**NONLINEAR BOUNDARY STABILIZATION OF A VON KÁRMÁN
PLATE VIA BENDING MOMENTS ONLY**

By

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NONLINEAR BOUNDARY STABILIZATION OF A VON KÁRMÁN PLATE VIA BENDING MOMENTS ONLY

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Continued study in the areas of exact controllability and uniform stabilization of partial differential equations has been spurred by current research problems such as the deployment of large scale flexible structures in space. How can vibrations be damped out or suppressed in a newly designed space station? Plate equations such as the von Kármán plate can be used as a model for this problem as well as others. At the same time, problems such as this, which are motivated by real world applications, raise new questions in the area of mathematics and, in particular, stability theory for partial differential equations.

Recently, much attention has been directed toward the problem of uniform stabilization of plate equations. A qualitative definition of uniform stability of a plate is as follows: *If a damping term is introduced into the system, preferably acting through all or part of the boundary, the energy of the system, defined in an appropriate function space, decays uniformly with respect to the energy of the initial state of the plate as time increases.* The preference for boundary control in the above definition, as opposed to interior control, is motivated by the fact that boundary controls, though mathematically more challenging, are more easily implemented as they need to act only on the boundary of the spatial domain.

1 Statement of the Problem

We consider a fully nonlinear von Kármán system with, in addition to the nonlinearity which appears in the equation, a nonlinear feedback control acting through the boundary

as a moment. Let Ω be an open bounded domain in R^2 with a sufficiently smooth boundary, Γ . In Ω , we consider the following von Kármán system in the variables $w(t, x)$ and $\chi(w(t, x))$ with a nonlinear feedback control, g :

$$w_{tt} - \gamma^2 \Delta w_{tt} + \Delta^2 w + b(x)w_t = [w, \chi(w)] \quad \text{in } Q_\infty = (0, \infty) \times \Omega \quad (1.a)$$

$$\left. \begin{aligned} w(0, \cdot) &= w_0 \\ w_t(0, \cdot) &= w_1 \end{aligned} \right\} \quad \text{in } \Omega \quad (1.b)$$

$$w = 0 \quad \text{on } \Sigma_\infty = (0, \infty) \times \Gamma \quad (1.c)$$

$$\Delta w + (1 - \mu)\mathcal{B}w = -g\left(\frac{\partial}{\partial \nu} w_t\right) \quad \text{on } \Sigma_\infty = (0, \infty) \times \Gamma, \quad (1.d)$$

where $b(x) \in L^\infty(\Omega)$ satisfies $b(x) > 0$ a.e. in Ω , $0 < \mu < \frac{1}{2}$ is Poisson's ratio, and the parameter, γ , is proportional to the thickness of the plate and is therefore assumed to be small. The operator $\mathcal{B}$ is given by

$$\mathcal{B}w \equiv -\frac{\partial^2}{\partial \tau^2} w - k \frac{\partial}{\partial \nu} w = -k \frac{\partial}{\partial \nu} w, \quad (1.2)$$

where k is the geodesic curvature of the boundary and the second equality follows from (1.1.c). Additionally, we assume the control, g , is a continuous, monotone increasing function and is subject to the following constraints:

$$\left. \begin{aligned} g(s)s &> 0 && \text{for } s \neq 0 \\ m|s| &\leq |g(s)| \leq M|s| && \text{for } |s| > 1. \end{aligned} \right\} \quad (H)$$

Notice that no growth assumptions are made on the behavior of g at the origin. This is in contrast to most of the literature related to the subject (see [6], [7], etc.).

In (1.1), $\chi(w)$ satisfies the system of equations

$$\left. \begin{aligned} \Delta^2 \chi &= -[w, w] && \text{in } \Omega \\ \chi &= \frac{\partial}{\partial \nu} \chi = 0 && \text{on } \Gamma, \end{aligned} \right\} \quad (1.3)$$

where

$$[\phi, \psi] = \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \phi}{\partial y^2} \frac{\partial^2 \psi}{\partial x^2} - 2 \frac{\partial^2 \phi}{\partial x \partial y} \frac{\partial^2 \psi}{\partial x \partial y}. \quad (1.4)$$

Physically, the boundary conditions represent a “simply supported” plate, i.e., the position of the boundary remains fixed while the plate is allowed to rotate about the tangent to the boundary. The term corresponding to γ^2 in (1.1.a) represents the rotational inertia of the plate. We note that without this term, i.e., when $\gamma \equiv 0$, only recently has the solution to system (1.1) been proven to be unique (see [3]).

