**STRUCTURED CONDITION NUMBERS FOR
LINEAR MATRIX STRUCTURES**

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STRUCTURED CONDITION NUMBERS FOR LINEAR MATRIX STRUCTURES*

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Abstract. Formulas for condition numbers of differentiable maps restricted to linearly structured subsets are given. These formulas are applied to some matrix maps on Toeplitz matrices. Other matrix examples are also indicated.

Key Words. Linear structure, structured condition number, Toeplitz matrix.

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1. Introduction. In this paper we consider structured condition numbers for some matrix maps on linearly structured classes of matrices, notably, Toeplitz matrices, which appear frequently in signal processing, see, for example, T. Kailath [10] and references therein.

To illustrate usefulness of structured condition numbers consider the matrix inversion map at the Hilbert matrix $A = \left((i+j-1)^{-1}\right)_{i,j=1}^n$. Its condition number which corresponds to perturbations of A in the set of all nonsingular matrices is $3 \cdot 10^{12}$ which is also equal to the condition number at A with respect to perturbations in the set of nonsingular Hankel matrices (see Section 4 for definition). As a consequence, if one attempts to invert A on a computer with unit round-off error $u > 10^{-12}$ using a general matrix solver, or a special algorithm defined on nonsingular Hankel matrices only, then one may expect the loss of all significant figures in A^{-1} . This was indeed observed in numerical experiments in Gohberg, Kailath, Koltracht and Lancaster [2]. On the other hand, the condition number of A with respect to perturbation in the class of Cauchy matrices, (matrices of the form $\left((t_i - s_j)^{-1}\right)_{i,j=1}^n$), is ≤ 740 . Therefore one may expect that a stable special algorithm defined on nonsingular Cauchy matrices only will give an accurate inverse of A . Supporting numerical evidence can be found in [2], and an explanation in Gohberg and Koltracht [3]. The numerical instability of a general matrix solver, or a Hankel solver, is understandable, namely, entries $(i+j-1)^{-1}$ are formed, thus introducing an ill-conditioned step in the course of solving a well conditioned problem.

We used for this illustration mixed structured condition numbers of A , (see Section 2, or Gohberg and Koltracht [4], for definition). For discussion of numerical stability of algorithms in general, we refer to Stoer and Bulirsch [13] and Golub and Van Loan [8].

We remark that the Cauchy structure for which we have such a large difference between structured and general condition numbers is not linear. For linear structures we expect that there will be little difference between the two condition numbers, although we can prove it for positive definite Toeplitz matrices only, see Section 3 below. Thus

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general purpose stable algorithms remain (forward) stable on positive definite Toeplitz matrices.

In Section 2 we give formulas for linear structured condition numbers based on explicit representation of a linear structure and a directional derivative of a map. In Section 3 we apply these formulas to some matrix maps at Toeplitz matrices. In Section 4 we give more examples of directional derivatives of some useful matrix maps, and of some linear structures other than Toeplitz. We follow concepts and definitions of [4]. A different approach to structured perturbations of matrices can be found in Higham and Higham, [9].

2. Linear Structures. Let $G : R^p \rightarrow R^q$ be a differentiable map defined on an open subset of R^p, D_G . The *usual* condition number of the map G at a point $A \in D_G, A \neq 0, G(A) \neq 0$, is given by:

$$(2.1) \quad k(G, A) = \frac{\|G'(A)\| \|A\|}{\|G(A)\|},$$

where $\|A\|$ is some norm on R^p , $\|G(A)\|$ is some norm on R^q , and $\|G'(A)\|$ is the corresponding operator norm of $G'(A)$, as a linear map from R^p to R^q . The *mixed* condition number of G at A is defined as follows:

$$(2.2) \quad m(G, A) = \frac{\|G'(A) D_A\|_\infty}{\|G(A)\|_\infty},$$

where $A = (A_1, \dots, A_p)$ and $D_A = \text{diag}\{A_1, \dots, A_p\}$. The mixed condition number relates normwise errors in $G(A)$ to componentwise errors in A , hence the term: mixed. To be more specific let X_i be the perturbed value of A_i such that

$$|X_i - A_i| \leq \epsilon |A_i|, \quad i = 1, \dots, p.$$

Then

$$\frac{\|G(X) - G(A)\|_\infty}{\|G(A)\|_\infty} \leq m(G, A) \epsilon + o(\epsilon).$$

