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Approximation Algorithms for Tours of Orientation-varying View Cones

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Abstract

This paper considers the problem of finding the shortest tour to cover a given set of inverted cone views with apex angle α and height H when their apex points lie on a planar surface. This is a novel variant of the 3D Traveling Salesman Problem with intersecting Neighborhoods (TSPN) called Cone-TSPN. When the cones are allowed to tilt by an angle ϵ we have the tilted Cone-TSPN problem, to which we present a polynomial time approximation algorithm. We demonstrate through simulations that our algorithm can be implemented in a practical way and by exploiting the structure of the cones we can achieve shorter tours. Finally, we present results from covering a reflective surface (lake area) that shows the importance of selecting different view angles under strong sunlight specularities.

1 Introduction

Consider the task of an aerial vehicle charged with collecting images of a given set of locations. Such tasks arise in many applications such as crop monitoring, animal tracking and road inspection. In this paper, we study a novel coverage problem inspired by this scenario. We associate each measurement with an inverted cone apexed at the location of the interest. The height of the cone is associated with the desired resolution and the apex angle corresponds to camera field of view. In other words, each cone encodes the set of view points from which a target can be imaged at a desired location. See Figure 1. The task is to visit a given set of cones so as to ensure that all locations are covered.

Coverage is a fundamental problem in robotics and has been extensively studied [11, 7]. The basic task of visiting a given set of locations is the Traveling Salesman Problem (TSP) which admits a Polynomial Time Approximation Scheme (PTAS) [2, 13]. If we incorporate sensor footprint as a set of given areas instead of points we wish to visit then we have the Traveling Salesman Problem with Neighborhoods (TSPN) which is APX-hard [8, 17]. TSPN in 2D is a well established problem and many researchers have provided ‘approximation algorithms for various cases [15].

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Figure 1: In order for a UAV to capture images of a target animal it needs to enter an inverted view cone C_i , positioned at X_i with orientation vector $\vec{a}_i$ and apex angle $\alpha \leq \frac{\text{camera}_{FOV}}{2}$. If visibility is occluded, the view cones C_i may need to be tilted by an angle ϵ_i .

Many variants of TSPN in 3D are open. A PTAS was provided in [4] for disjoint polygons of comparable size and a QuasiPolynomial Time Approximation Scheme (QPTAS) in [5] for α -fat, weakly-disjoint neighborhoods. Recently in [14], we presented a polynomial time approximation algorithm for the 3D TSPN with intersecting neighborhoods when the neighborhoods take the form of right angular cones and their apex points lie on a planar surface. We called this problem Cone-TSPN. Our approach was based on the idea that by intersecting the cones with a set of horizontal planes we can reduce the problem to that of equal size disks on the plane with a number of upper bounded detours. Carefully chosen tilted views might be required if visibility of the target area is limited (see Figure 1) or when covering reflective surfaces (see Figure 2). In this work we extend Cone-TSPN and provide a polynomial time approximation algorithm for input view cones of varying orientation (or tilt).

More recently, there has been significant interest in informative path planning problems in which mobile robots aims to maximize an objective function related to information gain [18, 12]. For complete coverage path planning the work in [6] provides a constant factor solution for a UAV covering an urban building. In contrast to our work, tilted views are not taken into consideration. A coverage tree structure was proposed in [16] allowing for online, non-uniform adaptive coverage. However, this strategy cannot provide any guarantees on the length of the tour which coupled with the UAVs limited battery might result in an incomplete tour. Complete coverage of a tree surface using right angular cones was used in [19] for a UAV flying at a low altitude. While the solution worked well for the problem of complete yield coverage, it did not consider differently orientated view cones. Similar to our cone sensor model in 3D, [3] used wedges in 2D in order to perform target localization of collared animals with a directional antenna. Our work provides a formulation that can be used for the 3D case.

The rest of the paper is organized in the following way. The problem statement is presented in Section 2. Our analysis starts with disjoint cones of the same height and

orientation in Section 3. Then, we study the cases where the cones do not have the same height in Section 4 and are disjoint in Section 5. We conclude our analysis with non-disjoint cones with different orientation in Section 6 and evaluate the performance of our algorithm in Section 7. Complete proofs for the analysis of Sections 3, 4, 5, 6 are included in the appendix. Finally, in Section 8 we show our experimental results while covering a lake area with sunlight reflection with different view angles and discuss future work in Section 9.

Our contributions can be summarized as follows:

1. We show that tilted cone views are useful for coverage applications on reflective surfaces and present experimental results where we analyze sunlight specularities for different views over a lake
2. We present a polynomial time approximation algorithm that solves the Cone-TSPN problem when the cones are titled.

