**NONEXISTENCE OF LYAPUNOV FUNCTIONS AND THE
INSTABILITY OF THE VON KARMAN VORTEX STREETS**

By

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Nonexistence of Lyapunov Functions and the Instability of the von Karman Vortex Streets

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Abstract

The instability of the von Karman vortex streets and the existence of a global Lyapunov function at the special aspect ratio $h/\ell = \frac{1}{\pi} \sinh^{-1}(1)$, are some of the difficulties with the well-known von Karman model. By consistently applying the principle of structural-stability, this author shows that a new family of near-equilibrium periodic solutions of the von Karman model for aspect ratios near $0.281 \dots$ supplies numerous theoretical candidates for observed vortex trails. This set of solutions implies that there is no global Lyapunov functions when $h/\ell \neq \frac{1}{\pi} \sinh^{-1}(1)$ which in turn leads to a rich variety of near-equilibrium solutions for the model.

1 Introduction

The von Karman model [1] introduced in 1911 predicts successfully many aspects of the nearly two-dimensional flows past long cylindrical objects [2], [3], [4], [5], [6], [7]. However, there are difficulties with this model: it is an infinite lattice consisting of two rows of vortices while the experimental situations are at best semi-infinite lattices (in superfluids for example), with complicated boundary conditions on the cylinder [8]. Moreover, three-dimensional effects such as Crow's instability should be factored in, albeit at a lower order.

The von Karman model [1], consists of a family of vortex streets [2], in which the main parameter is the aspect ratio h/ℓ of the separation h between the vortex rows, and the distance ℓ between adjacent vortices in the same row. Another parameter is the circulation $\mathcal{H}$ of the point vortices in the top row which has opposite sign to those in the bottom row. The fluid dynamics of uniform flows past the cylinder is governed by the Reynolds number. Regular vortex shedding begins at about $Re = 36$ and continues up to about $Re = 200$ after which the wakes become turbulent [9].

One of the serious difficulties of the von Karman model is its instability: the equilibrium is linearly (neutrally) stable if and only if the ratio h/ℓ of the separation h between the 2 vortex rows, and the distance ℓ between 2 adjacent vortices in the same row, has the isolated special value $\frac{1}{\pi} \sinh^{-1}(1) \sim 0.281$ [1]. In fact, the shortest wavelength periodic disturbance, i.e. the four-group (cell) in the von Karman lattice is the most unstable, and a small range of wavelengths around the shortest one, become unstable at any given ratio $\frac{h}{\ell} \neq 0.281 \dots$ [10].

Thus, the von Karman model is not structurally-stable [11] at $\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}$ and, its linear instability at other values of $\frac{h}{\ell}$ makes the equilibrium von Karman street unlikely theoretical candidates for observed vortex trails. To make matters worse, Kochin [12] showed that at the special value of $\frac{h}{\ell}$, the von Karman model has a Lyapunov function, the existence of which implies that arbitrarily small disturbances of this special equilibrium will eventually blow-up. This is even stronger than nonlinear instability.

On the other hand, vortex trails with 30 or more pairs of vortices frequently occur in normal fluids at approximately the special value of h/ℓ [13]. In superfluids, the vortex streets may contain hundreds of vortices because viscosity effects are small [14]. This paradoxical situation prompted many studies on the von Karman model in the past 80 years, in part due to the physical and historical significance of the von Karman vortex street.

At first, reseachers believed that the instability paradox could be resolved if one replaced the point vortices in von Karman's model with inviscid vortex cores. Saffman et. al. [15], [16] found that the difficulty remains: the equilibrium is neutrally stable only at one special value

$$\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1) + 0(a^2)$$

where a is the diameter of the vortex cores. Recently, Mackay [17] and Wan [18] confirmed this independently.

I will show in this paper that a new set of “small” (near-equilibrium) periodic solutions add more support to the physical relevance of the von Karman model by increasing significantly the number of theoretical candidates for observed vortex trails. I will sketch the Hamiltonian Hopf bifurcation which produces these small periodic solutions in section III, after the theoretical consequences of their existence are discussed in section II.

