**SCALING PROPERTIES OF VORTEX RING FORMATION
AT A CIRCULAR TUBE OPENING**

By

Monika Nitsche

IMA Preprint Series # 1376

January 1996

**INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
UNIVERSITY OF MINNESOTA**

**514 Vincent Hall
206 Church Street S.E.
Minneapolis, Minnesota 55455**

Scaling properties of vortex ring formation at a circular tube opening

Monika Nitsche

*Institute for Mathematics and its Applications
University of Minnesota, Minneapolis, MN 55455*

ABSTRACT. A vortex sheet model is used to study vortex ring formation at the edge of a circular tube. We determine properties of the vortex ring as a function of the generating piston motion and investigate the extent to which similarity theory for planar vortex sheet separation applies. We find that the ring diameter, core size and circulation are well predicted by the planar theory, even at large times when the ring has travelled significantly downstream. The axial ring translation is a superposition of an upstream component predicted by the theory and a downstream component which is linear in the piston stroke. The front of the fluid volume exiting the tube is also linear in the piston stroke and travels with 75% of the piston velocity.

I. INTRODUCTION

A typical experiment of vortex ring formation at a tube opening is shown in Fig. 1a. It consists of a circular tube immersed in fluid and a piston inside the tube which moves, ejecting fluid from the opening. This causes the boundary layer on the inner tube wall to separate at the edge as an axisymmetric shear layer. The separated layer rolls up and forms a vortex ring. One objective of vortex ring experiments has been to describe the ring properties as a function of the generating conditions (Shariff & Leonard¹). For the tube geometry, the relevant condition is the piston velocity. In this paper we use a numerical vortex sheet model to investigate the dependence of the ring trajectory, circulation, size and shape on piston velocities of the form $U_p(t) \sim t^m$.

Theoretical predictions based on similarity theory exist for the planar flow shown in Fig 1b. Here, flow past a semi-infinite flat plate causes the separation and roll-up of a planar shear layer. After an initial time-interval the viscous shear layer thickness is small relative to the length scales of the flow and the separated layer is well approximated by a vortex sheet. The vortex sheet separation and roll-up is known to be self-similar: for starting flows that satisfy a power law in time, the position of the spiral center, the spiral size and the shed circulation have known power law behaviour. Pullin² discusses the similarity theory and computes the self-similar planar separation using a numerical method.

The similarity results are based on the form of the potential flow past the plate. Near the edge, the axisymmetric potential flow out of a tube is similar to the planar flow past the plate. It is therefore plausible to assume that at small times the axisymmetric separation in Fig. 1a is approximated by the planar separation in Fig. 1b. The planar similarity theory can then be used to predict the vortex ring trajectory, shape and circulation, for the case of power law piston velocity (Saffman³, Pullin⁴). However, there is a significant difference between the two flows. In the planar case, the self-similar spiral center travels along a straight line with negative horizontal velocity (dotted line in Fig. 1b). The vortex ring on the other hand has a downstream velocity component and travels along a curved trajectory with positive axial velocity (dotted curve in Fig. 1a). The position of the vortex significantly affects for example the circulation shedding rate at the edge. It is thus not clear a priori to what extent the planar similarity theory describes the axisymmetric ring formation process.

Nitsche & Krasny⁵ developed a vortex sheet model for vortex ring formation at the edge of a circular tube and simulated an experiment by Didden⁶. Comparisons with the experimental measurements showed that the model accurately recovers the formation process. In this paper we apply the model to simulate the vortex ring formation for piston velocities $U_p \sim t^m$. We consider $m = 0, 1/2, 1, 2$. We investigate the flow and determine the extent to which it is described by the planar similarity theory.

