**WELLPOSEDNESS AND UNIFORM DECAY RATES FOR WEAK
SOLUTIONS TO A VON KÁRMÁN SYSTEM WITH NONLINEAR
DISSIPATIVE BOUNDARY CONDITIONS**

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Wellposedness and Uniform Decay Rates for Weak Solutions to a von Kármán System With Nonlinear Dissipative Boundary Conditions

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Abstract

A von Kármán system with nonlinear boundary feedback is considered. Existence and *uniqueness* of *weak* solutions is proven. Moreover, it is shown that these solutions *(i)* are limits (with respect to the thickness of the plate which is small) of solutions corresponding to a more general von Kármán model accounting for rotational forces, and *(ii)* decay to zero at a uniform rate.

1 Introduction

1.1 Statement of the Problem

Let Ω be a bounded smooth domain in R^2 with boundary Γ . In $\Omega \times (0, T)$, where $T > 0$ is given, we consider the following von Kármán system with nonlinear boundary conditions:

$$w_{tt} + \Delta^2 w + b(x)w_t = [\mathcal{F}(w), w] \quad \text{in } Q_T = (0, T) \times \Omega \quad (1.1.a)$$

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$$\left. \begin{aligned} w(0, \cdot) &= w_0 \\ w_t(0, \cdot) &= w_1 \end{aligned} \right\} \quad \text{in } \Omega \quad (1.1.b)$$

$$\Delta w + (1 - \mu)B_1 w = -h_1\left(\frac{\partial}{\partial \nu} w_t\right) \quad \text{on } \Sigma_T = (0, T) \times \Gamma \quad (1.1.c)$$

$$\frac{\partial}{\partial \nu} \Delta w + (1 - \mu)B_2 w - w = g(w_t) - \frac{\partial}{\partial \tau} h_2\left(\frac{\partial}{\partial \tau} w_t\right) \quad \text{on } \Sigma_T = (0, T) \times \Gamma, \quad (1.1.d)$$

and

$$\begin{aligned} \Delta^2 \mathcal{F}(w) &= -[w, w] \quad \text{in } (0, T) \times \Omega \\ \mathcal{F}|_\Gamma &= \frac{\partial}{\partial \nu} \mathcal{F}|_\Gamma = 0 \quad \text{on } (0, T) \times \Gamma, \end{aligned} \quad (1.2)$$

where

$$[u, v] \equiv u_{xx}v_{yy} + u_{yy}v_{xx} - 2u_{xy}v_{xy}.$$

In (1.1), the boundary operators B_1 and B_2 are given by

$$\begin{aligned} B_1 w &= 2n_1 n_2 w_{xy} - n_1^2 w_{yy} - n_2^2 w_{xx}, \\ B_2 w &= \frac{\partial}{\partial \tau} [(n_1^2 - n_2^2) w_{xy} + n_1 n_2 (w_{yy} - w_{xx})], \end{aligned} \quad (1.3)$$

$0 < \mu < \frac{1}{2}$ is Poisson's ratio, $b(x) \in L^\infty(\Omega)$, and the functions, h_i and g are differentiable, monotone increasing, real-valued functions, zero at the origin, and are subject to the following constraints:

$$\left. \begin{aligned} m &\leq h'_i(s) \leq M \\ g'(s) &\leq M(|s|^r + 1), \end{aligned} \right\} (H)$$

where r is any positive constant. This system describes the transversal displacement w and the Airy-stress function $\mathcal{F}(w)$ of a vibrating plate, whose boundary is subject to nonlinear damping in the form of moments and forces/shears applied to the edge of the plate.

Questions related to the existence of solutions for the von Kármán system (1.1.a), (1.1.b), (1.2),

with the *homogeneous* boundary conditions:

$$w|_{\Gamma} = \frac{\partial}{\partial \nu} w|_{\Gamma} = 0 \quad \text{on } (0, T) \times \Gamma, \quad (1.4)$$

have received considerable attention in the literature. Indeed, the existence of global *weak* (i.e., $(w(t), w_t(t)) \in H^2(\Omega) \times L_2(\Omega)$) solutions for (1.1.a), (1.1.b), (1.2), (1.4) has been proven by Faedo-Galerkin methods in Lions [16] and Vorovič [21]. In [19], [20], von Wahl gives a proof of existence *and* uniqueness of a local solution with higher regularity. Existence of *local classical* solutions has been established by Stahel in [17]. Arguments of [17] also prove the uniqueness property which is valid on a small time interval. More recently, Koch and Stahel [11] were able to establish appropriate a priori bounds for their classical solutions, hence proving the *global* existence of classical solutions. *Global* existence and uniqueness results for *strong* (i.e., $H^4(\Omega) \times H^2(\Omega)$) solutions to (1.1.a), (1.1.b), (1.2), (1.4) was proven by Chueskov in [3].

As already stated, the results quoted above refer to a situation when the boundary conditions are *homogeneous*, or, more precisely, of the form as in (1.4). The techniques used in these references can be adapted to treat the case of boundary conditions as in (1.1.c) and (1.1.d), but with *linear* boundary dissipation, i.e., g and h are linear (this fact was already noted by Lagnese in [12]). Instead, the presence of *nonlinear* functions in boundary conditions (1.1.c) and (1.1.d) raises a number of technical difficulties since the methods developed previously are not applicable any longer. Indeed, a standard Faedo-Galerkin method which has been used for proving existence of solutions of von Kármán systems with homogeneous boundary conditions (as in [3], [7], [21]) runs into difficulties (at the limit process) because of the appearance of nonlinearities on the boundary.

To our knowledge, the only results available in the literature and dealing with the *nonlinear boundary* damping are due to Lagnese and Leugering [13], where the one-dimensional model has

been treated, and in [7], [14], where the rotational inertia of the plate, which induces a regularizing effect on the velocity, was accounted for in the model. In this last case, the mathematical nature of the problem is, obviously, very different.

Thus, the main distinctive feature of our paper is that we treat a two dimensional von Kármán system (1.1), (1.2) with fully nonlinear boundary damping. We shall prove that there exists a *unique weak* solution to (1.1), (1.2). We note that the uniqueness result for weak solutions is *new* even in the case of linear homogeneous boundary conditions. Precise statements of these results are given below.

