Recent perturbative results on heavy quark decays

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Outline

- 1 Helicity fractions of W bosons from top quark decays
JHP, Andrzej Czarnecki,
Jürgen Körner

- 2 Mass effects in semi-leptonic bottom quark decays
Alexey Pak, Andrzej Czarnecki

Part I

Helicity fractions

JHP, Andrzej Czarnecki, Jürgen Körner

Introduction

- couplings are very susceptible to “new physics”
- dominant decay mode: $t \rightarrow b + W^+$
- polarisation allows for detailed study of decay process
- LHC will produce many top quarks
- total width is hard to measure directly at hadron colliders
- helicity fractions can be measured

$$t \rightarrow b + W^+$$

W has spin 1 $\rightsquigarrow$ 3 polarisation states

$\rightsquigarrow$ different contributions to angular distribution

$$\frac{d\Gamma}{d\cos\theta} :$$

W_-

W_L

W_+

Effect of radiative corrections

bottom quark has to be left-handed

additional gluon and $m_b \neq 0$ leads to $\Gamma_+ \neq 0$

Experimental results

leading order Standard Model predictions ($m_b = 0$)

$$\mathcal{F}_L = 0.697, \mathcal{F}_- = 0.303, \mathcal{F}_+ = 0$$

Tevatron

[CDF '07] [D0 '05, '07]

$$\mathcal{F}_L = 0.85 \pm 0.28, \mathcal{F}_+ = 0.05 \pm 0.12$$

$$\mathcal{F}_L = 0.56 \pm 0.31, \mathcal{F}_+ = 0.056 \pm 0.10$$

LHC with 10 fb^{-1}

[Aguilar-Saavedra *et al.* '07]

$$\Delta\mathcal{F}_L = 1.9\%, \Delta\mathcal{F}_- = 1.8\%, \Delta\mathcal{F}_+ = 0.21\%$$

State of the art

unpolarised decay

$\mathcal{O}(\alpha_s)$ correction with full m_b and m_W dependence

$\mathcal{O}(\alpha_s^2)$ correction as expansion in m_W/m_t

Jeżabek
Kühn
Czarnecki
Blokland
Ślusarczyk
Tkachov
Pak

polarised decay

$\mathcal{O}(\alpha_s)$ correction with full m_b and m_W dependence

subsequent decay of W boson

electroweak and finite width

Fischer
Groote
Körner
Mausser
Lampe
Do

How can we reach $\mathcal{O}(\alpha_s^2)$ for polarised decay?

apply optical theorem and
expansion by regions

do **not** sum over polarisations

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3-loop diagrams

$m_b = 0 \rightsquigarrow$ two scales: m_t , m_W

$\rightsquigarrow$ use expansion by regions to reduce to single scale integrals

Expansion by regions

on-shell integrals with two masses and large mass on-shell

$\rightsquigarrow$ two regions:

- hard: $q_i \sim m_t \rightsquigarrow$ expand in m_W

- soft: $q_i \sim m_W \rightsquigarrow$ expand in q

How does polarisation enter the game?

polarisation sum

$$\epsilon_{L\mu}^* \epsilon_{L\nu} + \epsilon_{+\mu}^* \epsilon_{+\nu} + \epsilon_{-\mu}^* \epsilon_{-\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}$$

$\rightsquigarrow$ numerator of W boson propagator

pick out individual terms:

$$\epsilon_{L\mu}^* \epsilon_{L\nu} = \frac{(m_W^2 p^\mu - p \cdot q q^\mu)(m_W^2 p^\nu - p \cdot q q^\nu)}{m_W^2 m_t^2 |\vec{q}|^2}$$

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complication: appearance of $|\vec{q}| = \sqrt{q_0^2 - m_W^2}$ in denominator

Soft region

$\rightsquigarrow |\vec{q}|^2 = q_0^2 - m_W^2$ cannot be expanded, but W loop factorises:

$\rightsquigarrow$ calculation just as in unpolarised case, but with different vacuum integral:

$$\int \frac{d^d q}{(q^2 - m_W^2)(q_0^2 - m_W^2)} = i\pi^{d/2} \frac{2\Gamma(\epsilon)}{1-2\epsilon} \frac{1}{(m_W^2)^\epsilon}$$

Hard region

rewrite $|\vec{q}|^2$ in terms of p^μ and q^μ

$$|\vec{q}|^2 = q_0^2 - m_W^2 = \frac{(2p \cdot q)^2}{4m_t^2} - m_W^2$$

we only need the imaginary part $\rightsquigarrow q^2 = m_W^2$:

$$|\vec{q}|^2 = \frac{1}{4m_t^2} (q^2 + 2p \cdot q - m_W^2)^2 - m_W^2$$

expansion in terms of top propagators

$$\frac{1}{|\vec{q}|^2} = \frac{4m_t^2}{(q^2 + 2p \cdot q)^2} \sum_{i=0}^{\infty} \left(\frac{\dots}{(q^2 + 2p \cdot q)^2} \right)^i$$