Our goal is to show that the boundary control, g , causes the energy of our system, $E_w(t)$, defined in some appropriate topology, to decay uniformly with respect to the initial energy as time increases. Although our model contains some light internal damping in the interior, represented by the term $b(x)w_t$, alone it is not enough to uniformly stabilize the model. However, this mild damping term plays a critical role in the proof of the compactness/uniqueness argument. Without it, there is no apparent way to show that the boundary feedback alone is sufficient to uniformly stabilize the model.

We define the energy functional by

$$E_w(t) \equiv \frac{1}{2} \int_{\Omega} \{|w_t|^2 + \gamma^2 |\nabla w_t|^2 + |\Delta w|^2 + |\Delta \chi(w)|^2\} d\Omega \equiv E_{w,1}(t) + E_{w,2}(t), \quad (1.5)$$

where $E_{w,2}(t)$ is defined by

$$E_{w,2}(t) \equiv \frac{1}{2} \int_{\Omega} |\Delta \chi(w)|^2 d\Omega. \quad (1.6)$$

In view of this, the associated space of finite energy is $\mathcal{H} \equiv H^2(\Omega) \times H_0^1(\Omega)$ with the norm

$$\|(w, w_t)\|_{\mathcal{H}}^2 \equiv \|w\|_{H^2(\Omega)}^2 + \|w_t\|_{L_2(\Omega)}^2 + \gamma^2 \|\nabla w_t\|_{L_2(\Omega)}^2. \quad (1.7)$$

The following well-posedness theorem for problem (1.1)-(1.3) is a very special case of the result in [8].

Theorem 1.1 *(See [8].) For any $w_0 \in H^2(\Omega)$, $w_1 \in H^1(\Omega)$, and $T > 0$, there exists a unique solution to (1.1), $w \in C(0, T; H^2(\Omega)) \cap C^1(0, T; H^1(\Omega))$, such that*

$$\frac{\partial}{\partial \nu} w_t|_{\Gamma} \in L_2(0, T; L_2(\Gamma)). \quad (1.8)$$

Remark: Notice that the regularity property in (1.8) does not follow from a priori interior regularity of w (i.e., $w_t \in H^1(\Omega)$).

1.1 Literature

Boundary stabilization of thin plates has attracted considerable attention in recent years (e.g., see [6], [10], [11], [12]). We shall briefly concentrate on results which apply to model (1.1).

In the context of control theory and, in particular, stabilization theory, the von Kármán model was introduced for the first time in [6]. In fact, in [6], the uniform decay

rates for the solutions to the von Kármán plate equation with $\gamma = 0$ and with *linear feedbacks* which act through higher boundary conditions than those in (1.1.c) and (1.1.d) (i.e., through moments and torques) were established. This result of [6] was derived under geometric conditions on Γ which required that Ω be “star-shaped.” Subsequently, in [2], the results of [6] were extended to the case when:

- i.) $\gamma \neq 0$, i.e., the rotational forces are taken into account;
- ii.) no geometric conditions are imposed on Γ .

These generalizations required techniques which were based on microlocal estimates combined with a nonlinear compactness/uniqueness argument, rather than the Lyapunov techniques which were used in [6]. In [5], further generalizations of the results of [2] are established by additionally allowing the feedback controls to be nonlinear, provided they satisfy appropriate growth conditions *away from the origin*.

Initial results for the von Kármán plate which has boundary conditions (1.1.c) and (1.1.d) can be found in [1], where uniform stabilization has been established assuming *no geometric conditions on the domain* when control is acting through the entire boundary.