Note that zero entries of A are not perturbed, so that X preserves the sparseness pattern of A . For a more detailed discussion of the condition numbers $k(G, A)$ and $m(G, A)$ see [4]. It is clear that if $k(G, A)$ is taken with respect to the ∞ -norm in R^p and R^q then

$$(2.3) \quad m(G, A) \leq k(G, A).$$

A *structured* subset of D_G is the range of another differentiable map, say, $H : R^n \rightarrow R^p$ with $n < p$. A *structured* condition number of G with respect to this structure is defined as the (usual or mixed) condition number of the restriction of G onto the

structured subset, or more formally, the structured condition number of G at $A = Ha$ is the condition number of $F = G \circ H$ at a , with the notation

$$m(F, a) = \mu(G, A),$$

$$k(F, a) = \kappa(G, A).$$

In this paper we only consider the case when H is a linear map, namely, when for $a = (a_1, \dots, a_n) \in D_H$,

$$Ha = a_1 h_1 + a_2 h_2 + \dots + a_n h_n,$$

where $h_1, \dots, h_n$ are some fixed vectors in R^p . We identify H with its matrix in standard bases of R^n and R^p , such that $h_1, \dots, h_n$ are the columns of H . For example, if $H : (a_1, \dots, a_n) \rightarrow \text{diag}\{a_1, \dots, a_n\}$ then h_k is an $n \times n$ matrix with 1 in (k, k) -th position, and zeros elsewhere, identified with a vector in R^{n^2} , (here $p = n^2$).

It follows from the chain rule of differentiation that the partial derivative of F with respect to a_k equals to the directional derivative of G with respect to h_k ,

$$\frac{\partial F}{\partial a_k} = \frac{\partial G}{\partial h_k}, \quad k = 1, \dots, n,$$

which is a vector in R^q . Therefore $F'(a) = \left[\frac{\partial G}{\partial h_1}, \dots, \frac{\partial G}{\partial h_n} \right]$, and hence

$$(2.4) \quad \kappa(G, A) = \frac{\left\| \left[\frac{\partial G}{\partial h_1}, \dots, \frac{\partial G}{\partial h_n} \right] \right\| \|a\|}{\|G(A)\|},$$

$$(2.5) \quad \mu(G, A) = \frac{\left\| \left[a_1 \frac{\partial G}{\partial h_1}, \dots, a_n \frac{\partial G}{\partial h_n} \right] \right\|_\infty}{\|G(A)\|_\infty}.$$

It is clear that if $\kappa(G, A)$ is taken with respect to the ∞ -norm in R^n and R^q then

$$(2.6) \quad \mu(G, A) \leq \kappa(G, A).$$

Suppose now that H is an isometry. In this case it is easy to see that

$$(2.7) \quad \kappa(G, A) \leq k(G, A).$$

Indeed, since $H'(a) = H$ for any a , it follows that $F'(a) = G'(A)H$ where $Ha = A$, and hence $\|F'(a)\| \leq \|G'(A)\|$. To obtain a similar inequality for mixed condition numbers we make an assumption about H which is satisfied for all linear structured classes of matrices considered in this paper.