Theorem 1.1 (*Uniqueness*) *Let (w, w_t) be any solution corresponding to (1.1) which satisfies $(w, w_t) \in C(0, T; H^2(\Omega) \times L_2(\Omega))$. Then, such a solution is unique.*

Theorem 1.2 (*Existence of weak solutions*) *For all initial data $(w_0, w_1) \in H^2(\Omega) \times L_2(\Omega)$, there exists a unique global solution, $(w, w_t) \in C(0, T; H^2(\Omega)) \times C(0, T; L_2(\Omega))$.*

Remark 1.1: The results of Theorems 1.1 and 1.2 hold true under somewhat weaker hypotheses imposed on h_i and g . Indeed, it would suffice to assume the growth condition (H) valid for the functions (rather than the derivatives) and holding for large values of s only. In order to keep the exposition as simple as possible and to focus attention on the key ideas, we shall dispense with this level of generality.

Our interest in studying wellposedness of this von Kármán system is motivated (among other things) by certain stabilization problems arising in control theory. Let us consider the following more general von Kármán model accounting for rotational forces (represented by the term with

γ^2).

$$\left\{ \begin{array}{l} w_{tt} - \gamma^2 \Delta w_{tt} + \Delta^2 w + bw_t = [w, \mathcal{F}(w)] \\ w(0) = w_0 \in H^2(\Omega); \quad w_t(0) = w_1 \in H^1(\Omega) \\ \Delta w + (1 - \mu)B_1 w = -h_1(\frac{\partial}{\partial \nu} w_t) \\ \frac{\partial}{\partial \nu} \Delta w + (1 - \mu)B_2 w - \gamma^2 \frac{\partial}{\partial \nu} w_{tt} - w = g(w_t) - \frac{\partial}{\partial \tau} h_2(\frac{\partial}{\partial \tau} w_t), \end{array} \right. \quad (1.5)$$

Here, $\mathcal{F}(w)$ satisfies (1.2), the functions h_i and g are under the same hypothesis (H), and $b(x) > 0$ a.e. in Ω .

With (1.5), we associate the energy

$$E_{\gamma,w}(t) \equiv \frac{1}{2} \int_{\Omega} \{ |w_t^2 + \gamma^2 |\nabla w_t|^2 + |\Delta \mathcal{F}(w)|^2 \} d\Omega + \frac{1}{2} a(w, w), \quad (1.6)$$

where

$$a(w, w) \equiv \int_{\Omega} \{ |\Delta w|^2 + (1 - \mu)(2w_{xy}^2 - w_{xx}^2 - w_{yy}^2) \} d\Omega + \frac{1}{2} \int_{\Gamma} w^2 d\Gamma. \quad (1.7)$$

The parameter γ in (1.1.a) is proportional to the thickness of the plate and it is usually assumed to be small. It can be shown that problem (1.5) admits a unique solution in the energy norm (notice the regularizing effect of the term $\gamma^2 \Delta w_{tt}$). Moreover a special case of the uniform decay result presented in [9] implies that for initial data in a bounded set of $H^2(\Omega) \times H^1(\Omega)$, the energy of weak solutions decays exponentially to zero. This is to say that: $\forall (w_0, w_1) \in \mathcal{B}(0, R) \subset H^2(\Omega) \times H^1(\Omega)$,

$$E_{\gamma,w}(t) \leq C e^{\omega t}, \quad t \geq 0, \quad (1.8)$$

where the constants $C, \omega > 0$ *do not* depend on γ . Thus the decay estimate in (1.8) is “robust” with respect to the small parameter γ .

An interesting question that arises is: What happens to solutions of (1.5) when $\gamma \rightarrow 0$? A

formal limit of system (1.5) is, of course, equation (1.1). Moreover, the uniformity (with respect to γ) of the estimate in (1.8) indicates that a similar estimate should hold for solutions to (1.1). However, this limit process is not obvious at all, due to nonlinearities appearing on the boundary. Indeed, (1.8) implies weak convergence (on a subsequence) of solutions $w_\gamma(t)$. This, however, is not sufficient to pass with the limit on nonlinear boundary conditions. On the other hand, techniques used in this paper will allow us to justify, in a rigorous manner, the limit process taken on (1.5) and leading to system (1.1). Precise formulation of this result is given below.

Theorem 1.3 *Let $w_\gamma(t)$ be any solution to (1.5) such that (without loss of generality) $E_{\gamma,w}(t) \leq 1$. Then, as $\gamma \rightarrow 0$,*

$$\begin{aligned} w_\gamma &\rightarrow w \quad \text{in } C(0, \infty; H^2(\Omega)) \\ w_{\gamma,t} &\rightarrow w_t \quad \text{in } C(0, \infty; L_2(\Omega)), \end{aligned} \tag{1.9}$$

where (w, w_t) satisfy equation (1.1). Moreover, the following uniform stability estimate holds:

$$\|w(t)\|_{H^2(\Omega)} + \|w_t(t)\|_{L_2(\Omega)} \leq C e^{-\omega t}, \tag{1.10}$$

for some constants $C, \omega > 0$.

Remark 1.2: A direct proof of uniform decay rates for *smooth* solutions of the von Kármán system with $\gamma = 0$ subject to *linear* dissipation acting on the boundary of Ω -star shaped domain was proven in [12]. Extension of this result to (i) *nonlinear* (sublinear) boundary feedbacks, (ii) general domains and (iii) all *weak* solutions of the von Kármán equation corresponding to $H^2(\Omega) \times L_2(\Omega)$ initial data is given in [10]. The advantage of the decay estimate in (1.10) in the context of equation (1.1) is that it also holds for a *superlinear* boundary feedback, g . The price to pay is a stronger smoothness requirement imposed on the initial velocity ($H^1(\Omega)$ rather than $L_2(\Omega)$).

Remark 1.3: Results related to an existence of compact (and, in some cases, finite dimensional)

attractors associated with a von Kármán system subject to nonlinear boundary dissipation are given in [15].

2 Uniqueness: Proof of Theorem 1.1

In what follows, in (1.1), we shall assume $b \equiv 0$ to simplify exposition. (Addition of this lower order term does not alter arguments.) We consider the map $G : H^2(\Omega) \times H^2(\Omega) \rightarrow L_2(\Omega)$ given by

$$\begin{aligned} \Delta^2 G &= -[u, v] \quad \text{in } \Omega \\ G &= \frac{\partial}{\partial \nu} G = 0 \quad \text{on } \Gamma \end{aligned} \tag{2.1}$$

The following result is critical to the proof.