Results

- checked expansion of $\mathcal{O}(\alpha_s)$ up to $(m_W/m_t)^{16}$
- obtained analytical results for all three-loop master integrals
- calculated $\mathcal{O}(\alpha_s^2)$ corrections up to $(m_W/m_t)^{10}$

Numerical evaluation

Helicity fractions

$$\mathcal{F}_L = 0.6971 (1 - 0.0108 - 0.0034) = 0.6872(6)$$

$$\mathcal{F}_- = 0.3029 (1 + 0.0214 - 0.0070) = 0.3115(5)$$

$$\mathcal{F}_+ = 0 + 0.00103 + 0.00023 = 0.00126(6)$$

estimated error from expansion: $\sim 10^{-6}$

expected uncertainties at LHC:

$$\Delta\mathcal{F}_L = 1.9\%, \quad \Delta\mathcal{F}_- = 1.8\%, \quad \Delta\mathcal{F}_+ = 0.21\%$$

$\mathcal{O}(\alpha_s^2)$ correction as function of m_W/m_t

$$\frac{\Gamma_L^{(2)}(0.466)}{\alpha_s^2 C_F G_0} = -\textcolor{red}{50.316} + \textcolor{green}{12.214} + \textcolor{blue}{1.736} - \textcolor{magenta}{0.381} - \textcolor{cyan}{0.044} - 0.003 + \dots$$

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$$\frac{\Gamma_L^{(2)}(0.466)}{\alpha_s^2 C_F G_0} = -50.316 + 12.214 + 1.736 - 0.381 - 0.044 - 0.003 + \dots$$

Part II

Mass effects in semi-leptonic b decays

Alexey Pak, Andrzej Czarnecki

arXiv: 0803.0960 [hep-ph]

Introduction

Semi-leptonic B-meson decay

- Clean decay
- HQET: $\Gamma(b \rightarrow c \ell \bar{\nu})$
- $\mathcal{O}(\alpha_s^2) \Rightarrow |\mathbf{V}_{cb}|, \dots$
- $m_c/M_b \sim 0.3$

Introduction

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Muon decay

- $\Gamma(\mu \rightarrow e\nu\bar{\nu}) \sim G_F^2$
- MuLan@PSI: $< \text{ppm}$
- Need $\mathcal{O}(\alpha^2)$
- $m_e/M_\mu \sim 0.005$

Studies of mass effects at NNLO

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- various expansions

Czarnecki, Melnikov, Blokland, ...

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- $m_q = 0$ van Ritbergen, Stuart

Studies of mass effects at NNLO

- various expansions
Czarnecki, Melnikov, Blokland,...
- $m_q = 0$ van Ritbergen, Stuart
- expansion in m_q/M_Q
Pak, Czarnecki
- agrees with Melnikov

Structure of the result

$$\begin{aligned}
 X_c = & -\frac{1009}{288} + \frac{8}{3} \zeta_3 + \frac{77}{216} \pi^2 \\
 & - \rho \left\{ \frac{5}{4} \pi^2 \right\} \\
 & + \rho^2 \left\{ \frac{145}{3} + \frac{16}{3} \pi^2 + \frac{52}{3} \ln \rho - 8 \ln^2 \rho \right\} \\
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- $1/\epsilon^3$ poles at ρ^4 and further
 $\rightsquigarrow$ additional terms of hardest masters integrals needed

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- $1/\epsilon^3$ poles at ρ^4 and further $\rightsquigarrow$ additional terms of hardest masters integrals needed
- linear term unexpected

Mass definition

Origin of linear term:
region with soft momentum ($k \sim m_q$)

Calculation: pole mass scheme

QCD: soft region $m_q \sim \Lambda_{\text{QCD}}$ cannot be treated perturbatively

Short-distance mass absorbs linear term:

$$\frac{G_F^2 |V_{cb}|^2 \cancel{M_Q^5}}{192\pi^3} (\dots) \quad \text{vs.} \quad \cancel{5} \frac{\pi^2}{4} \frac{m_q}{M_Q}$$

QED: perturbative at any scale $\rightsquigarrow$ pole mass for electron and muon

Correction of -0.43 ppm $\rightsquigarrow$ relevant for MuLan experiment

Summary

- applied optical theorem to polarised decay
- calculated helicity fractions of W at NNLO
- results relevant for LHC
- calculated NNLO mass effects for semi-leptonic decay rate and distribution moments
- results relevant for determination of V_{cb} and G_F