2 Nonexistence of Lyapunov Functions

The instability of the von Karman model leaves few theoretical candidates for the vortex trails that are observed in experiments. Between the von Karman street with $\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)$ which does not support bounded solutions other than the equilibrium, and the vortex streets with different aspect ratios which require exponential growth in a neighborhood of the equilibria, all studies in the past have focussed on the special case. Numerical experiments [5] show that small disturbances of this vortex street ($\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)$) grow slowly due to cubic nonlinearities in the equations (1).

Nonetheless, the traditional focus on the special vortex street suffers from a serious flaw because it is based on the isolated and non-structurally stable special case within the family of vortex streets that constitute the von Karman model. A scientifically correct approach should be based on the generic components of the model.

Departing from traditional approaches, I will focus on

(A) small (near-equilibrium) periodic solutions of the von Karman model instead of the equilibrium configurations,

(B) values of $\frac{h}{\ell}$ near 0.281 ... instead of the special value itself.

Apriori, it is not obvious that “small” non-equilibrium solutions exist in the von Karman model, although large amplitude quasi-periodic solutions have been found [19]. These invariant tori correspond to near-collision orbits which are far from the equilibrium vortex streets, and thus do not qualify to be in (A).

By Kochin’s analysis, arbitrarily small disturbances of the equilibrium vortex street will become unbounded eventually, provided the average ratio $\overline{(h/\ell)}$ of these disturbances equal the special value, 0.281... Thus, if small solutions (A) exists, they cannot have the special value of h/ℓ .

The discussion in section III shows that the exponential growth from equilibria actually saturates due to nonlinear effects. Thus,

(*) For each value of $\frac{h}{\ell}$ near $\frac{1}{\pi} \sinh^{-1}(1)$, there are two 1-parameter families of small periodic solutions of the von Karman model, at the shortest allowable wavelength, i.e., the four-group case [10].

These small periodic solutions (*), e.g. (24), (25) form two 2-parameter families with h/ℓ and energy, C being the parameters in the problem.

Since the existence result (*) extends to other wavelengths such as the six group case etc., the small periodic solutions in (A) form 3-parameter families, with wavelength λ as the third parameter.

I will now discuss the importance of the smallness and periodic properties of these solutions: A disturbance of the lattice which is small in the maximum norm, cannot be visually distinguished from the equilibrium street itself, for all practical purposes. In other words, a small disturbance is statically close to the corresponding equilibrium. Periodicity in time and space now implies that the small periodic solution is dynamically close to the equilibrium street, e.g., the average translation velocity of the lattice with a small periodic disturbance is close to the horizontal translation velocity of the unperturbed vortex street.

The above argument shows that the 3-parameter sets of solutions (A) are physically indistinguishable from the 1-parameter family of equilibrium vortex streets for which $\frac{h}{\ell}$ is the only parameter. Thus, the von Karman model now contains many more theoretical candidates for actual vortex trails when compared to previous estimates based on the equilibrium configurations.

The next result:

(**) For $\frac{h}{\ell}$ near but unequal to $\frac{1}{\pi} \sinh^{-1}(1)$, the von Karman model does not have a global Lyapunov function.

follows immediately from the existence of periodic solutions. The nonexistence of a Lyapunov function (**) means that the von Karman model can in principle support a much richer variety of bounded solutions. Indeed, one can show that:

(***) Subharmonic and tori bifurcations occur from the small periodic solutions (A) (cf. [20]).

By the same arguments as above, all these small periodic and quasi-periodic solutions are physically indistinguishable from the equilibrium streets. This further increases the number of theoretical candidates for observed vortex trails, which adds to the physical relevance of the von Karman model.