Lemma 2.1 *The map $(u, v) \rightarrow G[u, v]$ is bounded from $H^2(\Omega) \times H^2(\Omega) \rightarrow W_\infty^2(\Omega)$.*

As the proof of Lemma 2.1, based on the compensated compactness method, is technical and is relegated to section 5.

Remark 2.1: Notice that the result of Lemma 2.1 improves by “ ϵ ” a known regularity result stating that this map is bounded from $H^{2+\epsilon}(\Omega) \times H^2(\Omega) \rightarrow W_\infty^2(\Omega)$. As we shall see later, this improvement by “ ϵ ” is critical.

Let w_1 and w_2 be two solutions of (1.1). Set $\tilde{w} \equiv w_1 - w_2$. Then $\tilde{w}$ satisfies

$$\begin{cases} \tilde{w}_{tt} + \Delta^2 \tilde{w} = [\mathcal{F}(w_1) - \mathcal{F}(w_2), w_1] + [\mathcal{F}(w_2), \tilde{w}] & \text{in } Q_T \\ \Delta \tilde{w} + (1 - \mu)B_1 \tilde{w} = -[h_1(\frac{\partial}{\partial \nu} w_{1,t}) - h_1(\frac{\partial}{\partial \nu} w_{2,t})] & \text{on } \Sigma_T \\ \frac{\partial}{\partial \nu} \Delta \tilde{w} + (1 - \mu)B_2 \tilde{w} = g(w_{1,t}) - g(w_{2,t}) - \frac{\partial}{\partial \tau} [h_2(\frac{\partial}{\partial \tau} w_{1,t}) - h_2(\frac{\partial}{\partial \tau} w_{2,t})] & \text{on } \Sigma_T. \end{cases} \tag{2.2}$$

Multiplying (2.2) by $\tilde{w}_t$ and integrating (first formally) over Q_T yields

$$\begin{aligned}
& \|\tilde{w}(t)\|_{H^2(\Omega)}^2 + \|\tilde{w}_t(t)\|_{L_2(\Omega)}^2 + \int_0^t \int_{\Gamma} [h_1(\frac{\partial}{\partial \nu} w_{1,t}) - h_1(\frac{\partial}{\partial \nu} w_{2,t})](\frac{\partial}{\partial \nu} w_{1,t} - \frac{\partial}{\partial \nu} w_{2,t}) d\Gamma dt \\
& + \int_0^t \int_{\Gamma} [h_2(\frac{\partial}{\partial \tau} w_{1,t}) - h_2(\frac{\partial}{\partial \tau} w_{2,t})](\frac{\partial}{\partial \tau} w_{1,t} - \frac{\partial}{\partial \tau} w_{2,t}) d\Gamma dt \\
& + \int_0^t \int_{\Gamma} [g(w_{1,t}) - g(w_{2,t})](w_{1,t} - w_{2,t}) d\Gamma dt \\
& \leq \int_0^t \int_{\Omega} |[\mathcal{F}(w_1) - \mathcal{F}(w_2), w_1] \tilde{w}_t + [\mathcal{F}(w_2), \tilde{w}] \tilde{w}_t| d\Omega dt.
\end{aligned} \tag{2.3}$$

By Lemma 2.1, $\mathcal{F}(w_i) \in W_{\infty}^2(\Omega)$, hence the right-hand side of (2.2) belongs to $L_2(\Omega)$. Therefore, the formal computation leading to (2.3) can be made rigorous (see [6]) by treating the nonlinear bracket, $[w, \chi(w)]$ as a perturbation of a nonlinear semigroup of contractions generated by (1.1) with $\mathcal{F} = 0$ and applying the usual (see [2]) approximation arguments in the context of nonlinear semigroup theory.

From monotonicity of h and g and using the Cauchy-Schwartz inequality, we obtain

$$\|\tilde{w}(t)\|_{H^2(\Omega)}^2 + \|\tilde{w}_t(t)\|_{L_2(\Omega)}^2 \leq C \int_0^t \{ \|[\mathcal{F}(w_1) - \mathcal{F}(w_2), w_1]\|_{L_2(\Omega)}^2 + \|[\mathcal{F}(w_2), \tilde{w}]\|_{L_2(\Omega)}^2 \} dt. \tag{2.4}$$

From Lemma 2.1,

$$\|\mathcal{F}(w_2)\|_{W_{\infty}^2(\Omega)} \leq C \|w_2\|_{H^2(\Omega)} \|w_2\|_{H^2(\Omega)}. \tag{2.5}$$

Moreover,

$$\begin{aligned}
\|[\mathcal{F}(w_2), \tilde{w}]\|_{L_2(\Omega)} & \leq C \|\tilde{w}\|_{H^2(\Omega)} \|\mathcal{F}(w_2)\|_{W_{\infty}^2(\Omega)} \\
& \leq C \|\tilde{w}\|_{H^2(\Omega)} \|w_2\|_{H^2(\Omega)} \|w_2\|_{H^2(\Omega)}.
\end{aligned} \tag{2.6}$$

We can rewrite $\mathcal{F}(w_1) - \mathcal{F}(w_2)$ in the following way:

$$\mathcal{F}(w_1) - \mathcal{F}(w_2) = G([w_1, w_1] - [w_2, w_2]) = G[\tilde{w}, w_1 + w_2]. \tag{2.7}$$

Hence, again from Lemma 2.1,

$$\|G[\tilde{w}, w_1 + w_2]\|_{W_\infty^2(\Omega)} \leq C\|\tilde{w}\|_{H^2(\Omega)}[\|w_1\|_{H^2(\Omega)} + \|w_2\|_{H^2(\Omega)}], \quad (2.8)$$

$$\begin{aligned} \Rightarrow \|[\mathcal{F}(w_1) - \mathcal{F}(w_2), w_1]\|_{L_2(\Omega)} &\leq C\|w_1\|_{H^2(\Omega)}\|\mathcal{F}(w_1) - \mathcal{F}(w_2)\|_{W_\infty^2(\Omega)} \\ &\leq C\|w_1\|_{H^2(\Omega)}\|\tilde{w}\|_{H^2(\Omega)}[\|w_1\|_{H^2(\Omega)} + \|w_2\|_{H^2(\Omega)}]. \end{aligned} \quad (2.9)$$

Combining (2.3), (2.6), and (2.9) proves that the solution must be unique. $\square$