2 Problem Statement

Our problem, tilted Cone-TSPN can be formulated in the following way. We are given $C = (C_1, \dots, C_n)$, $\vec{A} = (\vec{a}_1, \dots, \vec{a}_n)$, $X = (X_1, \dots, X_n)$, $H = (h_1, \dots, h_n)$ where C is a set of cones $C_1, \dots, C_n$ with fixed apex angle α . Cone C_i has apex point X_i on the ground plane G with normal $\vec{n}$ and orientation vector $\vec{a}_i$ of length h_i such that $\vec{n} \angle \vec{a}_i = \epsilon_i$. The goal is to compute a minimum length trajectory T which intersects all cones in C .

We present a strategy called Orientation-Visit (Algorithm 5) which has an approximation factor

$$O\left(\frac{1 + \tan \alpha}{1 - \tan \epsilon \tan \alpha} \left(1 + \log \frac{\max(H)}{\min(H)}\right)\right) \quad (1)$$

with $|\epsilon| + |\alpha| < \frac{\pi}{2}$, which means that the cones (and by extension the UAV) do not touch the ground.

The tangent terms in Equation 1 behave well in practical situations. A camera has usually around $\frac{\pi}{2}$ Field of View (FOV) which translates to $\alpha \leq \frac{\pi}{4}$ and $\tan \alpha \leq 1$. If we set $\epsilon = \alpha - \frac{\pi}{30}$, which means that the view cones will almost touch the ground, we have $\tan \epsilon \leq 0.9$ and thus $\frac{1 + \tan \alpha}{1 - \tan \epsilon \tan \alpha} \leq 20$.

3 Disjoint Cones of Similar Height and Identical Orientation

We first present upper and lower bounds for the swept volume of a cone with a fixed orientation angle ϵ along a path. Then we use these bounds to compute the performance of our strategy outlined in Algorithm 1. The full proofs are provided in the appendix. For our problem, we assume that there exists an estimate height $\hat{h}$ in the direction of the cone orientation vector $\hat{a}$ such that $h^* \leq \hat{h} \leq 2h^*$ (its existence is proven in [14]).

Figure 2: When covering watery fields like wild rice, sunlight reflection can be a problem. By choosing our view angles carefully we can reduce sunlight specularities.

Figure 3: The sweep volume of a cone C along a path of length L_G^* can be split into three parts. The checkered pattern area of a semicircle swept by the upper half base of the cone. The triangle area PL_T swept by the largest inscribing triangle on cone C . The grey area $Vol(C)$ includes the points covered by two half cones located at the start and end of L_G^* .

Figure 4: The volume of intersection between two cones $C_{i'}$, $C_{j'}$ of equal height is proportional to the volume of $C_{i'}$, say by a constant number C_v .

Lemma 1. *Let the optimal tour T^* have length L^* and maximum height h^* . Its projection on plane G with normal $\vec{n}$ as T_G^* . For maximum cone height h_{max} and an estimate height $\hat{h}$ such that $\hat{h} \geq h_{max}$, $\hat{h} \leq 2h^*$, the Minkowski sum sweep volume $f(T_G^*, \hat{h})$ of a cone C with apex and orientation angles α and ϵ such that $|\epsilon| + \alpha \in (0, \frac{\pi}{2})$ traveling along T is upper bounded by $f(T_G^*, \hat{h}) \leq L^* \hat{h}^2 \tan \alpha (1 + \frac{5\pi}{6} \tan \alpha)$.*

The proof is based on computing the area covered by the upper half base of the cone, the area covered by largest inscribed triangle and the area covered by the remaining two half conic areas not covered by any other cone along the same path (see Figure 3).

Lemma 2. *Let the optimal tour T^* have length L^* and maximum height h^* . Its projection on plane G with normal $\vec{n}$ is T_G^* . For maximum cone height h_{max} and an estimate height $\hat{h}$ such that $\hat{h} \geq h_{max}$, $\hat{h} \leq 2h^*$, there exists a constant C_v such that the Minkowski sum sweep volume $f(T_G^*, \hat{h})$ of a cone C with apex and orientation angles α and $\epsilon = 0$ such that $|\epsilon| + \alpha \in (0, \frac{\pi}{2})$ traveling along T is lower bounded by $f(T_G^*, \hat{h}) \geq C_v \tan^2 \alpha \sum_{h_i \in H} h_i^3$.*

The proof takes advantage of the fact that the volume of a cone swept by a tour visit can be represented as the intersection between two cones $C_{i'}$, $C_{j'}$ of equal height h_i and apex angle α , with their apex points at distance $h_i \tan \alpha$. The volume of this intersection is proportional to the volume of $C_{i'}$ by a constant number C_v . For a single cone visit we have $\frac{\pi r_i^2 h_i}{3} = \frac{\pi h_i^3 \tan \alpha}{3}$ and the tour visits all cones with heights $h_i \in H$ (see Figure 4).

