 1-3, 1986, FFA Report, to appear.
- [19] Müller, B., and Rizzi, A.: "Navier-Stokes Simulation of Laminar Flow over the 65° Round Leading Edge Delta Wing at $M_\infty=0.85$ and $\alpha=10^\circ$ ". International Vortex Flow Symposium Stockholm, Oct. 1-3, 1986, FFA Report, to appear.
- [20] Carcaillet, R., Manie, F., Pagan, D., and Solignac, J.L.: "Leading Edge Vortex Flow over a 75 Degree Swept Delta Wing: Experimental and Computational Results". 15th ICAS Congress, London, Sept 7-12, 1986.

Table 1. Comparison of integral values for
 $M_\infty=0.85$, $\alpha=10^\circ$, $Re_{\infty, c_R} = 2.38 \times 10^6$
 (9×10^6 in experiment)

MESH	C_L	C_D	C_m
Coarse 33×13×17	0.3811	0.0633	-0.0088
Medium 65×25×33	0.4466	0.0741	-0.0108
Fine 129×49×65	0.4586	0.0752	-0.0098
-----	-----	-----	-----
Experiment [16]	0.4386	0.0755	-0.0073

Fig. 1 Basic hexahedron P and neighbouring cells 1 to 6 (2 and 5 set apart)

Fig. 2 Contours of constant modulus of growth factors of (12) and stability bounds RK and CFL

Fig. 3 Surface mesh of 65° swept cropped delta wing

a) global view

b) enlargement

Fig. 5 Mesh in surfaces I=33 and 97 seen from behind (at ~37% local chord on wing)

a) global view

b) enlargement

Fig. 4 Mesh in plane of symmetry

Fig. 6 Convergence history for $M_\infty=0.85$
 $\alpha=10^\circ$, $Re_\infty, c_R=2.38 \times 10^6$

Fig. 7 Stations $x/c_R=0.3, 0.6, 0.8, 0.95$ and $2y/b=0.55$

Fig. 8 Comparison of pressure coefficient for $M_\infty=0.85$, $\alpha=10^\circ$, $Re_{\infty, c_R}=2.38 \times 10^6$

— present Navier-Stokes solution
 o experiment [17]
 --- Euler solution [18]

Fig. 9 Pressure coefficient, Mach number and total pressure ($1-p_o/p_{o_\infty}$) contours, velocity vectors in global view and enlarged (in $2s(x)/b - 0.2 < 2y/b < 2s(x)/b$, $0 < 2z/b < 0.05$) for $M_\infty = 0.85$, $\alpha = 10^\circ$, $Re_\infty, c_R = 2.38 \times 10^6$

Fig. 10 Pressure coefficient, Mach number and total pressure ($1-p_o/p_{o\infty}$) contours, velocity vectors at $2s/b=0.55$ for $M_\infty=0.85$, $\alpha=10^\circ$, $Re_{\infty,c_R}=2.38 \times 10^6$

Fig. 11 Skin friction lines, and lines of constant C_p and $|\text{rot } \underline{u}|$ for $M_\infty=0.85$, $\alpha=10^\circ$, $Re_{\infty,c_R}=2.38 \times 10^6$

Fig. 12 Streamlines on leeward side seen from above for $M_\infty=0.85$, $\alpha=10^\circ$, $Re_{\infty,c_R}=2.38 \times 10^6$

Fig. 13 Velocity vectors in cells I=33 and 96 seen from behind for $M_\infty=0.85$, $\alpha=10^\circ$, $Re_{\infty,c_R}=2.38 \times 10^6$

Fig. 14 Velocity vectors in cells I=17 and 49 seen from behind for $M_\infty=0.85$, $\alpha=10^\circ$