We extend the results of [1] by additionally assuming that the feedback control can be nonlinear, provided it satisfies appropriate growth conditions *away from the origin*. In this fully nonlinear case, we do not have, in general, smooth solutions even if the initial data are assumed to be very regular. However, rigorous derivation of the estimates needed to solve the stabilization problem requires a certain amount of regularity of the solutions which is not guaranteed. To deal with this problem, we introduce a regularization/approximation procedure which leads to an “approximating” problem for which partial differential equation calculus can be rigorously justified. Passage to the limit on the approximation reconstructs the needed estimates for the original nonlinear problem.

1.2 Statement of Main Results

To state our stability result, we will need the following notation. Let the function $h(x)$ be a concave, strictly increasing function with $h(0) = 0$ such that

$$h(sg(s)) \geq s^2 + (g(s))^2 \quad \forall |s| < 1. \quad (1.9)$$

Such a function can be easily constructed (see [9]). Define

$$\tilde{h}(x) \equiv h\left(\frac{x}{mes\Sigma_T}\right). \quad (1.10)$$

Since $\tilde{h}$ is monotone increasing, for every $c \geq 0$, $cI + \tilde{h}$ is invertible. Setting

$$p(x) \equiv (cI + \tilde{h})^{-1}(Kx), \quad (1.11)$$

where K is a positive constant, we see that p is a positive, continuous, strictly increasing function with $p(0) = 0$.

We are now in a position to state our result.

Theorem 1.2 *Assume hypothesis (H) holds. Let w be the solution to system (1.1). Then for some $T_0 > 0$,*

$$E_w(t) \leq \mathcal{S}\left(\frac{t}{T_0} - 1\right) \text{ for } t > T_0, \quad (1.12)$$

where $\mathcal{S}(t) \rightarrow 0$ as $t \rightarrow \infty$ and is the solution (contraction semigroup) of the differential equation

$$\begin{cases} \frac{d}{dt}\mathcal{S}(t) + q(\mathcal{S}(t)) = 0 \\ \mathcal{S}(0) = E_w(0), \end{cases} \quad (1.13)$$

and $q(x)$ is given by

$$q(x) \equiv x - (I + p)^{-1}(x) \text{ for } x > 0. \quad (1.14)$$

In this case, the constant K will generally depend on $E_w(0)$ and the constant $c = \frac{1}{mes\Sigma_T}(m^{-1} + M)$, but will not depend on the parameter γ .

2 Preliminary Energy Estimates

Our goal is to prove energy decay rates for problem (1.1). In order to do this, we wish to use multiplier methods which require regularity of the solutions higher than is available from Theorem 1.1. Since our nonlinear problem may not have a sufficiently regular solution (even if the initial data are smooth), we resort to an approximation argument (this argument was used in the context of wave equations in [9]). In fact, the idea here is to approximate solutions to the nonlinear problem (1.1) by solutions to different (linear) problems. Since this linear problem admits regular solutions for smooth initial data, the partial differential equation calculations can be performed on this problem. Final passage

to the limit on the approximation problem allows us to obtain needed energy identities for the original nonlinear problem.

To follow our program, we start by defining the following approximations. Hypothesis (H) with (1.8) of Theorem 1.1 implies

Corollary 2.1 *Let w be the solution to (1.1). Then*

$$g(\frac{\partial}{\partial \nu} w_t) \in L_2(0, T; L_2(\Gamma)). \quad (2.1)$$

Let w be the solution of the original problem, (1.1). By using the regularity properties in (1.8) and (2.1), along with density of approximate (see below) Sobolev spaces, we are in a position to define

$$f_n \in H^{1,1}(Q_T); \quad \|f_n - [w, \chi(w)]\|_{L_2(0, T; H^{-1}(\Omega))} \longrightarrow 0 \quad (2.2)$$

$$g_n \in H^{1,1}(\Sigma_T); \quad \|g_n - g(\frac{\partial}{\partial \nu} w_t)\|_{L_2(\Sigma_T)} \longrightarrow 0 \quad (2.3)$$

$$\alpha_n \in H^{1,1}(\Sigma_T); \quad \|\alpha_n - \frac{\partial}{\partial \nu} w_t\|_{L_2(\Sigma_T)} \longrightarrow 0, \quad (2.4)$$

where $Q_T \equiv \Omega \times (0, T)$ and $\Sigma_T \equiv \Gamma \times (0, T)$. We consider the following approximating problem:

$$\begin{cases} w_{n,tt} - \gamma^2 \Delta w_{n,tt} + \Delta^2 w_n + b w_{n,t} = f_n \\ w_n(0) = w_{n,0}; \quad w_{n,t}(0) = w_{n,1} \\ w|_{\Gamma} = 0 \\ \Delta w_n + (1 - \mu) B_1 w_n + \frac{\partial}{\partial \nu} w_{n,t}|_{\Gamma} = -g_n + \alpha_n \end{cases} \quad (2.5)$$

where

$$\|w_{n,0} - w_0\|_{H^2(\Omega)} \rightarrow 0; \quad \|w_{n,1} - w_1\|_{H^1(\Omega)} \rightarrow 0, \quad (2.6)$$

and $(w_{n,0}, w_{n,1}) \in \mathcal{D}$, where $\mathcal{D}$, as dense set of $\mathcal{H}$, consists of $w_{n,0} \in H^4(\Omega)$, $w_{n,1} \in H^3(\Omega)$, where $w_{n,0}, w_{n,1}$ satisfy the appropriate compatibility conditions on the boundary. By standard linear semigroup methods, one easily shows that the linear problem, (2.5), admits a classical solution,

$$w_n \in C(0, T; H^4(\Omega)) \cap C^1(0, T; H^3(\Omega)). \quad (2.7)$$

The following proposition, which is analogous to Proposition 2.1 of [5] and is proven in the same manner, plays a critical role in our development.

Proposition 2.1 *Let w_n (respectively, w) be a solution of (2.5) (respectively, (1.1)). Then as $n \rightarrow \infty$, the following convergence holds.*

$$w_n \rightarrow w \text{ in } C(0, T; H^2(\Omega)) \cap C^1(0, T; H^1(\Omega)) \quad (2.8)$$

$$\frac{\partial}{\partial \nu} w_{n,t}|_{\Gamma} \rightarrow \frac{\partial}{\partial \nu} w_t \text{ in } L_2(\Sigma_T). \quad (2.9)$$

Multiplier methods and the convergence properties in Proposition 2.1 allow us to prove the following fundamental energy relation for problem (1.1).

Lemma 2.1 (*Energy Identity*) *Let w be the solution to (1.1). Then the following energy identity holds:*

$$E_w(T) - E_w(0) + \int_{\Sigma_T} g\left(\frac{\partial}{\partial \nu} w_t\right) \frac{\partial}{\partial \nu} w_t d\Gamma dt + \int_{Q_T} b(x) w_t^2 d\Omega dt = 0. \quad (2.10)$$

3 A Priori Estimates

To prove Theorem 1.2, the following inequality is needed.

Lemma 3.1 *Let w be the solution to (1.1), $0 < \alpha < T/2$ and $\epsilon > 0$ be arbitrary. Then there exist constants, C and $C(E_w(0))$ such that*

$$\int_{\alpha}^{T-\alpha} E_w(t) dt - C E_w(0) \leq C(E_w(0)) \{ \left\| \frac{\partial}{\partial \nu} w_t \right\|_{L_2(\Sigma_T)}^2 + \left\| g\left(\frac{\partial}{\partial \nu} w_t\right) \right\|_{L_2(\Sigma_T)}^2 + \int_{Q_T} b(x) w_t^2 d\Omega dt \}, \quad (3.1)$$

where $C(E_w(0))$ is an increasing function of $E_w(0)$.

Proof: For the sake of brevity, we include only a brief sketch of the proof, particularly as the proof is very similar to that of Lemma 3.1 in [5] with appropriate adjustments for the change in boundary conditions.

Step 1: Using the multipliers w_n and $(x - x_0) \cdot \nabla w_n$, where $x_0 \in R^2$ on the approximation problem, (2.5), we estimate the energy of that system in terms of the norms of f_n , g_n , α_n , w_n and $w_{n,t}$.