Lemma 3. *Given cones C_i , C_j with apex and orientation angles α and ϵ , height h_i , cap radius $r = h_i \tan \alpha$ and apex points $X_i, X_j \in$ plane G with normal $\vec{n}$. Let G_ϵ be a 2D plane with normal $\vec{n}_\epsilon$ passing through X_i such that $n \angle n_\epsilon = \epsilon$ and $\vec{a}_i \angle n_\epsilon = 0$. The relative arrangement of two cones tilted by angle ϵ with $\vec{a}_j$ intersecting the cap of c_i is identical to that of two non-tilted cones with their apex points $X_i, X_j \in G_\epsilon$ and one of them elevated from G_ϵ by $h_e = r \tan \epsilon$.*

Figure 5: Modified Slice-Visit. TSP will visit the cone at T_i , with $|X_i T_i| = h_t$. T_i is not further away than $h_i \tan \alpha$ from T_i^* projected along $\vec{a}$ onto P_{ht} .

The proof of Lemma 3 is based on Figure 5. $\triangle X_i E X_j$ is a right triangle with $\hat{E} = \frac{\pi}{2}$:
 $\tan \epsilon = \frac{|EX_j|}{|X_i E|} = \frac{|EX_j|}{r} \iff |EX_j| = r \tan \epsilon$.

Lemma 4. *Let the optimal tour T^* have length L^* and maximum height h^* . Its projection on plane G with normal $\vec{n}$ as T_G^* . For maximum cone height h_{max} and an estimate height $\hat{h}$ such that $\hat{h} \geq h_{max}$, $\hat{h} \leq 2h^*$, there exists a constant C_ϵ such that the Minkowski sum sweep volume $f(T_G^*, \hat{h})$ of a cone C with apex and orientation angles α and ϵ such that $|\epsilon| + \alpha \in (0, \frac{\pi}{2})$ traveling along T is lower bounded by $f(T_G^*, \hat{h}) \geq C_\epsilon \tan^2 \alpha \sum_{h_i \in H} h_i^3$, where $C_\epsilon = C_v(1 - \tan \alpha \cdot \tan \epsilon)^3$.*

Lemma 4 is a natural extension of Lemma 2 if the cones are tilted by ϵ . The volume of intersection between two cones will decrease if one of them gets elevated, equivalent to them being tilted (Lemma 3). This volume decrease is relative to the difference between the elevation and the cone height.

Algorithm 1 Modified Slice-Visit

Input: $x_0, C, H, \hat{A}, X, \epsilon, \alpha$

Output: tilted Cone-TSPN tour T

- 1: Define a plane P_{ht} that is parallel to G and elevated by coverage height $h_t = h_{min}$ in the direction of $\vec{a}$
 - 2: Intersect all orientation vectors with P_{ht} , the result is a number of points on P_{ht}
 - 3: Approximate TSP tour T_{ht} that visits all points on P_{ht} with starting point x_0 (using e.g. a PTAS for euclidean points)
 - 4: Connect P_{ht} with G using a vertical double line segment at x_0
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Lemma 5. *Given an input set C of n disjoint cones with orientation and apex angles ϵ, α , heights H and coverage height h_t , if tilted Cone-TSPN tour T is computed with algorithm Modified Slice-Visit using an $(1 + \beta)$ approximation, then on the coverage plane P_{ht} we have $\frac{L}{1+\beta} \leq 2h_t + \frac{L^* + 2n \tan \alpha}{\cos \epsilon} \text{mean}(H)$.*

The proof relies on the fact that the optimal tour T^* with length L^* can be at most $h_i \cdot \tan \alpha$ distance away from our strategy at height h_i (see Figure 5).

Theorem 6. *Given an input set C of n disjoint cones with orientation and apex angles ϵ, α and heights H . For an estimate height $\hat{h} \geq h_{max}$, $h^* \leq \hat{h} \leq 2h^*$ and $|\epsilon| + \alpha \in (0, \frac{\pi}{2})$, the strategy Modified Slice-Visit solves tilted Cone-TSPN with an approximation factor $(1 + \beta) \left(\frac{1}{\cos \epsilon} \left(\frac{\hat{h}}{\text{mean}(H)} \right)^2 \left(1 + \frac{5\pi}{6} \tan \alpha \right) \right)$.*

The proof uses the bounds from Lemmata 1, 4 and 5. Note that parameters $\epsilon, \alpha, \text{mean}(H)$ are provided as inputs and $\hat{h}$ is fixed.

4 Disjoint Cones of Identical Orientation

If the cones have different heights then Algorithm 1 has poor performance. This is addressed by splitting the cones into a number of height bins such that the requirements of Lemma 1 are met and perform modified Slice-Visit on each. The strategy is outlined in Algorithm 2.