II. SIMILARITY THEORY

Pullin² derives the similarity laws governing vortex sheet separation and roll-up at the edge of a flat plate. The vortex sheet travels in an otherwise potential flow and Pullin considers flow which grows in time like t^m . To leading order in an expansion near the edge of the plate, the velocity potential for attached flow is

$$\phi(r, \theta) = -i a t^m r^{1/2} \sin(\theta/2) . \quad (1)$$

Here, (r, θ) are polar coordinates centered at the edge and a is a dimensional constant. It follows from dimensional analysis that the center of the separated spiral vortex sheet, $z_c = x_c + i y_c$, and the total shed circulation Γ , satisfy

$$z_c(t) = \omega(m) \left[\frac{3a/4}{m+1} \right]^{2/3} t^{\frac{2}{3}(m+1)} , \quad \Gamma(t) = J(m) \left[\frac{3a^4/4}{m+1} \right]^{1/3} t^{\frac{4}{3}(1+m)-1} . \quad (2)$$

Here ω, J are nondimensional constants. Pullin then computes the shape of the self-similar vortex sheet roll-up. Under the scaling laws (2), the governing equations reduce to an integro-differential equation independent of time. Pullin solves this equation by approximating the shed vortex sheet by a finite number of outer turns and representing the remaining inner spiral core as a point vortex. He records the solution for a range of values for m . Krasny⁷ solves the time dependent problem using a planar vortex sheet model and finds good agreement with Pullin's solution.

Pullin⁴ applies the planar similarity theory to axisymmetric flow out of a circular tube. For piston velocities

$$U_p(t) = U_o t^m , \quad (3)$$

the velocity potential near the tube opening has the same form, to leading order, as in the planar case (1). Using the dimensions of the constant a in (1) and a computed estimate for the magnitude of the potential, Pullin finds that $a = U_o(D_o/2\pi)^{1/2}$, where D_o is the tube diameter. It is thus expected that the axisymmetric separation approximately satisfies (2) with this value of a .

We now change to the nondimensional variables

$$\hat{t} = \frac{\overline{U}_o t}{D_o} , \quad \hat{\Gamma} = \frac{\Gamma}{\overline{U}_o D_o} , \quad \hat{z}_c = \frac{z_c}{D_o} . \quad (4)$$

Here $\overline{U}_o$ is the average velocity at time t , $\overline{U}_o = \frac{1}{t} \int_0^t U_p(\xi) d\xi$, and $\hat{t}$ equals the nondimensional piston stroke at time t . With this change of variables and the given value of a , (2) reduces to

$$\hat{z}_c = C_{z_c} \hat{t}^{2/3} , \quad \hat{\Gamma} = C_{\Gamma} \hat{t}^{1/3} . \quad (5)$$

where $C_{z_c} = C_{x_c} + i C_{y_c} = \omega(m)(9/32\pi)^{1/3}$ and $C_{\Gamma} = J(m)(1+m)(3/16\pi^2)^{1/3}$. Throughout the rest of this paper, all variables are nondimensionalized as in (4).

III. NUMERICAL METHOD AND SOLUTION

The vortex sheet model and its numerical implementation are discussed in detail in Nitsche & Krasny⁵ and will only be briefly described here. The tube wall and back are modelled by a bound vortex sheet whose strength is such that the flow is tangent to the wall and equals the piston velocity in the rear of the tube. The separated shear layer is modelled by a free vortex sheet. Both the bound and the free vortex sheet are discretized by circular vortex filaments. The vortex shedding is simulated by releasing a filament from the edge at each time-step, with velocity equal

to the average velocity at the edge and circulation given by Prandtl's slip model for sharp edge separation⁸. The spiral roll-up of the free vortex sheet is resolved using the vortex blob method. Here, a smoothing parameter δ is introduced into the governing equations. The computations are performed with $\delta > 0$ and the vortex sheet is obtained from the limit $\delta \rightarrow 0$.

Figures 2 and 3 show the solution computed for impulsively started piston motion, $m = 0$. Each plot shows a section of the tube and a curve connecting the discrete shed vortex filaments. The coordinate system is centered as indicated in Fig. 1a. Figure 2 shows the solution computed with a fixed value of δ . As time increases, a spiral develops and the number of turns increases. To resolve the roll-up at small times, the flow is computed with smaller values of δ . As an example, Fig. 3 shows the solution at $\hat{t} = 0.0002$ computed with a decreasing sequence of δ . We note that δ , which scales as a length, has also been nondimensionalized.