3 Existence of Weak Solutions: Proof of Theorem 1.2

Lemma 3.1 (*Local existence*) *Assume hypothesis (H). Then there exists $T_0 > 0$ such that for any initial data $w_0 \in H^2(\Omega)$, $w_1 \in L_2(\Omega)$, there exists a unique solution to (1.1), $w \in C(0, T_0; H^2(\Omega))$, $w_t \in C(0, T_0; L_2(\Omega))$.*

Proof: We consider the following nonlinear equation:

$$\begin{aligned} w_{tt} + \Delta^2 w &= [f, w] \quad \text{in } Q_T = (0, T) \times \Omega \\ \left. \begin{aligned} w(0, \cdot) &= w_0 \\ w_t(0, \cdot) &= w_1 \end{aligned} \right\} & \quad \text{in } \Omega \\ \Delta w + (1 - \mu)B_1 w &= -h_1\left(\frac{\partial}{\partial \nu} w_t\right) \quad \text{on } \Sigma_T = (0, T) \times \Gamma \\ \frac{\partial}{\partial \nu} \Delta w + (1 - \mu)B_2 w - w &= g(w_t) - \frac{\partial}{\partial \tau} h_2\left(\frac{\partial}{\partial \tau} w_t\right) \quad \text{on } \Sigma_T = (0, T) \times \Gamma, \end{aligned} \quad (3.1)$$

where the function f is a given element of $L_1(0, T; W_\infty^2(\Omega))$.

Applying a standard perturbation theorem for nonlinear semigroups (see Barbu [2]), we obtain the existence and uniqueness of the solutions to (3.1) on the space $H^2(\Omega) \times L_2(\Omega)$. Multiplying (3.1) by w_t , integrating over Q_T and using nonlinear semigroup arguments to justify formal calculations

(see [14]) gives

$$\begin{aligned} \|w_t(t)\|_{L_2(\Omega)}^2 &+ a(w(t), w(t)) + 2 \int_0^t \int_{\Gamma} h_1\left(\frac{\partial}{\partial \nu} w_t\right) \frac{\partial}{\partial \nu} w_t d\Gamma dt + 2 \int_0^t \int_{\Gamma} g(w_t) w_t d\Gamma dt + 2 \int_0^t \int_{\Gamma} h_2\left(\frac{\partial}{\partial \tau} w_t\right) \frac{\partial}{\partial \tau} w_t d\Gamma dt \\ &= \|w_1\|_{L_2(\Omega)}^2 + a(w_0, w_0) + 2 \int_0^t \int_{\Omega} [f, w] w_t d\Omega dt, \end{aligned} \quad (3.2)$$

where

$$m_1 \|w\|_{H^2(\Omega)}^2 \leq a(w, w) \leq M_1 \|w\|_{H^2(\Omega)}^2. \quad (3.3)$$

We construct a map

$$v \longrightarrow \mathcal{T}v, \quad (3.4)$$

defined on a Banach space, X , where

$$X \equiv C(0, T; H^2(\Omega)) \cap C^1(0, T; L_2(\Omega)), \quad (3.5)$$

and $\mathcal{T}v$ is defined as the solution to (3.1) with $f(t)$ given by

$$f(t) \equiv \mathcal{F}(v(t)). \quad (3.6)$$

To prove Lemma 3.1, it suffices to construct a unique fixed point for the map $\mathcal{T}$ introduced in (3.4) and defined on $C(0, T_0; B_R)$, where $B_R \equiv \{u \in H^2(\Omega) \times L_2(\Omega) : \|u\|_{H^2(\Omega) \times L_2(\Omega)} \leq R\}$.

We shall prove that for sufficiently small values of T_0 and sufficiently large values of R , the map $\mathcal{T}$ is a contraction on $C(0, T_0; B_R)$. To accomplish this, we first note that by virtue of Lemma 2.1, the Airy's stress function $\mathcal{F}(w)$ is locally Lipschitz: $H^2(\Omega) \rightarrow W_{\infty}^2(\Omega)$. Indeed, from Lemma 2.1, it

follows that

$$\begin{aligned}\|\mathcal{F}(w_1) - \mathcal{F}(w_2)\|_{W_\infty^2(\Omega)} &= \|G[w_1 - w_2, w_1 + w_2]\|_{W_\infty^2(\Omega)} \\ &\leq C\|w_1 - w_2\|_{H^2(\Omega)}[\|w_1\|_{H^2(\Omega)} + \|w_2\|_{H^2(\Omega)}].\end{aligned}\tag{3.7}$$

Applying (3.2) with $w \equiv \mathcal{T}v$, where $v \in C(0, T; B_R)$, and $f \equiv \mathcal{F}(v)$, we obtain for $t \leq T_0$,

$$\begin{aligned}\|\mathcal{T}v(t)\|_{H^2(\Omega)}^2 + \|\frac{d}{dt}\mathcal{T}v(t)\|_{L_2(\Omega)}^2 &\leq C\{\|w_0\|_{H^2(\Omega)}^2 + \|w_1\|_{L_2(\Omega)}^2 + \int_0^t \|[\mathcal{F}(v(\tau)), w(\tau)]\|_{L_2(\Omega)}^2 d\tau\} \\ &\leq C\{\|w_0\|_{H^2(\Omega)}^2 + \|w_1\|_{L_2(\Omega)}^2 + \int_0^t \|v(\tau)\|_{H^2(\Omega)}^4 \|\mathcal{T}v(\tau)\|_{H^2(\Omega)}^2 d\tau\} \\ &\leq C\{\|w_0\|_{H^2(\Omega)}^2 + \|w_1\|_{L_2(\Omega)}^2 + R^4 T_0 \|\mathcal{T}v\|_{C(0, T_0; B_R)}^2\},\end{aligned}\tag{3.8}$$