Algorithm 2 Modified Height-Visit

Input: $x_0, C, H, \epsilon, \alpha$

Output: tiltedCone-TSPN tour T

- 1: $i = 0$
 - 2: **repeat**
 - 3: Create bin with height range $B_i = [2^i h_{min}, 2^{i+1} h_{min})$
 - 4: For all cones with height $\in B_i$ call Algorithm 1 and find tour T_{B_i} with starting point x_0
 - 5: $i = i + 1$
 - 6: **until** $2^i h_{min} > h_{max}$
 - 7: Connect all tours T_{B_i} with a vertical line segment at x_0
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Theorem 7. *Given an input set C of n disjoint cones with orientation and apex angles ϵ, α and heights H . For an estimate height $\hat{h} \geq h_{max}$, $h^* \leq \hat{h} \leq 2h^*$ and $|\epsilon| + \alpha \in (0, \frac{\pi}{2})$, the strategy Modified Height-Visit solves tilted Cone-TSPN with an approximation factor $(1 + \beta) \left(\frac{1}{\cos \epsilon} \left(1 + \frac{5\pi \tan \alpha}{6} \right) \left\lceil 1 + \log_2 \frac{\hat{h}}{h_{min}} \right\rceil \right)$.*

5 Non-Disjoint Cones of Identical Orientation

Since we already have a method for the disjoint case, we can handle intersections of cones with the same orientation simply by selecting their Maximal Independent Set (MIS) [9, 10]. The cones in the MIS intersect all cones in the set C . We then guarantee coverage of all cones not in the MIS by performing an extra detour on each cone in the MIS. The strategy is outlined in Algorithm 3.

Algorithm 3 Modified Height-Select

Input: C, H, ϵ, α

Output: tilted Cone-TSPN tour T

- 1: Sort cones into a set C_{sort} from shortest to tallest based on cone height h
 - 2: $MIS = \emptyset$
 - 3: **repeat**
 - 4: Select the first cone $C_1 \in C_{sort}$, this is the shortest cone in the set
 - 5: $MIS = MIS \cup C_1$
 - 6: Remove from C_{sort} cone C_1 and all cones intersecting with it
 - 7: **until** $C_{sort} = \emptyset$
 - 8: Call Algorithm 4 with input MIS
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Algorithm 4 Modified Height-Visit

Input: $C, H, x_0, \epsilon, \alpha, \vec{A}$

Output: tilted Cone-TSPN tour T

- 1: Truncate all cones to not be taller than $\hat{h} = h_{max}$
 - 2: $i = 0$
 - 3: **repeat**
 - 4: Create bin with height range $B_i = [2^i h_{min}, 2^{i+1} h_{min})$
 - 5: For all cones with height $\in B_i$ call Algorithm 1 and find tour T_{B_i} with starting point x_0 .
 - 6: **for** each cone c_i visited in T_{B_i} **do**
 - 7: Add $k = \lceil \frac{2}{1 - \tan \alpha \tan \epsilon} \rceil$ circumference detours perpendicular to $\vec{a}$ such that detour $j \in [1, k]$ is centered at T_i and has radius $j2(h_t - h_e) \tan \alpha$
 - 8: **end for**
 - 9: $i = i + 1$
 - 10: **until** $2^i h_{min} > h_{max}$
 - 11: Connect all tours T_{B_i} with a vertical line segment at x_0
-

Figure 6: The furthestmost cone C_j can be from C_i while still intersecting it results in have a single intersection point lying along the straight line s originating from its apex X_i .

Lemma 8. *Given an input set C with orientation and apex angles ϵ and α let the MIS be a subset of C such that the cones $\in$ MIS do not intersect one another and collectively intersect all cones $\in C$. If the MIS is selected using Modified Height-Select then Algorithm 4 adds $k = \lceil \frac{2}{1 - \tan \alpha \tan \epsilon} \rceil$ detours of length $\leq (8k\pi + 4)h_t \tan \alpha$ that visit every cone.*

The proof uses Lemmata 2 and 3 to compute the maximum distance between the center of two cones C_i, C_j tilted by ϵ and the diameter d_{je} of the elevated cone (see Figure 6).

Theorem 9. *Given an input set C of n disjoint cones with orientation and apex angles ϵ and α and heights H . For an estimate height $\hat{h} \geq h_{max}$, $h^* \leq \hat{h} \leq 2h^*$ and $|\epsilon| + \alpha \in (0, \frac{\pi}{2})$, the strategy Modified Height-Visit solves tilted Cone-TSPN with an approximation factor $(1 + \beta) \left(\frac{18k}{\cos \epsilon} \left(1 + \frac{5\pi \tan \alpha}{6} \right) \left\lceil 1 + \log_2 \frac{\hat{h}}{h_{min}} \right\rceil \right)$, where $k = \lceil \frac{2}{1 - \tan \alpha \tan \epsilon} \rceil$.*

6 Non-Disjoint Cones with different orientation

We create a number of orientation sets such that all cones have orientation angle difference θ such that $\epsilon_i \angle \epsilon_j = \theta \leq \frac{\alpha}{2}$, $\forall i, j$, $\theta + \alpha \in (0, \frac{\pi}{2})$. For each such set, we will show that we only need to add 1 additional circumference detour to our previous strategy. The strategy is outlined in Algorithm 5.