IV. COMPARISON WITH SIMILARITY THEORY

We investigate the behaviour of various quantities. Figure 4 shows the definition of the vortex center coordinates x_c, y_c , the spiral half diameter d_s , and the distance L travelled by a particle initially on the axis in the tube exit plane. The dashed curve denotes the position of a material curve initially across the opening. Together with the free vortex sheet (solid curve), the dashed curve therefore bounds the fluid initially inside the tube from the fluid initially outside the tube. Γ is the total circulation shed from the edge at time t . Figures 5-7 describe the computed behaviour of these quantities, for impulsively started piston motion, $m = 0$. The variables y_c , d_s , L and Γ are monotonically increasing in time and are treated first; the axial ring coordinate x_c is treated separately. We then discuss the shape of the sheet with varying m .

A. Radial coordinate y_c , spiral diameter d_s , circulation Γ and distance L

Figure 5 shows log-log plots of the computed quantities $\hat{y}_c$, $\hat{d}_s$, $\hat{\Gamma}$ and $\hat{L}$, as functions of the piston stroke $\hat{t}$. For each quantity, various curves are shown, corresponding to values of $\delta \in [0.00002, 0.02]$. For each value of δ the curve is not plotted at early times for which no spiral turns have yet formed. The curves converge to an "enveloping" curve, which appears linear for $\hat{t} < 0.5$. Thus, the quantities obey a power law at these times,

$$\hat{q} = C_q \hat{t}^{p_q} . \quad (6)$$

Each subplot also shows a line whose slope closely approximates the one of the enveloping curve. In the cases (a,b,c) the slope of the line is the value predicted by planar similarity theory. The slope shown in (d) is the one that best fits the data. The computations for $m = 1/2, 1, 2$ are not shown here. The behaviour is qualitatively the same as the case $m = 0$ shown, with the curves differing by a vertical shift.

To quantify the power law behaviour observed in Fig. 5, we find the values p_q and C_q which best approximate the data in a least square sense. The least square approximation is performed over the interval $\hat{t} \in [0.000005, 0.5]$, which is the same interval over which the lines in Fig. 5 are drawn. The results for $m = 0, 1/2, 1, 2$ are shown in Table I.

Columns 2-5 in Table I record the power p_q , columns 6-9 record the constant C_q for the indicated quantities q . The values in parentheses are the values computed by Pullin² for the planar separation at the edge of a flat plate. They are obtained from his Figs. 8-9 (for d_s) and Figs. 11-13 (for J and $\omega = \rho_\nu e^{i\chi_{nu}}$). Note that Table I describes the behaviour of nondimensional variables as a function of the piston stroke $\hat{t}$. The behaviour of the dimensional variables in time t can be recovered from (4) and (5) and the definition of $\bar{U}_o$.

m	p_{y_c}	p_{d_s}	p_{Γ}	p_L	C_{y_c}	C_{d_s}	C_{Γ}	C_L
0	0.673	0.658	0.327	1.003	0.199 (0.18)	0.138 (0.15)	0.70 (0.70)	0.756
1/2	0.679	0.658	0.324	1.001	0.178 (0.16)	0.121	0.93 (0.93)	0.739
1	0.684	0.661	0.321	1.004	0.172 (0.14)	0.116 (0.13)	1.16 (1.17)	0.732
2	0.686	0.659	0.320	1.007	0.165 (0.13)	0.111	1.63 (1.67)	0.736

TABLE I. Exponents p_q and constants C_q for the indicated variables $\hat{q} = C_q \hat{t}^{p_q}$.

The computed powers p_{y_c} , p_{d_s} and p_{Γ} in Table I agree well with the similarity theory predictions of $2/3$, $2/3$ and $1/3$ respectively. The constants C_{y_c} , C_{d_s} , C_{Γ} are within 25%, 10% and 2% respectively of the values for planar separation computed by Pullin. The agreement holds until large times when the piston stroke is of the order of the tube diameter.