where the first inequality follows from Lemma 2.1. Selecting R so that

$$\|w_0\|_{H^2(\Omega)}^2 + \|w_1\|_{L_2(\Omega)}^2 \leq \frac{R^2}{4C},$$

and taking T_0 sufficiently small yields

$$\mathcal{T}(C(0, T_0; B_R)) \subset C(0, T_0; B_R).\tag{3.9}$$

Let $v_1, v_2 \in C(0, T; B_R)$. Similar arguments as above yield

$$\begin{aligned}\|(\mathcal{T}v_1 - \mathcal{T}v_2)(t)\|_{H^2(\Omega)}^2 + \|\frac{d}{dt}(\mathcal{T}v_1 - \mathcal{T}v_2)(t)\|_{L_2(\Omega)}^2 \\ \leq C \int_0^t \{\|\mathcal{F}(v_1) - \mathcal{F}(v_2)\|_{W_\infty^2(\Omega)}^2 \|\mathcal{T}v_1\|_{H^2(\Omega)}^2 + \|\mathcal{F}(v_2)\|_{W_\infty^2(\Omega)}^2 \|\mathcal{T}v_1 - \mathcal{T}v_2\|_{H^2(\Omega)}^2\} dt \\ \leq C R^4 T_0 \{\|v_1 - v_2\|_{C(0, T_0; H^2(\Omega))}^2 + \|\mathcal{T}v_1 - \mathcal{T}v_2\|_{C(0, T_0; H^2(\Omega))}^2\},\end{aligned}\tag{3.10}$$

where we have used (2.1), (2.2) and Lemma 2.1. Hence,

$$\|\mathcal{T}v_1 - \mathcal{T}v_2\|_{C(0, T_0; B_R)}^2 \leq \frac{CR^4T_0}{1 - CR^4T_0} \|v_1 - v_2\|_{C(0, T_0; H^2(\Omega))}^2. \quad (3.11)$$

Taking T_0 small enough yields contraction property for the map $\mathcal{T}$. The result of Lemma 3.1 now follows from the Contraction Mapping Principle. $\square$

To complete the proof of Theorem 1.2, it suffices to establish the following a priori bound.

Lemma 3.2 (*A priori bounds*) *Assume hypothesis (H). Let (w, w_t) be any local solution to (1.1) such that $w \in C(0, T_0; H^2(\Omega))$, $w_t \in C(0, T_0; L_2(\Omega))$. Then the following a priori bound holds.*

$$\|w(t)\|_{H^2(\Omega)}^2 + \|w_t(t)\|_{L_2(\Omega)}^2 \leq C(\|w_0\|_{H^2(\Omega)}^2, \|w_1\|_{L_2(\Omega)}^2) \quad t \geq 0, \quad (3.12)$$

where $C(u, v)$ denotes a continuous function of u, v .

Proof: By using energy methods, the a priori bound in (3.12) can be proven in a standard way for “smooth” solutions. On the other hand, the existence of such smooth solutions for “smooth” initial data has been shown in [6]. (Here is the point where we use the differentiability of f and g .) We need to extend this bound to hold for all *weak* solutions. To this end, we select a suitable approximation of the initial data $w_0 \in H^2(\Omega)$, $w_1 \in L_2(\Omega)$ such that

$$\begin{cases} H^4(\Omega) \ni w_{0m} \rightarrow w_0 & \text{in } H^2(\Omega) \\ H^2(\Omega) \ni w_{1m} \rightarrow w_1 & \text{in } L_2(\Omega), \end{cases} \quad (3.13)$$

and w_{0m}, w_{1m} satisfy appropriate compatibility conditions. Let $w_m(t)$ denote a solution to (1.1) corresponding to initial data (w_{0m}, w_{1m}) . Then by the result of Theorem 1.3 in [6], $w_m(t)$ is

“smooth,” hence the following inequality holds.

$$\begin{aligned}
& \|w_m(t)\|_{H^2(\Omega)}^2 + \|w_{m,t}(t)\|_{L_2(\Omega)}^2 + \int_0^t \int_{\Gamma} h_1\left(\frac{\partial}{\partial \nu} w_{m,t}\right) \frac{\partial}{\partial \nu} w_{m,t} d\Gamma dt \\
& + \int_0^t \int_{\Gamma} g(w_{m,t}) w_{m,t} d\Gamma dt + \int_0^t \int_{\Gamma} h_2\left(\frac{\partial}{\partial \tau} w_{m,t}\right) \frac{\partial}{\partial \tau} w_{m,t} d\Gamma dt \\
& \leq C(\|w_0\|_{H^2(\Omega)}, \|w_1\|_{L_2(\Omega)}).
\end{aligned} \tag{3.14}$$

We shall show that

$$\begin{cases} w_m \rightarrow w & \text{in } C(0, T_0; H^2(\Omega)) \\ w_{m,t} \rightarrow w_t & \text{in } C(0, T_0; L_2(\Omega)), \end{cases} \tag{3.15}$$

where, we recall, w is a weak solution to (1.1).

Indeed,

$$\begin{aligned}
& \|(w_n - w_m)(t)\|_{H^2(\Omega)}^2 + \|(w_{n,t} - w_{m,t})(t)\|_{L_2(\Omega)}^2 + \int_0^t \int_{\Gamma} [h_1\left(\frac{\partial}{\partial \nu} w_{n,t}\right) \\
& - h_1\left(\frac{\partial}{\partial \nu} w_{m,t}\right)] \frac{\partial}{\partial \nu} (w_{n,t} - w_{m,t}) d\Gamma dt + \int_0^t \int_{\Gamma} [g(w_{n,t}) - g(w_{m,t})] (w_{n,t} - w_{m,t}) d\Gamma dt \\
& + \int_0^t \int_{\Gamma} [h_2\left(\frac{\partial}{\partial \tau} w_{n,t}\right) - h_2\left(\frac{\partial}{\partial \tau} w_{m,t}\right)] \frac{\partial}{\partial \tau} (w_{n,t} - w_{m,t}) d\Gamma dt \\
& \leq C(\|w_{0n} - w_{0m}\|_{H^2(\Omega)}^2, \|w_{1n} - w_{1m}\|_{L_2(\Omega)}^2) \\
& + C \int_0^t \|\mathcal{F}(w_n) - \mathcal{F}(w_m)\|_{W_{\infty}^2(\Omega)}^2 \|w_n\|_{H^2(\Omega)}^2 d\tau \\
& + C \int_0^t \|\mathcal{F}(w_m)\|_{W_{\infty}^2(\Omega)}^2 \|w_n - w_m\|_{H^2(\Omega)}^2 d\tau.
\end{aligned} \tag{3.16}$$

From (3.7) and (3.14),

$$\int_0^t \|\mathcal{F}(w_n) - \mathcal{F}(w_m)\|_{W_{\infty}^2(\Omega)}^2 \|w_n\|_{H^2(\Omega)}^2 d\tau \leq C(\|w_0\|_{H^2(\Omega)}, \|w_1\|_{L_2(\Omega)}) \int_0^t \|w_n(\tau) - w_m(\tau)\|_{H^2(\Omega)}^2 d\tau. \tag{3.17}$$