Lemma 10. *Given two intersecting cones $C_i \in$ MIS, $C_j \notin$ MIS with apex angle α , heights h_i, h_j and orientation angles ϵ_i and ϵ_j , with $\theta = \epsilon_i \angle \epsilon_j \leq \frac{\alpha}{2}$. If coverage height $h_t \geq h_i \frac{\sin \theta}{\sin(2\alpha + \theta)} + r \tan \epsilon$, we only need to add 1 additional circumference detour to the strategy outlined on Algorithm 3 centered at the center of the cone at height h_t , at a right angle with respect to $\vec{a}_i$ and at distance $4h_t \tan \alpha + 2(h_t - r \tan \epsilon) \tan \alpha$ to guarantee coverage of all cones intersecting the ones $\in$ MIS (see Algorithm 5 and Figure 7).*

Algorithm 5 Orientation-Visit

Input: $C, H, \vec{A}, \epsilon, \alpha$

Output: tilted Cone-TSPN tour T

- 1: **for** each orientation vector $\vec{a}_k \in \vec{A}$ **do**
 - 2: Create cone orientation set O_k with representative orientation $\vec{a}_k$
 - 3: Put all cones with orientation $\vec{a} \in \vec{A}$ such that $(\vec{a} \angle \vec{a}_k) \leq \frac{\alpha}{2}$ into set O_k
 - 4: Call Algorithm 3 with input O_k and perform $k + 1$ detours
 - 5: **end for**
-

Figure 8: The computed tour, colored blue, for a set of 100 randomly generated cones with apex angle $\pi/5$, tilt angle $\pi/4$ at coverage height $10 m$. S denotes the start and end position of the tour. The ellipsoids represent the cones $\in MIS$. The red points show the apex points of all the cones.

7 Simulations

In this section we will show through simulations that a practical implementation of our algorithm can compute efficient tours. We implemented our strategy with heuristics that improve performance while keeping the theoretical guarantees similar to [14]. Instead of performing the whole ellipsoidal detours on every cone in the MIS , we will only perform the part of the detour that visits another cone based on the cone intersections. After identifying a set of points that cover all cones, we use the concorde TSP solver [1] in order to compute an optimized tour (see Figure 8). To evaluate the efficiency of our implementation we compare it with the TSP tour on the cones apex points. This tour, the cone apex tour, does not use any information about the cones and can be used as an upper bound on the length of any TSPN tour on our view cone problem.

We generated sets of 100 view cones with an apex angle of $\pi/5$ positioned uniformly with $U(0, 100)$ over a $100 \times 100 m^2$ area. In total, 9 sets were generated with varying coverage heights $\in \{10, 20, 40\}$ and cone tilt angles $\in \{\frac{\pi}{4}, \frac{\pi}{5}, \frac{\pi}{10}\}$. The performance evaluation was

based on the average tour length difference between our implementation and the cone apex tour over 100 simulations. Table 1 presents how much shorter our tour is when compared with the cone apex tour for different coverage heights (columns) and cone tilt angles (rows). Our strategy performs better as the cones get taller and the performance stays consistent over multiple tilting angles. These results show that our algorithm can be used to provide shorter tours by exploiting the structure of the cones.

	$\pi/4 \text{ rad}$	$\pi/5 \text{ rad}$	$\pi/10 \text{ rad}$
10 <i>m</i>	17.23%	17.29%	16.26%
20 <i>m</i>	27.98%	27.83%	26.04%
40 <i>m</i>	36.54%	34.46%	35.44%

Table 1: Average performance improvement of our strategy over the cone apex tour for sets of varying coverage heights $\{10, 20, 40\}$ and cone tilting angles $\{\pi/4, \pi/5, \pi/10\}$.

8 Field Experiments

Currently, most applications that perform visual coverage of an area with a UAV are restricted to non-tilted, top-down views. However, this can result in significant loss of information and sometimes failure. To demonstrate this we consider two different scenarios. First, we consider the problem of visual coverage of reflective surfaces. We demonstrate that by choosing our angle views carefully we can reduce sunlight specularities. Second, we consider the problem of visual inspection of a target in difficult to see areas in cases where top-down coverage might fail. We show a scenario where tilted view angles can successfully obtain visual information of a target object or area even if it lies below another object blocking our view (e.g. bridges, trees).

8.1 Reflective Surface Coverage

We performed a set of experiments that verify our claim that coverage of reflective areas requires tilted view cones to avoid direct sunlight reflection. We covered a $30 \times 30 \text{ m}^2$ area over a lake at an altitude of 10 *m* with three different view angles. We will refer to the view angle with the least amount of sunlight reflection in the camera as the best view angle. Similarly, the worst view angle is the one with the most amount of sunlight reflection. Using the 0 deg, non-tilted views as baseline (down-looking views), we compared the amount of sunlight in the images using pixel intensities for over 1000 images (see Figure 2).