The front of the volume of fluid exiting the tube satisfies $\hat{L} \approx 0.75 \hat{t}$ for all values of m presented. Equivalently, it travels with approximately 75% of the piston velocity. Since the front is fairly flat, this implies that approximately 25% of the column of fluid leaving the tube opening is displaced and entrained by the vortex ring. There is currently no theoretical explanation for the observed behaviour of L .

B. Axial coordinate x_c

A sketch of the vortex ring trajectory is shown in Fig. 6 (solid curve). Almost immediately after leaving the edge the ring travels downstream, with positive velocity. In a small initial time-interval however the ring travels upstream, with negative velocity. The axial coordinate x_c is therefore negative at these small times, and positive at later times. Figure 7a) shows a log-log plot of the absolute value $|\hat{x}_c|$ vs. $\hat{t}$, computed with $m = 0$ and various values of $\delta \in [0.00002, 0.02]$. The dashed vertical line indicates the piston stroke $\hat{t} = \hat{t}_o$ at which the ring center crosses the tube exit plane after its initial upstream motion. This crossing time is small, with $\hat{t}_o = O(10^{-3})$.

Figure 7a also shows several lines with the indicated slopes. The initial upstream motion of x_c is seen to satisfy $\hat{x}_c \sim \hat{t}^{2/3}$, as predicted by similarity theory. The large time downstream motion appears to approach linear behaviour in the piston stroke. Note that in the transition from $\hat{x}_c \approx 0$ to the large time linear behaviour, the ring may appear to satisfy $\hat{x}_c \sim \hat{t}^p$ for various p , depending on what time-interval is considered. This may partially explain the differing experimental results observed. Didden⁶ quotes a rough estimate of $p = 1.5$, Weigand & Gharib⁹ observe $p \approx 1$. One needs to consider that in these experiments, the piston undergoes an initial acceleration period and does not satisfy $U_p \sim t^m$ over the time interval studied.

The initial upstream self-similar behaviour observed in Fig. 7a is indicated by the dashed line in Fig. 6. The difference between the observed trajectory and the initial self-similar trajectory is denoted by arrows. This difference $\hat{\alpha} = \hat{x}_c - C_{x_c} \hat{t}^{2/3}$ is shown in Fig. 7b, vs. $\hat{t}$. The value of C_{x_c} used is a rough estimate obtained from Fig. 7a. It is recorded in Table II together with the values computed by Pullin (in brackets). The curves in Fig. 7b are almost linear, suggesting that $\hat{\alpha}$ also satisfies a power law, $\hat{\alpha} = C_{\alpha} \hat{t}^{p_{\alpha}}$. A least squares approximation of the “enveloping curve”, defined to be the minimum over all values of δ , gives estimates for C_{α} , p_{α} . The values p_{α} recorded in Table II suggest that the downstream component is linear in the piston stroke and

$$\hat{x}_c \approx -C_{x_c} \hat{t}^{2/3} + C_{\alpha} \hat{t} . \quad (7)$$

Two factors contribute to the downstream velocity component. The potential flow near the tube opening has, to second order, a uniform downstream velocity component (Graham¹⁰) and the vortex ring has a self-induced downstream velocity. These components are not present in self-similar planar vortex sheet separation. An explanation for the linear behaviour observed for the overall downstream component remains to be found.

m	C_{x_c}	C_α	p_α
0	0.027 (0.041)	0.34	1.07
1/2	0.038 (0.061)	0.25	1.05
1	0.040 (0.064)	0.22	1.06
2	0.041 (0.070)	0.19	1.07

TABLE II. Estimates for axial component (6).

We remark that the times during which upstream ring motion is observed in our computations are so small that in a typical experiment, the flow is most likely dominated by viscosity at these times. It is however observed in experiments that the ring does not leave the exit plane at the edge of the tube but at a slightly larger diameter. The values C_{x_c} , C_{y_c} , C_α , p_α recorded in Tables I and II imply that the piston stroke $\hat{t}_o$ at which the ring leaves the exit plane and the diameter at this time increases with m . The values also imply that the upward angle at which the ring leaves the edge decreases with increasing m . This can be observed in Fig. 8a.