PROPOSITION 1. *Suppose that the columns of $H, h_1, \dots, h_n$, have entries equal to zero or one only. Furthermore, suppose that $h_1, \dots, h_n$ are mutually orthogonal (or equivalently, that indices of 1's in $h_1, \dots, h_n$ are mutually disjoint). Then*

$$(2.8) \quad \mu(G, A) \leq m(G, A).$$

Proof. We need to show that the infinity norm of $F'(a) D_a = G'(A) H D_a$ is less than that of $G'(A) D_A$. Observe that D_A is a $p \times p$ diagonal matrix whose diagonal entries are $a_1, \dots, a_n$ in some order and with repetitions, (recall that $n < p$). Next note that $H D_a$ is a $p \times n$ matrix whose k -th column equals to the sum of all columns of D_A which contain a_k as an entry. Therefore the k -th column of $G'(A) H D_a$ equals to the sum of all those columns of $G'(A) D_A$ which have indices of those columns of D_A which contain a_k as an entry. Since each column of $F'(a) D_a$ is a sum of some columns of $G'(A) D_A$ such that each column of $G'(A) D_A$ is used exactly once, it follows that $\|F'(a) D_a\|_\infty \leq \|G'(A) D_A\|_\infty$. $\square$

We see from (2.4) and (2.5) that in order to find a structured condition number of G at $A = Ha$, given the structure map H , one needs directional derivatives of G . In the next section we consider some matrix maps with known directional derivatives and find their structured condition numbers at Toeplitz matrices.

3. Symmetric Toeplitz Matrices. In this section G is a map defined on $n \times n$ matrices and $H : R^n \rightarrow R^{n \times n}$,

$$H(a_1, \dots, a_n) = A = \begin{bmatrix} a_1 & a_2 & \ddots & a_n \\ a_2 & a_1 & \ddots & \ddots \\ \ddots & \ddots & \ddots & a_2 \\ a_n & \ddots & a_2 & a_1 \end{bmatrix}.$$

We identify $R^{n \times n}$ and R^{n^2} using any fixed ordering of matrix elements, say row by row. Thus k -th column of H, h_k , is an element of R^{n^2} which corresponds to the $n \times n$ matrix with ones in positions of a_k in A and zeros elsewhere, e.g. h_1 corresponds to the identity matrix. Next we consider some specific maps defined on $R^{n \times n}$.

3.1. Matrix inversion, $G : A \rightarrow A^{-1}$. The directional derivative of G in the direction h is given by ([8], Section 2.5):

$$\frac{\partial G}{\partial h} = -A^{-1} h A^{-1},$$

and hence $F'(a) = -[A^{-1} h_1 A^{-1}, \dots, A^{-1} h_n A^{-1}]$. Since the (i, j) -th entry of $A^{-1} h_k A^{-1}$ equals to $c_i^T h_k c_j$ where c_i is the i -th column of A^{-1} , it follows that

$$(3.1) \quad \|F'(a)\|_\infty = \max_{i,j=1,\dots,n} \sum_{k=1}^n |c_i^T h_k c_j|,$$

$$(3.2) \quad \|F'(a) D_a\|_\infty = \max_{i,j=1,\dots,n} \sum_{k=1}^n |a_k c_i^T h_k c_j|.$$

The corresponding condition numbers are now readily obtained. We remark that the computation of $\|F'(a)\|_\infty$ or $\|F'(a) D_a\|_\infty$ requires here $O(n^4)$ flops. This can be reduced to $O(n^3 \log n)$ by the use of FFT. If only one column of A^{-1} is required, e.g. the last column which gives the solution of Yule-Walker equations, then for the corresponding map, $F_n : a \rightarrow c_n$ we have

$$\|F'_n(a)\|_\infty = \max_{i=1,\dots,n} \sum_{k=1}^n |c_i^T h_k c_n|.$$

This can be computed in $O(n^2 \log n)$ flops, see Gohberg, Koltracht and Xiao [6]. When A is positive definite, $\kappa(G, A)$ and $\mu(G, A)$ can be estimated faster, with the speed of solving $Ax = b$.