At the time of the experiments, the sun polar angle was 35 deg and the sun azimuthal angle was 250 deg from magnetic north. By using the law of reflection, we calculated the sun angles and identified that the best view angle was the one with a polar angle of 55 deg and azimuthal angle of 70 deg from magnetic north. The worst view angle had a polar angle of 55 deg and azimuthal angle of 250 deg. By covering the area with the best view angles, the average amount of sunlight specularities in the images was reduced by 81.57%. However, coverage with the worst view angles increased the average amount of sunlight specularities

Figure 9: The red ball was located underneath a cluster of trees. It was not clearly visible from all side views.

in the images by 671.75%. These results validate our claim that we can reduce sunlight specularities when covering reflective surfaces by choosing our coverage views carefully.

8.2 Visual Inspection of a Target

We performed a set of experiments that verify our claim that we can obtain visual information of a target lying under other objects. During this set of experiments we chose a red ball as our target object for easy detection in RGB images. We chose the target location to be a cluster of trees where visibility of the center was only possible with a couple of side views (see Figure 9). In order to compare our side view coverage (cone-TSPN) with top-down coverage, we placed the target (red ball) such that it was partially visible with top-down views (non-tilted view cones). Our measure of comparison was how much of the ball is visible in number of pixels relative to the distance between the camera and the target.

The camera we used had $\frac{\pi}{2}$ and $\frac{2\pi}{3}$ Field Of View (FOV) angles and we chose our coverage plane to be at 20m altitude for both top-down and side view coverage. Top-down views were acquired with a simple grid coverage pattern and the maximum number of pixels detected was 317. Tilted views were generated uniformly around the tree cluster with their bisectors spaced by $\frac{2\pi}{\alpha}$. This resulted in 14 tilted view cones, each centered at the tree cluster with $\alpha = \frac{\pi}{15}$, $\epsilon = \frac{\pi}{4}$ (see Figure 10). By choosing a smaller apex angle than the camera FOV means we restricted the target to be closer to the center of the images. In our setting the target was only visible from 3 views. The maximum number of pixels detected with side-view coverage was 664.

We calculated the ratio of the number of maximum red ball pixels between the two different coverage plans as the ratio of the maximum ground area covered with respect to red ball size. The side views at the same altitude are slightly further away when compared to the top-downs views and the number of maximum pixels is reduced by a ratio of at most 49%. However, the side views were able to see the entire ball and detected 209% more pixels compared to the top-down views. This means that the top-down were able to see about

Figure 10: Cone-TSPN tour of the visual inspection experiment. A red ball was placed underneath a cluster of trees. Visibility was limited with top-down views. A number of tilted view cones were generated and a cone-TSPN tour was found to cover them (left image). The cone-TSPN was converted to a GPS waypoint mission (right image).

23% of the red ball (see Figure 11). These results depend on the position of the target and the environment. It was possible to choose the location of the red ball such that it was not visible with top-down views (i.e. the target could not be located with top-down coverage). It was also possible to place the red ball on a location clearly visible with both coverage plans in which case tilted side views provide 49% less pixels (they are further away).

9 Conclusion and Future Work

In this work we studied the optimization problem tilted Cone-TSPN which is an extension of Cone-TSPN. We demonstrated that tilted view cones are necessary in real world applications. Field experiments demonstrated the importance of carefully selecting our view angles when covering reflective surfaces. Our main contribution is a polynomial time approximation algorithm that solves tilted Cone-TSPN and guarantees a solution that only depends on the cone apex angle α , height h and tilt angle ϵ . In addition to presenting the mathematical bounds, we implemented our strategy with heuristics that improve performance without sacrificing the theoretical guarantees. Simulations over large numbers of cones indicated that our strategy produced a tour that was shorter than the tour on the cone apex points which did not exploit the cone structure.

There are multiple venues for future work. Our practical implementation performed well for the individual cone sets; however, there might be further improvements we can achieve by optimizing the tour over multiple height and orientation sets. We would like to explore the extreme case where the cones touch the ground. A different method and analysis might be required for this case, and a ground robot collaborating with a UAV might be a more appropriate option. In addition, multiple UAVs might be able to cover different height and orientation sets at the same time improving the coverage speed. Finally, we would like to determine whether a UAV can autonomously decide on the best coverage resolution for a given target and choose the best coverage height online.

Figure 11: An example of the red ball detection. The left images are RGB and the right images are binary for the detection of the red ball. The maximum number of pixels detected for the tilted side views (top image) were 664 pixels. The maximum number of pixels detected for the non-tilted top-down views (bottom) were 317. These images

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10 Appendix

10.1 Proof of Lemma 1

The volume swept by a cone C with apex angle α , orientation angle ϵ and height h along a path of length $L_G^* \in G$ can be split into 3 parts (see Figure 3). First, we have $vol(C)$ as two halves of the volume of a cone covering the points at the starting and ending locations of L_G^* not covered by any other cone along L^* . Second we have PL_C , the area of a semicircle along the path L_G^* . PL_C is the area swept by the upper half base of the cone in the direction of the normal of G . If L_G^* is a straight line its volume looks like that of a slanted half cylinder. Third we have PL_T , the area of a triangle along L_G^* . PL_T is the area swept by the largest inscribing triangle on cone C . Finally, we note that $\hat{h} \geq h_n = \vec{a} \cdot \vec{n}$.