From Lemma 2.1 and (3.14),

$$\int_0^t \|\mathcal{F}(w_m(\tau))\|_{W_\infty^2(\Omega)}^2 \|w_n(\tau) - w_m(\tau)\|_{H^2(\Omega)}^2 d\tau \leq C(\|w_0\|_{H^2(\Omega)}, \|w_1\|_{L_2(\Omega)}) \int_0^t \|w_n(\tau) - w_m(\tau)\|_{H^2(\Omega)}^2 d\tau. \quad (3.18)$$

Inserting (3.17) and (3.18) into (3.16) and applying Gronwall's inequality with (3.13) gives

$$\begin{cases} w_m \rightarrow w^* & \text{in } C(0, T; H^2(\Omega)) \\ w_{m,t} \rightarrow w_t^* & \text{in } C(0, T; L_2(\Omega)). \end{cases} \quad (3.19)$$

Moreover, from (H) and (3.16), (3.19), we also obtain that

$$\begin{aligned} \lim_{n,m \rightarrow \infty} \{ \int_0^t \int_\Gamma [h(\frac{\partial}{\partial \nu} w_{n,t}) - h(\frac{\partial}{\partial \nu} w_{m,t})] \frac{\partial}{\partial \nu} (w_{n,t} - w_{m,t}) d\Gamma dt \\ + \int_0^t \int_\Gamma [h_2(\frac{\partial}{\partial \tau} w_{n,t}) - h_2(\frac{\partial}{\partial \tau} w_{m,t})] \frac{\partial}{\partial \tau} (w_{n,t} - w_{m,t}) d\Gamma dt \} \rightarrow 0. \end{aligned} \quad (3.20)$$

Hence, by (H),

$$\|\nabla(w_{n,t} - w_{m,t})\|_{L_2(\Sigma)} \rightarrow 0. \quad (3.21)$$

From (3.7), (3.19), and properties of the bracket, $[\cdot, \cdot]$, it follows that

$$[\mathcal{F}(w_m(t)), w_m(t)] \rightarrow [\mathcal{F}(w^*(t)), w^*(t)] \text{ in } L_2(\Omega). \quad (3.22)$$

The result in (3.21), (3.22) allows us to deduce (by standard arguments) that w^* satisfies the original equation (1.1). Thus, by Theorem 1.1, $w^* \equiv w$ is a unique solution to (1.1).

Convergence in (3.15) together with passage to the limit on inequality (3.14) yields the desired a priori bound in Lemma 3.2. $\square$

4 Proof of Lemma 2.1

Let $f = G[u, v]$, i.e.,

$$\Delta^2 f = [u, v]; \quad f = 0, \quad \frac{\partial}{\partial \nu} f = 0 \text{ on } \Gamma. \quad (4.1)$$

By the Closed Graph Theorem, it suffices to prove that for $u, v \in H^2(\Omega)$, $f \in W_\infty^2(\Omega)$. By using partition of unity, it is enough to consider the case when both u and v are supported in the neighborhood of the point $x_0 \in \bar{\Omega}$. We shall consider two cases: (i) $x_0 \in \text{Int}(\Omega)$, (ii) $x_0 \in \Gamma$.

Case (i): We introduce a bilinear continuous operator $\mathcal{B}$ from $C_0^\infty(R^2) \times C_0^\infty(R^2)$ into $\mathcal{D}'(R^2)$ defined as

$$\mathcal{B}(u, v) \equiv [\Delta^{-1}u, \Delta^{-1}v],$$

where Δ^{-1} denotes the inverse of the Laplacian on R^2 . Let $b(\xi, \eta)$ be a symbol associated with $\mathcal{B}$ (see [4], page 29) defined by

$$\mathcal{B}(e^{i\xi x} e^{i\eta x}) = b(\xi, \eta) e^{i(\xi+\eta)x}, \quad \xi \in R^2, \eta \in R^2, x \in R^2.$$

Then

$$b(\xi, \eta) = \frac{\xi_1^2 \eta_2^2 + \xi_2^2 \eta_1^2 - 2\xi_1 \xi_2 \eta_1 \eta_2}{(\xi_1^2 + \xi_2^2)(\eta_1^2 + \eta_2^2)}. \quad (4.2)$$

Since $b(\xi, -\xi) = 0$ for all $\xi \neq 0$, the result of Theorem VI in [4] applies and tells us that $\mathcal{B}$ is bounded from $L_2(R^2) \times L_2(R^2) \rightarrow H_1(R_2)$, where H_1 is a real Hardy space (see [5]). This means that for $u \in H^2(\Omega)$, $v \in H^2(\Omega)$ (where, we recall, u and v are supported away from Γ),

$$[u, v] \in H_1. \quad (4.3)$$

Let $f_0 \equiv G_0[u, v]$, where $G_0 \in OPS^{-4}$ with a symbol $g_0 = \frac{1}{(\xi_1^2 + \xi_2^2)^2}$. By OPS^n , we denote, as

usual (see [18]), a class of pseudodifferential operators of order n .

Since $[u, v]$ is supported away from Γ , f_0 differs from f by a C^∞ function. From

$$D^\alpha G_0 \in OPS^0 \text{ for } |\alpha| \leq 4, \quad (4.4)$$

where D^α stands for the differential operator of order $|\alpha|$, we obtain, by Theorem 26 of [5] (see page 121), (4.3) and (4.4), that

$$D^\alpha G_0[u, v] \subset L_1(R^2), \text{ for } |\alpha| \leq 4. \quad (4.5)$$

By Sobolev's Imbedding (see [1], page 97, (7)),

$$f_0 \in W_1^4(\Omega) \subset W_\infty^2(\Omega) \text{ and } f \in W_\infty^2(\Omega). \quad (4.6)$$

Case (ii): Let $x_0 \in \Gamma$. By a local change of coordinates, we flatten the boundary and consider the extended functions $u, v \in H^2(R^2)$ with compact support (i.e., supported in the neighborhood of x_0). We denote by $\mathcal{L}$ the differential operator (with variable coefficients) defined on R^2 which corresponds to Δ^2 in new coordinates. Writing $[u, v]$ in new coordinates produces a symbol whose principal part satisfies the conditions of Theorem VI in [4] and the lower-order terms are the products of $D^\alpha u D^\beta v$, $|\alpha| + |\beta| \leq 3$. Thus, the contribution of lower-order terms is in $L_{1+\epsilon}(R^2)$, where $\epsilon > 0$. Therefore, a straightforward modification (to account for variable coefficients) of the compensated compactness result of [4] applied to the principal part of the symbol of $[u, v]$ gives

$$[u, v] = w + z, \quad w \in H_1, \quad z \in L_{1+\epsilon}(R^2). \quad (4.7)$$