PROPOSITION 2. *Let A be a positive definite Toeplitz matrix and let G be the map of matrix inversion. Then*

$$\mu(G, A) \leq \left\{ \begin{array}{c} \kappa(G, A) \\ m(G, A) \end{array} \right\} \leq k(G, A) \leq n^2 \mu(G, A),$$

where $k(G, A) = \|A\|_\infty \|A^{-1}\|_\infty$ and $\kappa(G, A)$ is taken with respect to the infinity norm in R^n and R^{n^2} .

Proof. All inequalities, except for the last one are just (2.3), (2.6), (2.7) and (2.8). To prove the last one denote $A^{-1} = (\sigma_{ij})_{i,j=1}^n$ and let

$$\sigma_{mm} = \max_{i,j=1,\dots,n} |\sigma_{ij}|.$$

Thus $\|A^{-1}\|_\infty \leq n\sigma_{mm}$ and $\|A\|_\infty \leq na_1$. On the other hand

$$\|F'(a) D_a\|_\infty = \max_{i,j=1,\dots,n} \sum_{k=1}^n |a_k c_i^T h_k c_j| \geq \sum_{k=1}^n |a_k c_m^T h_k c_m| \geq a_1 c_m^T c_m \geq a_1 \sigma_{mm}^2.$$

Since the norm of $A^{-1} = G(A)$ as a vector in R^{n^2} equals to σ_{mm} it follows that

$$\mu(G, A) = \frac{\|F'(a) D_a\|_\infty}{\|G(A)\|_\infty} \geq a_1 \sigma_{mm} \geq \frac{\|A\|_\infty \|A^{-1}\|_\infty}{n^2}.$$

□

It can be seen from the above proof that the factor n^2 in the last inequality is a result of a sequence of rude estimates. Moreover, a large number of experiments accompanying those reported in [6] and [7] show that the ratio of $\mu(G, A)$ and $k(G, A)$ is of order unity. Also, if $A^{-1} = |A^{-1}|$ where $|\cdot|$ denotes array of absolute values, then in fact, $\mu(G, A) = m(G, A)$. Indeed, in this case, for all i and j ,

$$\sum_{k=1}^n |a_k c_i^T h_k c_j| = \sum_{k=1}^n |a_k| c_i^T h_k c_j = c_i^T \left[\sum_{k=1}^n |a_k| h_k \right] c_j = c_i^T |A| c_j.$$

Hence $\|F'(a) Da\|_\infty = \|A^{-1} |A| A^{-1}\|_v$ where $\|\cdot\|_v$ equals to the largest absolute value among entries of an array. Since

$$m(G, A) = \frac{\|A^{-1} \cdot |A| \cdot A^{-1}\|_v}{\|A^{-1}\|_v}$$

(see, for example [4]), it follows that $\mu(G, A) = m(G, A)$. On the basis of all this evidence we claim that for practical purposes all condition numbers of Proposition 2 are equal to each other. Thus one can estimate $\|A^{-1}\|_\infty$ instead of (3.1) or (3.2) which can be done with the speed of solving $Ax = b$, see Dongarra, Bunch, Moler and Steward [1] for a lower bound and Koltracht and Lancaster [11], for an upper bound.

The analysis of this section remains true for banded Toeplitz matrices. The only difference would be that the upper summation limit, n , in (3.1) or (3.2) is replaced by the bandwidth.

3.2. Solution of $Ax = b$. It is convenient to consider here $G = G_1 \oplus G_2$ defined on $R^{n \times n} \oplus R^n$, such that

$$G[A, b] = x,$$

where $Ax = b$. Instead of one condition number we suggest to use a pair, corresponding to G_1 and G_2 respectively. For example,

$$k(G, [A, b]) = [k(G_1, A), k(G_2, b)] = \left[\|A\| \|A^{-1}\|, \frac{\|A^{-1}\| \|b\|}{\|A^{-1}b\|} \right].$$