I will now apply the above results to a particular problem discussed by Sirovich [5], namely the comparison of von Karman lattice vibrational frequencies with those found in

the wake of cylinders [2], [21], that are different from the shedding frequency. An important question concerning the identification of the theoretical object that carries these vibrations in the von Karman model, remains to be satisfactorily answered. Before the discovery of the small periodic solutions (A), the only known theoretical candidates that could generate these frequencies are:

(i) a growing oscillatory solution whose averaged aspect ratio, $\overline{\left(\frac{h}{\ell}\right)} = \frac{1}{\pi} \sinh^{-1}(1)$,
and,

(ii) exponentially growing oscillations with $\overline{\left(\frac{h}{\ell}\right)} \neq \frac{1}{\pi} \sinh^{-1}(1)$,

Since candidate (i) stays reasonably small in the time for 100 pairs of vortices to be shed (but blows-up after that), it was chosen [5]. But there are two difficulties with this choice. First, candidate (i) is not structurally-stable and second, it does not work very well for superfluids [22] where the slowly growing oscillation would have to remain small for hundreds of pairs of vortices.

The generic small periodic and quasi-periodic solutions with $\overline{\left(\frac{h}{\ell}\right)}$ near $\frac{1}{\pi} \sinh^{-1}(1)$ (cf. (*) and (**)) are natural candidates for the generators of the frequencies measured because

(i) Hopf bifurcation implies that the frequencies of the periodic solutions (*) are close to those calculated from the eigenvalues of the von Karman street $\left(\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)\right)$ and are thus, comparable to the measured frequencies, [5]

(ii) the tori bifurcation (**) implies that the resulting small quasi-periodic solutions also have frequencies close to those of the special equilibrium street $\left(\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)\right)$ and are thus, good candidates when the wake supports multiple frequencies for certain values of Reynolds number.

(iii) the small periodic (*) and quasi-periodic solutions (**) are both statically and dynamically close to the equilibrium vortex streets, i.e. they look and behave like the regular vortex trails seen in experiments.

To summarize, the results in this section were obtained by consistently applying the principle of structural-stability (genericity) to the von Karman model (cf. the working assumptions (A) and (B)). The special case $\left(\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)\right)$ is not only structurally-unstable with respect to linear stability but it turns out that its structural-instability with respect to the existence of a global Lyapunov function (cf. (**)) makes the other (generic) vortex streets in the von Karman model more relevant physically.

3 Hamiltonian Hopf Bifurcation

The following equations [1]

$$\frac{d\bar{Z}_m^\pm}{dt} = \frac{i\mathcal{H}}{2\pi} \left(\sum_{k \neq m} \frac{1}{Z_m^\pm - Z_k^\pm} \pm \sum_{\ell=-\infty}^{\infty} \frac{1}{Z_m^\pm - Z_\ell^\mp} \right) \quad (1)$$

govern the dynamics of the von Karman model. Here $Z_m^\pm(t)$ denote the location of the m -th vortex in the upper and lower rows respectively, $\mathcal{H}$ is the circulation of the upper row, and the bar denotes complex conjugation. The equilibrium vortex street is a double row of vortices of opposite circulation located at

$$Z_m^\pm = (m \pm \frac{1}{4})\ell + U_s t \pm ih/2 \quad (2)$$

where

$$U_s = \frac{\mathcal{H}}{2\ell} \tanh\left(\frac{\pi h}{\ell}\right) \quad (3)$$

is the horizontal street velocity, and $\frac{h}{\ell}$ is the aspect ratio.

A N -periodic disturbance of a vortex street, (2) can be viewed in terms of repeated groups of $2N$ vortices. The corresponding vortices in each group form an infinite vortex row whose complex velocity potential is given by

$$W_j(Z) = \frac{(-1)^j}{2\pi i} \mathcal{H} \log \sin \left[\frac{\pi}{N\ell} (Z - Z_j) \right] \quad (4)$$

where Z_j is the location of a typical vortex in the j -th row. Following Kochin [12], the equations for the N -periodic (or $2N$ group) problem are given by

$$\frac{d\bar{Z}_k}{dt} = \sum_{\substack{j=1 \\ j \neq k}}^{2N} (-1)^j \frac{\mathcal{H}}{2N\ell i} \cot \frac{\pi}{N\ell} (Z_k - Z_j), k = 1, \dots, 2N \quad (5)$$

This is a hamiltonian system with $2N$ degrees of freedom and the hamiltonian is

$$Re \left[\sum_{k=1}^N \sum_{\substack{j=1 \\ j \neq k}}^{2N} W_j(Z_k) \right] \quad (6)$$

The equations (5) has the integral

$$K = \sum_{i \text{ odd}} Z_i - \sum_{j \text{ even}} Z_j \quad (7)$$

which is easily shown to be equal to the complex quantity

$$N(b + ih) \quad (8)$$

where $b = \ell/2$ is the stagger and h is the separation between the two vortex rows in equilibrium, (2).