C. Shape of the roll-up

Figure 8a shows the computed vortex sheet at a small time during which the ring is still travelling upstream, for $m = 0, 1/2, 1, 2$. For comparison, Fig. 8b shows Pullin's³ solution for the self-similar planar roll-up at the edge of a flat plate, with $m = 0, 1, \infty$. The outer spiral turns in Figs. 8a and 8b are in good agreement. In both cases the ellipticity of the roll-up increases with m , the size of the roll-up decreases, and the angle at which the vortex leaves the edge decreases. The angle in our computations however is larger than in Pullin's planar solution.

The features observed in Fig. 8a are also observed at large times, when the ring has travelled downstream. The solution for piston stroke $\hat{t} = 0.5$ is shown in Fig. 9, together with the material curve which initially lies across the opening. The ellipticity and the tilt of the elliptical roll-up increases with m . The front of the fluid volume exiting the tube changes little with m , but the size of the roll-up decreases so that the distribution in the roll-up of fluid initially inside and initially outside the tube changes. As m increases, the entrained outer fluid is more concentrated near the spiral center.

V. SUMMARY

A numerical vortex sheet model was used to study vortex ring formation at the edge of a circular tube. We investigated the dependence of the ring trajectory, circulation, size and shape on the piston velocity, for velocities $U_p \sim t^m$, and $m = 0, 1/2, 1, 2$. The radial ring coordinate y_c , the core size d_s and the circulation Γ satisfy scaling laws predicted by similarity theory for planar vortex sheet separation. The values are in quantitative agreement with Pullin's computations for separation at the edge of a flat plate. The scaling laws hold until large times when the piston stroke is half the tube diameter and the ring has travelled a significant distance downstream. The front of the fluid exiting the tube, L , also satisfies a scaling law and travels with approximately 75% of the piston velocity. The axial component x_c is a superposition of an upstream component predicted by similarity theory and a downstream component which is linear in the piston stroke. The downstream component is associated to the ring's self-induced velocity and to the downstream component of the starting potential flow.

ACKNOWLEDGMENTS

The author thanks Alex Weigand for a discussion of the scaling behaviour and Robert Krasny for comments on the original manuscript. This research was partially supported by NSF Grant DMS-9408697 and by an industrial postdoctoral membership at the Institute for Mathematics and its Applications. The computations were performed at the IMA.

REFERENCES

- ¹K. Shariff & A. Leonard, "Vortex rings," Ann. Rev. Fluid Mech. **24**, 235 (1992).
- ²D.I. Pullin, "The large-scale structure of unsteady self-similar rolled-up vortex sheets", J. Fluid Mech. **88**, 401 (1978).
- ³P.G. Saffman, "The number of waves on unstable vortex rings", J. Fluid Mech. **84**, 625 (1978).
- ⁴D.I. Pullin, "Vortex ring formation at tube and orifice openings", Phys. Fluids **22**, 401 (1979).
- ⁵M. Nitsche & R. Krasny, "A numerical study of vortex ring formation at the edge of a circular tube", J. Fluid Mech. **276**, 139 (1994).
- ⁶N. Didden, "On the formation of vortex rings: rolling-up and production of circulation", J. of Applied Math. and Phys. (Z. Angew. Math. Phys.) **30**, 101. 30,101 (1979).
- ⁷R. Krasny, "Vortex sheet computations: roll-up, wakes, separation", Lect. Appl. Math. **28**, 385 (1991).
- ⁸L. Prandtl & O.G. Tietjens, *Fundamentals of hydro- and aeromechanics*, (McGraw-Hill, 1934), §93.
- ⁹A. Weigand & M. Gharib, "On the evolution of laminar vortex rings", preprint.
- ¹⁰J.M.R. Graham, 1977 Vortex shedding from sharp edges, Imperial College Report 77-06, ISSN 0308 7247 (1977).

Fig. 1 : Sketch. (a) Vortex ring formation at the edge of a circular tube. (b) Vortex sheet separation at the edge of a flat plate.