Step 2: Second order traces of the w_n in the estimate derived in Step 1 may be bounded as follows (see [4], Proposition 3.1): Assume $0 < \alpha < T/2$. Then

$$\begin{aligned} \left\| \frac{\partial^2 w_n}{\partial \tau \partial \nu} \right\|_{L_2(\alpha, T-\alpha, \Gamma)}^2 &\leq C \{ \left\| \frac{\partial}{\partial \nu} w_{n,t} \right\|_{L_2(\Sigma_T)}^2 + \left\| g_n \right\|_{L_2(\Sigma_T)}^2 + \left\| \alpha_n - \frac{\partial}{\partial \nu} w_{n,t} \right\|_{L_2(\Sigma_T)}^2 \\ &\quad + \left\| f_n \right\|_{H^{-3/2+\epsilon}(Q_T)}^2 + l.o.(w_n) \}, \end{aligned}$$

where $l.o.(w_n) \equiv \|w_n\|_{L_2(0,T;H^{2-\epsilon}(\Omega))} + \|w_{n,t}\|_{L_2(0,T;H^{-\epsilon}(\Omega))}$ and $f \equiv -bw_{n,t} + f_n$.

Step 3: The convergence properties of Proposition 2.1 allow us to obtain the following estimate for the original problem, (1.1), by passing with the limit as $n \rightarrow \infty$ on the estimate obtained for w_n and noting (2.10):

$$\begin{aligned} \int_0^T E_w(t)dt - C_1 E_w(0) &\leq C_2 \int_{\Sigma_T} \left| \frac{\partial}{\partial \nu} w_t \right|^2 d\Gamma dt + C_3 E_w^2(0) \int_{\Sigma_T} |\Delta \chi(w)| d\Gamma dt \\ &\quad + C_4 \int_{Q_T} b(x) w_t^2 d\Omega dt + C_5 l.o.(w). \end{aligned}$$

Step 4: Remaining terms, i.e., those involving $\chi(w)$ and $l.o.(w)$, are absorbed by using a nonlinear compactness/uniqueness argument as in [5]. $\square$

4 Final Estimates

Rewriting the L_2 -norm of the control in the following manner,

$$\int_{\Sigma_T} \left| g\left(\frac{\partial}{\partial \nu} w_t\right) \right|^2 d\Gamma dt = \int_{\Sigma_{A_1}} \left| g\left(\frac{\partial}{\partial \nu} w_t\right) \right|^2 d\Gamma dt + \int_{\Sigma_{B_1}} \left| g\left(\frac{\partial}{\partial \nu} w_t\right) \right|^2 d\Gamma dt, \quad (4.1)$$

where $\Sigma_{A_1} \equiv \{(t, x) \in \Sigma_T : \left| \frac{\partial}{\partial \nu} w_t \right| \leq 1\}$ and $\Sigma_{B_1} \equiv \Sigma_T \setminus \Sigma_{A_1}$, we use hypothesis (H) on Σ_{B_1} and the properties of the function $h(x)$ to find

$$\begin{aligned} \int_{\Sigma_T} \left| \frac{\partial}{\partial \nu} w_t \right|^2 d\Gamma dt &+ \int_{\Sigma_T} \left| g\left(\frac{\partial}{\partial \nu} w_t\right) \right|^2 d\Gamma dt \\ &\leq \int_{\Sigma_{A_1}} \left[\left| \frac{\partial}{\partial \nu} w_t \right|^2 + \left| g\left(\frac{\partial}{\partial \nu} w_t\right) \right|^2 \right] d\Gamma dt + \left(M + \frac{1}{m}\right) \int_{\Sigma_{B_1}} g\left(\frac{\partial}{\partial \nu} w_t\right) \frac{\partial}{\partial \nu} w_t d\Gamma dt \\ &\leq \int_{\Sigma_{A_1}} h\left(\frac{\partial}{\partial \nu} w_t g\left(\frac{\partial}{\partial \nu} w_t\right)\right) d\Gamma dt + \left(M + \frac{1}{m}\right) \int_{\Sigma_T} g\left(\frac{\partial}{\partial \nu} w_t\right) \frac{\partial}{\partial \nu} w_t d\Gamma dt. \end{aligned} \quad (4.2)$$