It follows from the definition of H_1 space and boundedness of the Riesz transform on $L_p(R^2)$ ($p > 1$) spaces, that the element $z \in L_{1+\epsilon}(R^2)$ with compact support can be written as $z = z_1 + z_2$, where $z_1 \in H_1$, $z_2 \in C^\infty$. Thus, without loss of generality, we may write that

$$[u, v] = w + z, \quad w \in H_1, \quad z \in C^\infty. \quad (4.8)$$

We denote by W a pseudodifferential operator in OPS^{-4} whose symbol is compactly supported in R^2 and is an approximation of $\mathcal{L}^{-1}$ in a small neighborhood of x_0 . This is to say that $(W\mathcal{L} - I)u \in C^\infty$ whenever u is supported in a neighborhood of x_0 . (Recall $\mathcal{L}$ represents the biharmonic operator written in the new coordinates and extended to R^2 .) We define

$$g \equiv W[u, v]. \quad (4.9)$$

By (4.8),

$$g = W[u, v] = Ww + Wz \in Ww + C^\infty, \quad w \in H_1. \quad (4.10)$$

From the construction of W and $\mathcal{L}$, it follows that

$$\mathcal{L}f - \mathcal{L}g \in C^\infty(R^2). \quad (4.11)$$

We shall show that

$$g \in W_\infty^2(R^2). \quad (4.12)$$

Indeed, since $W \in OPS^{-4}$, $D^\alpha W \in OPS^0$ for $|\alpha| \leq 4$. Theorem 4 in [8] then gives

$$D^\alpha Ww \in h_1, \quad |\alpha| \leq 4, \quad (4.13)$$

where h_1 is a local Hardy space (see [8]). By using Lemma 4 in [8] together with (4.13), we obtain

$$D^\alpha W w \in H_1 + C^\infty, \quad |\alpha| \leq 4. \quad (4.14)$$

Combining (4.10) with (4.14) gives

$$D^\alpha g \in H_1 + C^\infty, \quad |\alpha| \leq 4. \quad (4.15)$$

This, in particular, implies

$$g \in W_1^4(R^2) \subset W_\infty^2(R^2), \quad (4.16)$$

which proves (4.12).

We next consider the following elliptic problem:

$$\begin{cases} \mathcal{L}h = 0 & \text{in } \Omega_{x_0} \\ h = g_1 & \text{on } \Gamma_{x_0} \\ \frac{\partial}{\partial \nu} h = g_2 & \text{on } \Gamma_{x_0}, \end{cases} \quad (4.17)$$

where $g_1 \equiv -g|_\Gamma$, $g_2 \equiv -\frac{\partial}{\partial \nu} g|_\Gamma$, and Ω_{x_0} denotes $\Omega \cap U(x_0)$, $\Gamma_{x_0} = \Gamma \cap U(x_0)$. From (4.15) we infer $D^\alpha g \in L_1(R^2)$, $|\alpha| \leq 4$, and from (4.10), $D^\alpha R_\tau g \in L_1(R^2)$, $|\alpha| \leq 4$, where $R_\tau \in OPS^0$ denotes the Riesz transform in the tangential direction (see [5]). Hence, both $g, R_\tau g \in W_1^4(R^2)$. Applying trace theory on L_1 spaces yields $g|_\Gamma \in W_1^3(R)$, $R_\tau g|_\Gamma \in W_1^3(R)$, $\frac{\partial}{\partial \nu} g|_\Gamma \in W_1^2(R)$, $\frac{\partial}{\partial \nu} R_\tau g|_\Gamma \in W_1^2(R)$. From the definition of Hardy's spaces [5], we infer

$$\begin{cases} D^\alpha g_1 \in H_1(R^1) + C^\infty(R^1), & |\alpha| \leq 3 \\ D^\alpha g_2 \in H_1(R^1) + C^\infty(R^1), & |\alpha| \leq 2. \end{cases} \quad (4.18)$$

Proposition 4.1 *Let h satisfy (4.17) with g_1, g_2 subject to (4.18). Then $h \in C^2(\Omega)$.*

Proof: We shall use a decoupling procedure as in [18], Chapter 5. Let x represent the normal outward direction to the boundary Γ and y represent the tangential direction. By the collar neighborhood theorem, it suffices to consider $x \in [0, 1], y \in R^1$.

By Proposition 2.1 in [18], the solution h to (4.17) can be written as

$$h(x, y) = A_1(x, y, D_y)P^{cx}g_1 + A_2(x, y, D_y)P^{cx}g_2 + h_0, \quad (4.19)$$

where $h_0 \in C^\infty(\Omega_{x_0})$ and

$$(P^{cx}g_i)(y) \equiv \int_{R^1} e^{-x|\xi|} e^{iy\xi} \hat{g}_i(\xi) d\xi,$$

$\hat{g}_i$ are Fourier transforms of g_i , and the constant $c \geq 0$ is determined from the ellipticity constant of $\mathcal{L}$.

The pseudodifferential operators, A_1, A_2 satisfy (see Proposition 2.1 in [19])

$$\begin{cases} D_x^j A_1(x, y, D_y) \in OPS^j \\ D_x^j A_2(x, y, D_y) \in OPS^{j-1} \end{cases} \quad j = 0, 1, 2, \dots \quad (4.20)$$

Taking derivatives up to the order 3 in (4.19) yields

$$D^\alpha h = B_1(x, y, D_y)P^{cx}D_y^3g_1 + B_2(x, y, D_y)P^{cx}D_y^2g_2, \quad |\alpha| \leq 3, \quad (4.21)$$

and $B_i(x, y, D_y) \in OPS^0$.