10.2 Proof of Lemma 2

The volume of a cone swept out by a tour visit can be represented as the intersection between two cones $C_{i'}$, $C_{j'}$ of equal height h_i and apex angle α , with their apex points at distance $h_i \tan \alpha$. The volume of this intersection is proportional to the volume of $C_{i'}$, say by a constant number C_v . For a single cone visit we have $\frac{\pi r_i^2 h_i}{3} = \frac{\pi h_i^3 \tan^2 \alpha}{3}$ and the tour visits all cones with heights $h_i \in H$.

10.3 Proof of Lemma 3

On Figure 5, $\triangle X_i E X_j$ is a right triangle with $\hat{E} = \frac{\pi}{2}$: $\tan \epsilon = \frac{|EX_j|}{|X_i E|} = \frac{|EX_j|}{r} \iff |EX_j| = r \tan \epsilon$

10.4 Proof of Lemma 4

Given two tilted by ϵ cones C_i , C_j , by Lemma 3 this is equivalent to two non-tilted cones where C_j is elevated by $h_e = r \tan \epsilon$. Create a new cone $C_{j'}$ with apex point X_j , apex angle α , height $h_i - h_e = h(1 - \tan \alpha \tan \epsilon)$ and radius $r_e = h \tan \alpha (1 - \tan \alpha \tan \epsilon)$ (see Figure 4). Create a new cone $C_{i'}$ with apex point $X_{i'}$ lying at the intersection between h_e and $|X_i D|$, with radius r_e , apex angle α and height $h_i - h_e$. The volume of intersection between C_i , C_j is larger than the volume of intersection between $C_{i'}$, $C_{j'}$. Applying Lemma 2 on $C_{i'}$, $C_{j'}$ and noting that $C_e = C_v(1 - \tan \alpha \tan \epsilon)^3$ produces the desired lower bound.

10.5 Proof of Lemma 5

Let the optimal tour T^* with length L^* visit cone C at point T_i^* , lying at cone height h_i . T_i^* cannot be further than $r = h_i \tan \alpha$ from the orientation vector $\vec{a}$ (see figure 5). Our strategy T with length L visit the cone at point T_i , which is a point on the orientation vector $\vec{a}$. Define plane P such that its normal is parallel to $\vec{a}$ and it passes through X_i . Project T_i^* along $\vec{a}$ onto P and call the resulting point $T_{i,P}^*$. On P $|T_{i,P}^* T_i| \leq h_i \tan \alpha$, then

$$L_P - L_P^* \leq \sum_{\forall c_i \in C} h_i \tan \alpha \quad (2)$$

The projection of a tour with length L^* onto P cannot make it longer than

$$L_P^* \leq L^* \quad (3)$$

Furthermore, the projection of tour on P with length L_P onto P_{h_t} such that $P \angle P_{h_t} = \epsilon$ cannot get longer than a factor of $\cos \epsilon$:

$$L \leq \frac{L_P}{\cos \epsilon} \quad (4)$$

Combining Equations 2, 3 and 4 we get

$$L \leq \frac{L^* + \sum_{\forall c_i \in C} h_i \tan \alpha}{\cos \epsilon} \quad (5)$$

Connecting $x_0 \in G$ to P_{h_t} requires 2 additional line segments of length at most $2h_t$. Finally, note that $\sum_{h_i \in H} h_i = n \text{mean}(H)$.

10.6 Proof of Theorem 6

From Lemmata 1 and 4 and noting that $\sum_{h_i \in H} h_i^3 \geq n \cdot \text{mean}(H)^3$, we have :

$$n \tan \alpha \leq \frac{L^* \hat{h}^2 (1 + \frac{5\pi}{6} \tan \alpha)}{C_\epsilon \text{mean}(H)^3} \quad (6)$$

Substituting to Lemma 5 we get :

$$\frac{L}{1 + \beta} \leq 2h_t + \frac{L^*}{\cos \epsilon} + 2 \frac{\text{mean}(H)}{\cos \epsilon} \frac{L^* \hat{h}^2 (1 + \frac{5\pi}{6} \tan \alpha)}{C_\epsilon \text{mean}(H)^3} \quad (7)$$

Noting that $h_t \leq \hat{h}$, $h_t \leq 2h^* \leq L^*$ gives the resulting bounds.

10.7 Proof of Theorem 7

Given coverage height h_t , and the approximation factor from Theorem 6, we have $\left\lceil 1 + \log_2 \frac{\hat{h}}{h_{\min}} \right\rceil$ such detours. Finally, note that $\hat{h} \leq 2h_t$ and $\text{mean}(H) \leq \frac{3}{2}h_t$.