Each N -periodic problem of the von Karman model, (1) now corresponds to a $2N$ degrees of freedom Hamiltonian system, (5), (6). Since it is well-known that the four-group case (shortest wavelength) is the most unstable [10], I will discuss only this case here. The Hamiltonian Hopf bifurcation result for the higher degrees of freedom problems can be obtained in a similar way, [23].

Defining the normalized perturbations ρ_k by :

$$Z_k(t) = U_s t + Z_k(0) + \frac{2\ell}{\pi} \rho_k, \quad k = 1, 2, 3, 4$$

where $Z_k(0)$ gives a snapshot of the equilibrium street, (2) at time $t = 0$, and setting

$$\alpha = 2(\rho_2 - \rho_1), \quad \beta = (\rho_2 - \rho_3) \quad (9)$$

one obtains the reduced flow of (5) (with $N = 2$) on the manifolds of constant K , (7):

$$\begin{aligned} \frac{d\bar{\alpha}}{dt} &= -\sin \beta \left[\frac{1}{\cos \alpha + \cos \beta} - \frac{1}{\cos \beta + \cos G} \right] \\ \frac{d\bar{\beta}}{dt} &= -\sin \alpha \left[\frac{1}{\cos \alpha + \cos \beta} - \frac{1}{\cos \alpha - \cos G} \right] \end{aligned} \quad (10)$$

This family of equations has the Hamiltonian function

$$H = \log \left| \frac{(\cos \alpha - \cos G)(\cos \beta + \cos G)}{\cos \alpha + \cos \beta} \right| \quad (11)$$

where the complex parameter G is given by

$$G = \frac{\pi}{2\ell} K \quad (12)$$

In other words equations (10) can be written in the form

$$\begin{aligned} \frac{d\alpha_r}{dt} &= \frac{\partial H}{\partial \beta_i}, \quad \frac{d\beta_i}{dt} = -\frac{\partial H}{\partial \alpha_r} \\ \frac{d\beta_r}{dt} &= \frac{\partial H}{\partial \alpha_i}, \quad \frac{d\alpha_i}{dt} = -\frac{\partial H}{\partial \beta_r} \end{aligned} \quad (13)$$

in terms of

$$\alpha = \alpha_r + i\alpha_i, \quad \beta = \beta_r + i\beta_i$$

Note that time has been scaled as follows

$$\tau = \frac{\mathcal{H}\pi t}{2\ell^2} \quad (14)$$

Since the normalized stagger, $\frac{b}{\ell}$ of the von Karman streets is fixed at the value $\frac{1}{2}$, the parameter G (12) is essentially one real parameter proportional to the aspect ratio h/ℓ . The other free parameters of the von Karman model, (1) namely $\mathcal{H}$ and ℓ have been absorbed into the new time, τ (14). Thus the equations (10), (11) constitute a one-parameter family of Hamiltonian systems of two degrees of freedom.

In addition to spatially N -periodic disturbances which corresponds to rational values of a normalized wavenumber ϕ/π , the von Karman model supports other spatially periodic disturbances with irrational values of ϕ/π as well as non-periodic disturbances which have compact support or decay fast enough along the lattice to be in the Hilbert spaces, $\ell_p, 1 \leq p < \infty$ [24]. While some linear results are available for these cases [24], the nonlinear problems remain open, and cannot be adequately discussed here.