This pair has the following meaning. If $\|\tilde{A} - A\| \leq \epsilon_1 \|A\|$ and $\|\tilde{b} - b\| \leq \epsilon_2 \|b\|$ then

$$\frac{\|\tilde{x} - x\|}{\|x\|} \leq k(G_1, A) \epsilon_1 + k(G_2, b) \epsilon_2 + o(\max(\epsilon_1, \epsilon_2)).$$

By [8] Section 2.5, for any direction h in R^{n^2} we have

$$\frac{\partial G_1}{\partial h} = -A^{-1} h x.$$

Therefore $G'(A) = -[A^{-1} h_1 x, \dots, A^{-1} h_n x]$ and

$$\|G'(A)\|_\infty = \max_{i=1, \dots, n} \sum_{k=1}^n |c_i^T h_k x|,$$

$$\|G'(A) H D_a\|_\infty = \max_{i=1, \dots, n} \sum_{k=1}^n |a_k c_i^T h_k x|.$$

These norms can be computed in $O(n^2 \log n)$ flops as explained in [6]. Perturbations in b are not structured and the corresponding condition numbers are

$$k(G_2, b) = \frac{\|A^{-1}\| \|b\|}{\|A^{-1}b\|},$$

$$m(G_2, b) = \frac{\| |A^{-1}| |b| \|_\infty}{\|A^{-1}b\|_\infty},$$

where $|\cdot|$ denotes array of absolute values. For $m(G_2, b)$ see Skeel [12].

The relation between $\kappa, \mu(G_1, A)$ and $k, m(G_1, A)$ requires additional study.

3.3. A Simple Eigenvalue. Let $G : A \rightarrow \lambda \neq 0$, where λ is a simple eigenvalue of A . Let x be the appropriately normalized eigenvector. Then, [8] Section 7.2,

$$\frac{\partial G}{\partial h} = x^T h x,$$

and hence $F'(a) = x^T [h_1, \dots, h_k] x$,

$$\|F'(a)\|_\infty = \sum_{k=1}^n |x^T h_k x|,$$

$$\|F'(a) D_a\|_\infty = \sum_{k=1}^n |a_k x^T h_k x|.$$

It is clear that $\|F'(a) D_a\|_\infty \leq |x^T A x| \leq \|A\|_\infty$. An open question is therefore to see if for small λ , the structured condition number

$$\mu(G, A) = \frac{1}{\lambda} \sum_{k=1}^n |a_k x^T h_k x|$$

could be much smaller than $k(G, A) = \frac{1}{\lambda} \|A\|_\infty$.

4. More Examples. In this section we list some other matrix maps for which directional derivatives are available and some, other than Toeplitz, linear matrix structures. Structured condition numbers for these maps and matrices can be readily obtained using techniques described above.

4.1 Maps.

i. *Exponential*, $G : A \rightarrow e^A$:

$$\frac{\partial G}{\partial h} = \int_0^1 e^{(1-s)A} h e^{sA} ds,$$

see [8], Section 11.3. For example, let A be a symmetric Toeplitz matrix and let $\sigma_1(t), \dots, \sigma_n(t)$ denote the columns of e^{tA} . Then

$$\|F'(a)\|_\infty = \max_{i,j=1,\dots,n} \sum_{k=1}^n \left| \int_0^1 \sigma_i^T(1-s) h_k \sigma_j(s) ds \right|,$$

and

$$\|F'(a) D_a\|_\infty = \max_{i,j=1,\dots,n} \sum_{k=1}^n \left| \int_0^1 \sigma_i^T(1-s) a_k h_k \sigma_j(s) ds \right|,$$

where the matrices $h_1, \dots, h_k$ are defined as in Section 3.