Fig. 2 : Computed solution at the indicated values of $\hat{t}$, with $\delta = 0.001$ and $m = 0$.

Fig. 3 : Computed solution at $\hat{t} = 0.0002$, with the indicated values of δ ($m = 0$).

Fig. 4 : Sketch.

Fig. 5 : Log-log plots, for values of $\delta \in [0.00002, 0.02]$ and $m = 0$, vs. $\hat{t}$. a) $\hat{y}_c$, b) $\hat{d}_s$, c) $\hat{\Gamma}$, d) $\hat{L}$.

Fig. 6 : Sketch. Observed vortex ring trajectory (solid curve) and extension of the initial upstream trajectory (dashed line). The horizontal line denotes the edge of the tube.

Fig. 7 : Log-log plots vs. $\hat{t}$, and $m = 0$. a) $|\hat{x}_c|$, b) $\hat{x}_c - C_{x_c} \hat{t}^{2/3}$, where C_{x_c} is estimated from the small time behaviour in Fig. 7a.

Fig. 8 : a) Computed solution at $\hat{t} = 0.0002$, with $\delta = 0.00004$ and the indicated values of m . b) Pullin's solution for the self-similar planar roll-up at the edge of a plate (reproduced from Pullin 1978).

Fig. 9 : Computed solution at $\hat{t} = 0.5$, with $\delta = 0.01$ and the indicated values of m .