Denoting $\mathcal{F} \equiv \int_{\Sigma_T} g\left(\frac{\partial}{\partial \nu} w_t\right) \frac{\partial}{\partial \nu} w_t d\Gamma dt + \int_{Q_T} b(x) w_t^2 d\Omega dt$, we obtain from Lemma 3.1 and (4.2), using the monotonicity of $\tilde{h}$ to include $\int_{Q_T} b(x) w_t^2 d\Omega dt$,

$$\int_{\alpha}^{T-\alpha} E_w(t)dt - C_1 E_w(0) \leq C_{T,\alpha,\epsilon}(E_w(0))[\mathcal{F} + \tilde{h}(\mathcal{F})]. \quad (4.3)$$

Since

$$\int_0^{\alpha} E_w(t)dt + \int_{T-\alpha}^T E_w(t)dt \leq 2\alpha E_w(0), \quad (4.4)$$

we find

$$\int_0^T E_w(t)dt - C_{1,\alpha} E_w(0) \leq C_{T,\alpha,\epsilon}(E_w(0))[\mathcal{F} + \tilde{h}\mathcal{F}], \quad (4.5)$$

and by Lemma 2.1,

$$\begin{aligned} \int_0^T E_w(t)dt &\leq C_{T,\alpha,\epsilon}(E_w(0))[\mathcal{F} + \tilde{h}\mathcal{F}] + C_{1,\alpha} E_w(0) \\ \implies (T - C_{1,\alpha}) E_w(T) &\leq C_{T,\alpha,\epsilon}(E_w(0))[\mathcal{F} + \tilde{h}(\mathcal{F})] \\ \implies E_w(T) &\leq C_T(E_w(0))[\mathcal{F} + \tilde{h}(\mathcal{F})]. \end{aligned} \quad (4.6)$$

Hence, recalling (2.1),

$$(I + \tilde{h})^{-1} \left(\frac{E_w(T)}{C_T(E_w(0))} \right) \leq \mathcal{F} = E_w(0) - E_w(T). \quad (4.7)$$

Setting

$$p(s) \equiv (I + \tilde{h})^{-1} \left(\frac{s}{C_T(E_w(0))} \right), \quad (4.8)$$

we have proven the following proposition.

Proposition 4.1 *Let w be the solution to (1.1) and $E_w(t)$ be the corresponding energy at time t . If T is sufficiently large, then there exists a monotone increasing function, p , such that*

$$p(E_w(T)) + E_w(T) \leq E_w(0). \quad (4.9)$$

To arrive at the conclusion of Theorem 1.2, we need to apply the result of Lemma 3.3 in [9].

Lemma 4.1 ([9], **Lemma 3.3**) *Let p be a positive, increasing function such that $p(0) = 0$. Since p is increasing, we can define a function q such that $q(x) = x - (I + p)^{-1}(x)$. Notice that q is also an increasing function. Consider a sequence s_n of positive numbers which satisfy:*

$$s_{m+1} + p(s_{m+1}) \leq s_m. \quad (4.10)$$

Then $s_m \leq \mathcal{S}(m)$, where $\mathcal{S}(t)$ is a solution of a differential equation

$$\begin{cases} \frac{d}{dt} \mathcal{S}(t) + q(\mathcal{S}(t)) = 0 \\ \mathcal{S}(0) = s_0. \end{cases} \quad (4.11)$$

Moreover, if $p(x) > 0$ for $x > 0$, then $\lim_{t \rightarrow \infty} \mathcal{S}(t) = 0$.

Applying the result of Proposition 4.1, we obtain

$$E_w(m(T+1)) + p(E_w(m(T+1))) \leq E_w(mT), \quad (4.12)$$

for $m = 0, 1, \dots$. Thus, applying Lemma 4.1 with

$$s_m \equiv E_w(mT), \quad (4.13)$$

yields

$$E_w(mT) \leq \mathcal{S}(m), \quad m = 0, 1, \dots \quad (4.14)$$

Setting $t = mT + \tau$, $0 \leq \tau < T$, and recalling the evolution property gives

$$E_w(t) \leq E_w(mT) \leq \mathcal{S}(m) \leq \mathcal{S}\left(\frac{t-\tau}{T}\right) \leq \mathcal{S}\left(\frac{t}{T} - 1\right) \text{ for } t > T, \quad (4.15)$$

which completes the proof of Theorem 1.2. $\square$

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