From (4.18) and the structure of the operator P^{cx} , it follows that

$$\begin{cases} P^{cx} D_y^3 g_1 \in C([0, 1]; H_1(R^1)) \\ P^{cx} D_y^2 g_2 \in C([0, 1]; H_1(R^1)). \end{cases} \quad (4.22)$$

By Theorem 26 in [5], and (4.21), (4.22), we infer

$$D^\alpha h \in C([0, 1]; L_1(R^1)), \quad |\alpha| \leq 3. \quad (4.23)$$

Hence, by standard Sobolev's Imbeddings, $D^2 h \in C([0, 1]; C(\Gamma_{x_0}))$, which implies $h \in C^2(\Omega_{x_0})$, as desired. $\square$

From (4.11), it follows that

$$\begin{cases} \mathcal{L}(f - g) \in C^\infty(\Omega_{x_0}) \\ (f - g)|_\Gamma = g_1 & \text{on } \Gamma_{x_0} \\ \frac{\partial}{\partial \nu}(f - g)|_\Gamma = g_2 & \text{on } \Gamma_{x_0}. \end{cases} \quad (4.24)$$

Hence, by the result of Proposition 8.1 and standard elliptic regularity,

$$f - g \in C^2(\Omega_{x_0}). \quad (4.25)$$

Combining (4.25) with (4.12) yields $f \in W_\infty^2(\Omega_{x_0})$. $\square$

5 Uniform Decay Rates: Proof of Theorem 1.3

Let $w_\gamma(t)$ be a solution to (1.5) corresponding to the initial data $(w_0, w_1) \in H^2(\Omega) \times H^1(\Omega)$.

Without loss of generality, we assume $E_{\gamma, w}(0) \leq 1$.

By arguments identical to those used in Lemma 3.2 (see (3.16)-(3.19)), we obtain

$$\begin{aligned}
& \|w_{\gamma_1}(t) - w_{\gamma_2}(t)\|_{H^2(\Omega)}^2 + \gamma^2 \|\nabla(w_{\gamma_1,t}(t) - w_{\gamma_2,t}(t))\|_{L_2(\Omega)}^2 + \|w_{\gamma_1,t}(t) - w_{\gamma_2,t}(t)\|_{L_2(\Omega)}^2 \\
& + \int_0^t \int_{\Gamma} [h_1(\frac{\partial}{\partial \nu} w_{\gamma_1,t}) - h_1(\frac{\partial}{\partial \nu} w_{\gamma_2,t})] \frac{\partial}{\partial \nu} (w_{\gamma_1,t} - w_{\gamma_2,t}) d\Gamma dt \\
& + \int_0^t \int_{\Gamma} [h_2(\frac{\partial}{\partial \tau} w_{\gamma_1,t}) - h_2(\frac{\partial}{\partial \tau} w_{\gamma_2,t})] \frac{\partial}{\partial \tau} (w_{\gamma_1,t} - w_{\gamma_2,t}) d\Gamma dt \\
& + \int_0^t \int_{\Gamma} [g(w_{\gamma_1,t}) - g(w_{\gamma_2,t})] (w_{\gamma_1,t} - w_{\gamma_2,t}) d\Gamma dt \\
& \longrightarrow 0 \quad \text{as } \gamma_1, \gamma_2 \longrightarrow 0.
\end{aligned} \tag{5.1}$$

Hence, in particular,

$$w_{\gamma} \rightarrow w^* \quad \text{in } C(0, T; H^2(\Omega)) \tag{5.2}$$

$$w_{\gamma,t} \rightarrow w_t^* \quad \text{in } C(0, T; L_2(\Omega)),$$

$$\lim_{\gamma_1, \gamma_2 \rightarrow 0} \int_0^T \int_{\Gamma} [h_1(\frac{\partial}{\partial \nu} w_{\gamma_1,t}) - h_1(\frac{\partial}{\partial \nu} w_{\gamma_2,t})] \frac{\partial}{\partial \nu} (w_{\gamma_1,t} - w_{\gamma_2,t}) d\Gamma dt \longrightarrow 0, \tag{5.3}$$

$$\lim_{\gamma_1, \gamma_2 \rightarrow 0} \int_0^T \int_{\Gamma} [h_2(\frac{\partial}{\partial \tau} w_{\gamma_1,t}) - h_2(\frac{\partial}{\partial \tau} w_{\gamma_2,t})] \frac{\partial}{\partial \tau} (w_{\gamma_1,t} - w_{\gamma_2,t}) d\Gamma dt \longrightarrow 0, \tag{5.4}$$

$$\lim_{\gamma_1, \gamma_2 \rightarrow 0} \int_0^T \int_{\Gamma} [g(w_{\gamma_1,t}) - g(w_{\gamma_2,t})] (w_{\gamma_1,t} - w_{\gamma_2,t}) d\Gamma dt \longrightarrow 0. \tag{5.5}$$

Moreover, from (H), we also obtain

$$\|\nabla(w_{\gamma_1,t} - w_{\gamma_2,t})\|_{L_2(\Sigma_T)} \rightarrow 0. \tag{5.6}$$

Hence,

$$h_1(\frac{\partial}{\partial \nu} w_{\gamma,t}) \longrightarrow h_1(\frac{\partial}{\partial \nu} w_t^*) \quad \text{in } L_2(0, T; \Gamma) \tag{5.7}$$

$$h_2(\frac{\partial}{\partial \tau} w_{\gamma,t}) \longrightarrow h_2(\frac{\partial}{\partial \tau} w_t^*) \quad \text{in } L_2(0, T; \Gamma) \tag{5.8}$$

From $E_{w_\gamma}(t) \leq 1$ and from Sobolev's imbeddings, we obtain

$$\sup_{t \geq 0} \|w_{\gamma,t}(t)\|_{L_p(\Gamma)} \leq C \quad \text{for all } p \geq 1. \quad (5.9)$$

Moreover, (5.6) and Sobolev's imbeddings give

$$\int_0^T \|w_{\gamma_1,t} - w_{\gamma_2,t}\|_{L_p(\Gamma)}^2 dt \rightarrow 0 \quad \text{for all } p \geq 1. \quad (5.10)$$

This together with (5.9) and hypothesis (H) gives

$$g(w_{\gamma,t}) \rightarrow g(w_t^*) \quad \text{in } L_2(\Sigma_T). \quad (5.11)$$

Convergence in (5.2), (5.7), (5.8), and (5.11), together with

$$[\mathcal{F}(w_\gamma(t)), w_\gamma(t)] \longrightarrow [\mathcal{F}(w^*(t)), w^*(t)], \quad (5.12)$$

(which in turn follows from (5.2) and (3.7), (3.19)), allow us to pass with a limit on equation (1.5).

Thus, we conclude that $w^* = w$ satisfies the original equation (1.1).

Convergence in (5.2) together with (1.8) then implies (1.10). $\square$

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