#	Author/s	Title
1282	V. Jakšić & C.-A. Pillet,	On model for quantum friction II. Fermi's golden rule and dynamics at positive temperatures
1283	V. M. Malkin,	Kolmogorov and nonstationary spectra of optical turbulence
1284	E.G. Kalnins, V.B. Kuznetsov & W. Miller, Jr.,	Separation of variables and the XXZ Gaudin magnet
1285	E.G. Kalnins & W. Miller, Jr.,	A note on tensor products of q -algebra representations and orthogonal polynomials
1286	E.G. Kalnins & W. Miller, Jr.,	q -algebra representations of the Euclidean, pseudo-Euclidean and oscillator algebras, and their tensor products
1287	L.A. Pastur,	Spectral and probabalistic aspects of matrix models
1288	K. Kastella,	Discrimination gain to optimize detection and classification
1289	L.A. Peletier & W.C. Troy,	Spatial patterns described by the Extended Fisher-Kolmogorov (EFK) equation: Periodic solutions
1290	A. Friedman & Y. Liu,	Propagation of cracks in elastic media
1291	A. Friedman & C. Huang,	Averaged motion of charged particles in a curved strip
1292	G. R. Sell,	Global attractors for the 3D Navier-Stokes equations
1293	C. Liu,	A uniqueness result for a general class of inverse problems
1294	H-O. Kreiss,	Numerical solution of problems with different time scales II
1295	B. Cockburn, G. Gripenberg, S-O. Londen,	On convergence to entropy solutions of a single conservation law
1296	S-H. Yu,	On stability of discrete shock profiles for conservative finite difference scheme
1297	H. Behncke & P. Rejto,	A limiting absorption principle for separated Dirac operators with Wigner Von Neumann type potentials
1298	R. Lipton B. Vernescu,	Composites with imperfect interface
1299	E. Casas,	Pontryagin's principle for state-constrained boundary control problems of semilinear parabolic equations
1300	G.R. Sell,	References on dynamical systems
1301	J. Zhang,	Swelling and dissolution of polymer: A free boundary problem
1302	J. Zhang,	A nonlinear nonlocal multi-dimensional conservation law
1303	M.E. Taylor,	Estimates for approximate solutions to acoustic inverse scattering problems
1304	J. Kim & D. Sheen,	A priori estimates for elliptic boundary value problems with nonlinear boundary conditions
1305	B. Engquist & E. Luo,	New coarse grid operators for highly oscillatory coefficient elliptic problems
1306	A. Boutet de Monvel & I. Egorova,	On the almost periodicity of solutions of the nonlinear Schrödinger equation with the cantor type spectrum
1307	A. Boutet de Monvel & V. Georgescu,	Boundary values of the resolvent of a self-adjoint operator: Higher order estimates
1308	S.K. Patch,	Diffuse tomography modulo Graßmann and Laplace
1309	A. Friedman & J.J.L. Velázquez,	Liouville type theorems for fourth order elliptic equations in a half plane
1310	T. Aktosun, M. Klaus & C. van der Mee,	Recovery of discontinuities in a nonhomogeneous medium
1311	V. Bondarevsky,	On the global regularity problem for 3-dimensional Navier-Stokes equations
1312	M. Cheney & D. Isaacson,	Inverse problems for a perturbed dissipative half-space
1313	B. Cockburn, D.A. Jones & E.S. Titi,	Determining degrees of freedom for nonlinear dissipative equations
1314	B. Engquist & E. Luo,	Convergence of a multigrid method for elliptic equations with highly oscillatory coefficients
1315	L. Pastur & M. Shcherbina,	Universality of the local eigenvalue statistics for a class of unitary invariant random matrix ensembles
1316	V. Jakšić, S. Molchanov & L. Pastur,	On the propagation properties of surface waves
1317	J. Nečas, M. Ružička & V. Šverák,	On self-similar solutions of the Navier-Stokes equations
1318	S. Stojanovic,	Remarks on $W^{2,p}$ -solutions of bilateral obstacle problems
1319	E. Luo & H-O. Kreiss,	Pseudospectral vs. Finite difference methods for initial value problems with discontinuous coefficients
1320	V.E. Grikurov,	Soliton's rebuilding in one-dimensional Schrödinger model with polynomial nonlinearity
1321	J.M. Harrison & R.J. Williams,	A multiclass closed queueing network with unconventional heavy traffic behavior
1322	M.E. Taylor,	Microlocal analysis on Morrey spaces
1323	C. Huang,	Homogenization of biharmonic equations in domains perforated with tiny holes
1324	C. Liu,	An inverse obstacle problem: A uniqueness theorem for spheres
1325	M. Luskin,	Approximation of a laminated microstructure for a rotationally invariant, double well energy density
1326	Rakesh & P. Sacks,	Impedance inversion from transmission data for the wave equation
1327	O. Lafitte,	Diffraction for a Neumann boundary condition
1328	E. Sobel, K. Lange, J.R. O'Connell & D.E. Weeks,	Haplotyping algorithms
1329	B. Cockburn, D.A. Jones & E.S. Titi,	Estimating the number of asymptotic degrees of freedom for nonlinear dissipative systems
1330	T. Aktosun,	Inverse Schrödinger scattering on the line with partial knowledge of the potential