I will now sketch the Hamiltonian Hopf bifurcation for the 1-parameter family of Hamiltonian system (10), (11), leaving the mathematical details for another paper [23]. At the special value of the aspect ratio, $\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)$, $\cos G = -i$ and the Hamiltonian (11) is in $1 : -1$ semi-simple resonance. This follows easily from the leading terms of the Taylor expansion for (11). The Hamiltonian (11) becomes:

$$\begin{aligned} H(q_1, q_2, p_1, p_2; \lambda) &= H_2^\lambda + H_4 + \dots \\ H_2^\lambda &= \frac{1}{2}(q_1^2 + p_1^2) - \frac{1}{2}(q_2^2 + p_2^2) + \frac{1}{2}\lambda_1(q_1^2 + p_1^2) - \frac{1}{2}\lambda_2(q_2^2 + p_2^2) \\ &\quad + \lambda_4(q_1 p_2 - q_2 p_1) \\ H_4 &= \frac{1}{8} [4q_1 q_2 p_1 p_2 - q_1^2 q_2^2 - p_1^2 p_2^2 + q_1^2 p_2^2 + q_2^2 p_1^2] \end{aligned} \quad (15)$$

in the new canonical variables, q_1, q_2, p_1, p_2 which are defined as follows:

$$\begin{aligned} \alpha_r &= (q_1 + p_2)/\sqrt{2} \quad , \quad \alpha_i = (p_2 - q_1)/\sqrt{2} \\ \beta_r &= (q_2 + p_1)/\sqrt{2} \quad , \quad \beta_i = (p_1 - q_2)/\sqrt{2} \end{aligned} \quad (16)$$

The first two terms in H_2^λ (15), usually denoted by

$$H_2 = \frac{1}{2}(q_1^2 + p_1^2) - \frac{1}{2}(q_2^2 + p_2^2) \quad (17)$$

implies that $H(q_1, q_2, p_1, p_2; \lambda)$ is in $1 : -1$ resonance when the parameters $\lambda \equiv (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ given by

$$\begin{aligned} \lambda_1 = \lambda_2 &= 2 \left(\frac{g}{1+g} - \frac{1}{2} \right) \sim -\frac{1}{2}(g - g_*)^2 + \dots \\ \lambda_3 \equiv 0, \lambda_4 &= \left(1 - \frac{2}{1+g^2} \right) \sim (g - g_*) + \dots \end{aligned} \quad (18)$$

are identically zero at $g = g_*$.

The 1-parameter family (15) is now parametrized by

$$g \equiv \sinh \left(\pi \frac{h}{\ell} \right) \quad (19)$$

and $g^* = 1$ is the special value of the parameter g at $\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)$. Moreover the remaining quadratic terms in H_2^λ make it a generic family (or versal family in the language of Arnold [25]). It follows from (18) that the coefficient λ_4 of the last term in H_2^λ is zero at $\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)$, which implies that the $1 : -1$ resonance is semi-simple. After a near-identity canonical transformation which leaves H_2^λ unchanged, one removes all order 4 terms in $H(q_{1i}, q_{2i}, p_{1i}, p_{2i}; \lambda)$ except H_4 . Thus the 4th order Birkhoff normal form for (15) is

$$H(q_1, q_2, p_1, p_2; \lambda) = H_2^\lambda + H_4.$$

The matrix for the quadratic part H_2^λ of the Hamiltonian (or the linearized Hamiltonian system).

$$\mathbf{J}[\mathbf{A} + \mathbf{B}(\lambda)] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \lambda_4 & \lambda_1 & 0 \\ \lambda_4 & 0 & 0 & -\lambda_2 \\ -\lambda_1 & 0 & 0 & -\lambda_4 \\ 0 & \lambda_2 & -\lambda_4 & 0 \end{bmatrix} \quad (20)$$

has eigenvalues which are given by:

$$\eta = \pm(\lambda_4 \pm i(1 + \lambda_1)). \quad (21)$$

The behavior of these eigenvalues, described in figure 2, signals a Hamiltonian Hopf bifurcation [8], [27], [28], [29], [30], [31], at the $1 : -1$ semi-simple resonance.