BLM

estimate of higher order effects: [Brodsky, Lepage, Mackenzie '82]

$$\Gamma_L^{(3)} = -0.0186_{\text{BLM}} + 0.0043 = -0.0143$$

$$\Gamma_-^{(3)} = -0.0052_{\text{BLM}} + 0.0013 = -0.0039$$

$$\Gamma_+^{(3)} = 0.00029_{\text{BLM}} - 0.00013 = 0.00016$$

Master integrals

analytical results via:

- cutting rules
- differential equations

[Cutkosky '60]

[Kotikov '91] [Gehrmann, Remiddi '00]

Optical theorem

Optical theorem relates decay width to imaginary part of top quark self-energy:

Advantages:

- real radiation is automatically taken into account
- “technology” for multi-loop calculations can be used

Expansion by regions

general rules:

- use dimensional regularisation
- identify all regions which contribute to the given integral
- expand integrand in small quantities of each region
- set scaleless integrals to zero
- integrate loop momenta from $-\infty$ to $+\infty$
- sum contributions from all regions

[Smirnov, *Applied Asymptotic Expansions in Momenta and Masses*]

Higher orders

consider both regions for all loop momenta

↪ integrals become scaleless if gluon momentum is soft:

$$\int \frac{d^d k d^d q}{k^2 (k^2 + 2p \cdot k) (p + k + q)^2 (p + q)^2 (q^2 - m_W^2)}$$

$$\sim \int \frac{d^d k}{k^2 (2p \cdot k)} \times \begin{cases} \int \frac{d^d q}{((p+q)^2)^2 q^2} & q \text{ hard} \\ \frac{1}{(p^2)^2} \int \frac{d^d q}{(q^2 - m_W^2)} & q \text{ soft} \end{cases} + \dots$$

Treatment of γ_5

- replace γ_5 with ε tensor:

$$\gamma_\mu \gamma_5 \rightarrow \frac{i}{3!} \varepsilon_{\mu\alpha\beta\delta} \gamma^\alpha \gamma^\beta \gamma^\delta$$

- contract ε tensor with projector $\rightsquigarrow$ metric tensors
- renormalise axial-vector current
- restore anti-commutativity with additional finite renormalisation

[Larin, Vermaseren '91]

Unpolarised case

average over spin states of top quark

$$\bar{u}(p)[\dots]u(p) \rightarrow \frac{1}{2} \text{Tr} \{ (\not{p} + m_t) [\dots] \}$$

sum over polarisations of W boson

$$\sum_{\text{polarisations}} \epsilon_{\mu}^*(q) \epsilon_{\nu}(q) = - \left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{m_W^2} \right)$$

→ numerator of W propagator in unitary gauge

Longitudinally polarised W boson

kinematics

top quark is at rest: $p^\mu = (m_t, 0, 0, 0)$

W momentum in z direction: $q^\mu = (q_0, 0, 0, |\vec{q}|)$

polarisation vector: $\epsilon_L^\mu(q) = (|\vec{q}|, 0, 0, q_0) / m_W$

$$\Rightarrow \epsilon_L^\mu = - \frac{m_W^2 p^\mu - p \cdot q q^\mu}{m_W m_t |\vec{q}|}$$

$$\rightsquigarrow \epsilon_L^{\mu*}(q) \epsilon_L^\nu(q) = \frac{(m_W^2 p^\mu - p \cdot q q^\mu)(m_W^2 p^\nu - p \cdot q q^\nu)}{m_W^2 m_t^2 |\vec{q}|^2}$$

Polarisation

use projection operators:

$$\mathbb{P}_0^{\mu\nu}(q) = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2}$$

$$\mathbb{P}_L^{\mu\nu}(q) = \frac{(m_W^2 p^\mu - p \cdot q q^\mu)(m_W^2 p^\nu - p \cdot q q^\nu)}{m_W^2 m_t^2 |\vec{q}|^2}$$

$$\mathbb{P}_F^{\mu\nu}(q) = -\frac{i}{m_t |\vec{q}|} \varepsilon^{\mu\nu\sigma\rho} p_\sigma q_\rho$$

$$\mathbb{P}_\pm^{\mu\nu} = \mathbb{P}_0^{\mu\nu} - \mathbb{P}_L^{\mu\nu} \pm \mathbb{P}_F^{\mu\nu}$$

complications:

- appearance of $|\vec{q}| = \sqrt{q_0^2 - m_W^2}$ in denominator
- treatment of γ_5 in dimensional regularisation