ii. *Logarithm*, $G : A \rightarrow \log(I + A)$:

$$\frac{\partial G}{\partial h} = \int_0^1 (I + sA)^{-1} h (I + sA)^{-1} ds,$$

see Gohberg and Koltracht [5]. Here let $\sigma_i(s), i = 1, \dots, n$ denote the columns of $(I + sA)^{-1}$. Then

$$\|F'(a)\|_\infty = \max_{i,j=1,\dots,n} \sum_{k=1}^n \left| \int_0^1 \sigma_i^T(s) h_k \sigma_j(s) ds \right|,$$

and similarly for $\|F'(a) D_a\|_\infty$.

iii. *Full rank least squares*, $Ax_{ls} = b, A$ is $m \times n, m > n, \text{rank } A = n, G = [G_1, G_2], h = [E, f]$ as in Section 3.2. Then, see [8] Section 6.1,

$$\frac{\partial G_1}{\partial E} = (A^T A)^{-1} [E^T (Ax_{ls} - b) + A^T E x_{ls}],$$

$$\frac{\partial G_2}{\partial f} = (A^T A)^{-1} A^T f.$$

iv. *Full rank underdetermined system*, $Ax_{mn} = b, m \leq n, \text{rank } A = m, x_{mn}$ is the minimal norm solution. Again, $G = [G_1, G_2], G : [A, b] \rightarrow x_{mn}$. Then, see [8] Section 6.7,

$$\frac{\partial G_1}{\partial E} = [E^T - A^T (AA^T)^{-1} [AE^T + EA^T]] (AA^T)^{-1} b,$$

$$\frac{\partial G_2}{\partial f} = A^T (AA^T)^{-1} f.$$

v. *Eigenvector of an $n \times n$ matrix with n different eigenvalues $\lambda_1, \dots, \lambda_n$ and corresponding right $x_1, \dots, x_n$ and left $y_1, \dots, y_n$ eigenvectors*, $G : A \rightarrow x_k$, eigenvector number k . Then, see [8] Section 7.2,

$$\frac{\partial G}{\partial h} = \sum_{\substack{i=1 \\ i \neq k}}^n \frac{y_i^H h x_k}{(\lambda_k - \lambda_i) y_i^H x_i} x_i,$$

where H denotes hermitian transposed.

Norms of the derivatives in iii) - v) can be expressed in the same way as in i), ii).

4.2. Linear structures.

i. *Hankel matrices:*

$$A = \begin{bmatrix} a_1 & a_2 & a_3 & \cdot & a_n \\ a_2 & a_3 & \cdot & \cdot & a_{n+1} \\ a_3 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_n & a_{n+1} & \cdot & \cdot & a_{2n-1} \end{bmatrix}.$$

As in the Toeplitz case the formulas (3.1) and (3.2) etc. apply with the only difference that the summation is from 1 to $2n - 1$. Here, (apart from the example of the Hilbert matrix reported in the introduction) we do not have much evidence about the relationship between usual and structured condition numbers for the inversion of Hankel matrices.

ii. *Circulant matrices:*

$$A = \begin{bmatrix} a_1 & a_2 & \cdot & \cdot & a_n \\ a_n & a_1 & a_2 & \cdot & a_{n-1} \\ a_{n-1} & a_n & a_1 & a_2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_2 & a_3 & \cdot & a_n & a_1 \end{bmatrix}$$

iii. *Brownian matrices*

$$A = \begin{bmatrix} a_1 & a_2 & a_3 & \cdot & a_n \\ a_2 & a_2 & a_3 & \cdot & a_n \\ a_3 & a_3 & a_3 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_n \\ a_n & a_n & \cdot & a_n & a_n \end{bmatrix}$$

iv. *Matrices with a fixed sparseness pattern.* Their structured condition number is given, however, by (2.2).

v. *Block matrices.* All of the above with entries a_k replaced by matrices. These matrices can be structured themselves, e.g. Toeplitz block-Toeplitz matrices.

vi. *Linear combinations of the above,* e.g. Toeplitz plus Hankel, Toeplitz plus diagonal, etc.

vii. Additional examples can be found in Van Loan [14].

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