- 1331 T. Aktosun & C. van der Mee, Partition of the potential of the one-dimensional Schrödinger equation
- 1332 B. Engquist & E. Luo, Convergence of the multigrid method with a wavelet coarse grid operator
- 1333 V. Jakšić & C.-A. Pillet, Ergodic properties of the Spin-Boson system
- 1334 S.K. Patch, Recursive solution for diffuse tomographic systems of arbitrary size
- 1335 J.C. Bronski, Semiclassical eigenvalue distribution of the non self-adjoint Zakharov-Shabat eigenvalue problem
- 1336 J.C. Cockburn, Bitangential structured interpolation theory
- 1337 S. Kichenassamy, The blow-up problem for exponential nonlinearities
- 1338 F.A. Grünbaum & S.K. Patch, How many parameters can one solve for in diffuse tomography?
- 1339 R. Lipton, Reciprocal relations, bounds and size effects for composites with highly conducting interface
- 1340 H.A. Levine & J. Serrin, A global nonexistence theorem for quasilinear evolution equations with dissipation
- 1341 A. Boutet de Monvel & R. Purice, The conjugate operator method: Application to DIRAC operators and to stratified media
- 1342 G. Michele Graf, Stability of matter through an electrostatic inequality
- 1343 G. Avalos, Sharp regularity estimates for solutions of the wave equation and their traces with prescribed Neumann data
- 1344 G. Avalos, The exponential stability of a coupled hyperbolic/parabolic system arising in structural acoustics
- 1345 G. Avalos & I. Lasiecka, A differential Riccati equation for the active control of a problem in structural acoustics
- 1346 G. Avalos, Well-posedness for a coupled hyperbolic/parabolic system seen in structural acoustics
- 1347 G. Avalos & I. Lasiecka, The strong stability of a semigroup arising from a coupled hyperbolic/parabolic system
- 1348 A.V. Fursikov, Certain optimal control problems for Navier-Stokes system with distributed control function
- 1349 F. Gesztesy, R. Nowell & W. Pötz, One-dimensional scattering theory for quantum systems with nontrivial spatial asymptotics
- 1350 F. Gesztesy & H. Holden, On trace formulas for Schrödinger-type operators
- 1351 X. Chen, Global asymptotic limit of solutions of the Cahn-Hilliard equation
- 1352 X. Chen, Lorenz equations, Part I: Existence and nonexistence of homoclinic orbits
- 1353 X. Chen, Lorenz equations Part II: "Randomly" rotated homoclinic orbits and chaotic trajectories
- 1354 X. Chen, Lorenz equations, Part III: Existence of hyperbolic sets
- 1355 R. Abeyaratne, C. Chu & R.D. James, Kinetics of materials with wiggly energies: Theory and application to the evolution of twinning microstructures in a Cu-Al-Ni shape memory alloy
- 1356 C. Liu, The Helmholtz equation on Lipschitz domains
- 1357 G. Avalos & I. Lasiecka, Exponential stability of a thermoelastic system without mechanical dissipation
- 1358 R. Lipton, Heat conduction in fine scale mixtures with interfacial contact resistance
- 1359 V. Odisharia & J. Peradze, Solvability of a nonlinear problem of Kirchhoff shell
- 1360 P.J. Olver, G. Sapiro & A. Tannenbaum, Affine invariant edge maps and active contours
- 1361 R.D. James, Hysteresis in phase transformations
- 1362 A. Sei & W. Symes, A note on consistency and adjointness for numerical schemes
- 1363 A. Friedman & B. Hu, Head-media interaction in magnetic recording
- 1364 A. Friedman & J.J.L. Velázquez, Time-dependent coating flows in a strip, part I: The linearized problem
- 1365 X. Ren & M. Winter, Young measures in a nonlocal phase transition problem
- 1366 K. Bhattacharya & R.V. Kohn, Elastic energy minimization and the recoverable strains of polycrystalline shape-memory materials
- 1367 G.A. Chechkin, Operator pencil and homogenization in the problem of vibration of fluid in a vessel with a fine net on the surface
- 1368 M. Carme Calderer & B. Mukherjee, On Poiseuille flow of liquid crystals
- 1369 M.A. Pinsky & M.E. Taylor, Pointwise Fourier inversion: A wave equation approach
- 1370 D. Brandon & R.C. Rogers, Order parameter models of elastic bars and precursor oscillations
- 1371 H.A. Levine & B.D. Sleeman, A system of reaction diffusion equations arising in the theory of reinforced random walks
- 1372 B. Cockburn & P.-A. Gresh, A priori error estimates for numerical methods for scalar conservation laws. Part II: Flux-splitting monotone schemes on irregular Cartesian grids
- 1373 B. Li & M. Luskin, Finite element analysis of microstructure for the cubic to tetragonal transformation
- 1374 M. Luskin, On the computation of crystalline microstructure
- 1375 J.P. Matos, On gradient young measures supported on a point and a well
- 1376 M. Nitsche, Scaling properties of vortex ring formation at a circular tube opening
- 1377 J.L. Bona & Y.A. Li, Decay and analyticity of solitary waves
- 1378 V. Isakov, On uniqueness in a lateral Cauchy problem with multiple characteristics
- 1379 M.A. Kouritzin, Averaging for fundamental solutions of parabolic equations
- 1380 T. Aktosun, M. Klaus & C. van der Mee, Integral equation methods for the inverse problem with discontinuous wave speed