Indeed, the near-equilibrium periodic solutions of the system (10), (11) that bifurcate as the aspect ratio, $\frac{h}{\ell}$ changes from the special value, correspond to the critical points of the gradient system for the function

$$h(\mathbf{a}, \lambda, \eta) = H_2(\mathbf{a}) - \eta H(\mathbf{a}; \lambda) \quad (22)$$

on the constant energy surface given by

$$H(\mathbf{a}; \lambda) = C. \quad (23)$$

Here $\mathbf{a} \equiv (a_1, a_2, a_3, a_4)$ and, $H_2(\mathbf{a})$ and $H(\mathbf{a}; \lambda)$ are given in (15). It can be shown that the function $h(\mathbf{a}, \lambda, \eta)$ (22) is invariant under the transformation that interchanges a_1 with a_3 and a_2 with a_4 . It, (22) is also invariant under the related transformation which interchanges a_1 with $-a_3$, and a_2 with $-a_4$. Using the first of these symmetries, one obtains the following critical points: (a) for $c \geq 0$,

$$\begin{aligned} a_1^2 &= \frac{c}{1 + \lambda_1} + \sqrt{\frac{c^2}{(1 + \lambda_1)^2} + 16\lambda_4^2} \\ a_2 &= \frac{-2\lambda_4}{a_1}, a_3 = a_1, a_4 = a_2 \end{aligned} \quad (24)$$

(b) for $c < 0$,

$$\begin{aligned}
a_1^2 &= \sqrt{\frac{c^2}{(1+\lambda_1)^2} + 16\lambda_4^2} + \frac{c}{1+\lambda_1} \\
&\sim 8\lambda_4^2 \frac{(1+\lambda_1)}{|c|} + O(\lambda_4^4) \\
a_2 &= \frac{-2\lambda_4}{a_1}, a_3 = a_1, a_4 = a_2.
\end{aligned} \tag{25}$$

One obtains similarly, critical points which have the second symmetry, i.e. $a_1 = -a_3, a_2 = -a_4$. In summary, one has obtained the following existence result.

Theorem: For each value of the aspect ratio $\frac{h}{\ell}$ near $\frac{1}{\pi} \sinh^{-1}(1)$, there are two 1-parameters families of small periodic solutions of the Hamiltonian systems (10), (11), where the parameter is the energy, c .

Moreover, it can be shown that when $\frac{h}{\ell} = \frac{1}{\pi} \sinh^{-1}(1)$, the gradient systems, (22) has only the trivial critical point $\mathbf{a} \equiv 0$. This agrees with the existence of a global Lyapunov function for (10) when $\frac{h}{\ell}$ equals the special value.

Similarly one obtains small periodic solutions for other N -periodic cases (cf. [23]). The stability of these nonlinear normal modes, and the subharmonic and torus bifurcations from them will be discussed in [20].

In conclusion, the saturation of exponentially growing oscillatory solutions due to nonlinear effects, is established in the von Karman model for $\frac{h}{\ell} \neq \frac{1}{\pi} \sinh^{-1}(1)$ by the Hamiltonian Hopf bifurcation theory [27], [28]. The existence of small periodic solutions implied the nonexistence of a global Lyapunov function when $\frac{h}{\ell} \neq \frac{1}{\pi} \sinh^{-1}(1)$, i.e. Kochin's global Lyapunov function [12] is not generic. Then the nonexistence of a global Lyapunov function leads to the possibility of a rich variety of bounded near-equilibrium solutions in the von Karman model. These includes subharmonics, quasi-periodic solutions and transition chains of whiskered tori, (Arnold Diffusion). Finally, the analysis in this paper can be extended to the mixing layer case where the two parallel vortex rows are not staggered [9]. In that case, the corresponding equilibria are linearly ^{o.k.}unstable for all values of the aspect ratio $\frac{h}{\ell}$ and ~~yet,~~ ^{but} vortex trails are commonly found in jets [9], [13]. This makes the need for a Hamiltonian Hopf bifurcation even more compelling than in the von Karman model for vortex trails in wakes

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Figure Captions

Figure 1 The Bifurcation at a $1 : -1$ semi-simple resonance.

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