10.8 Proof of Lemma 8

At the coverage height h_t , the maximum distance between the orientation vectors $\vec{a}_i, \vec{a}_j$ of a cone $c_i \in MIS$ and a cone intersecting it $c_j \notin MIS$ is $4h_t \tan \alpha$, since $2h_t \leq h_i$. If the cones are tilted by ϵ then from Lemma 3 the relative arrangement of two such cones is identical to that of two non-tilted cones with one of them $\notin MIS$, say c'_j , elevated from the plane by $h_e = h_i \tan \alpha \tan \epsilon$. The maximum distance between the orientation vectors $\vec{a}_i, \vec{a}'_j$ is $4h_t \tan \alpha$. At height h_t , cone c'_j has diameter $d_{je} = 2(h_t - h_e) \tan \alpha$, thus we can guarantee coverage from $\vec{a}_i$ by adding circumference paths at a right angle, originating at T_i and having radius that are incrementals of d_{je} . The total length is less than $k8\pi h_t \tan \alpha + 4h_t \tan \alpha$, where k is a constant integer such that $k = \left\lceil \frac{4h_t \tan \alpha}{2(h_t - h_e) \tan \alpha} \right\rceil = \left\lceil \frac{2}{1 - \tan \alpha \tan \epsilon} \right\rceil$. (see Figure 6).

10.9 Proof of Theorem 9

Similar to Theorem 6 we can add the new detours to the length of the detour in Lemma 5 :

$$\frac{L}{1+\beta} \leq 2h_t + \frac{L^*}{\cos \epsilon} + 2 \frac{\text{mean}(H) + h_t k(8\pi+4)}{\cos \epsilon} \frac{L^* \hat{h}^2 \left(1 + \frac{5\pi}{6} \tan \alpha\right)}{C_\epsilon \text{mean}(H)^3}$$

And similar to Theorem 7 we have $\left\lceil 1 + \log_2 \frac{\hat{h}}{h_{\min}} \right\rceil$ such tours.

10.10 Proof of Lemma 10

First we note that if the cones are tilted by ϵ then from Lemma 3 the relative arrangement of two such cones is identical to that of two non-tilted cones with one of them $\notin MIS$, say c'_j , elevated from the plane by h_e . Thus, without loss of generality we can assume that on the relative arrangement between cones C_i and C_j , $\epsilon_i = \frac{\pi}{2}$. On Figure 7, applying law of sines on $\triangle IX_k X_j$ gives $|X_k X_j| = |IX_k| \frac{\sin \epsilon}{\cos(\alpha+\theta)} = \frac{r_i}{\sin \alpha} \frac{\sin \theta}{\cos(\alpha+\theta)}$. Applying law of sines on $\triangle X_k M X_j$, gives $|X_k M| = \frac{r_i}{\sin(2\alpha+\theta)} \frac{\sin \theta}{\sin \alpha}$. Triangle $\triangle X_k H M$ is a right triangle, thus $|HX_k| = h_i \frac{\sin \theta}{\sin(2\alpha+\theta)}$, where $r_i = h_i \tan \alpha$. Now, if X_k and X_j are elevated by h_e then we have $|HX_k| = h_i \frac{\sin \theta}{\sin(2\alpha+\theta)} + h_e$. Thus, if coverage height $h_t \geq h_i \frac{\sin \theta}{\sin(2\alpha+\theta)} + h_e$ then these detours guarantee coverage of all cones intersecting C_i .

10.11 Proof of Lemma 11

On Figure 7, let cone c_k have the same height h_i and intersect cone $c_i \in MIS$ at it's right most cap point. The added detour from Lemma 10 visits c_k at it's left most and right most cap points at coverage height h_t . Let cone c_j be tilted at an angle θ with respect to c_i and intersect c_i at it's right most cap point. Let W be the point of intersection between the 2 right most rays of c_k and c_j . Triangle $\triangle X_k W X_j$ has $\hat{W} = \theta$ and $\hat{X}_k = \frac{\pi}{2} - \alpha$. From law of sines we have $|X_j W| = |X_k X_j| \frac{\cos \alpha}{\sin \theta}$. From the proof of Lemma 10 we know $|X_k X_j| = \frac{r_i}{\sin \alpha} \frac{\sin \theta}{\cos(\alpha+\theta)}$. Combining the two we have $|X_j W| = \frac{h_i}{\cos(\alpha+\theta)}$. Since $\frac{\pi}{2} > \theta > 0$ then $|X_j W| > \frac{h_i}{\cos(\alpha)}$ which is the length of the right most ray of cone c_k . Also note that point $M \in c_i \in c_j$ and point $W \notin c_i \in c_j$ for $\theta > 0$. And since points M, W belong to the same line $|MW| \in c_j$ it follows that the right most ray of cone c_k above height $h_i \frac{\sin \theta}{\sin(2\alpha+\theta)} + h_e$ is fully contained in cone c_j . Thus, the outermost detour always visits a tilted cone at distance greater than $4h_t \tan \alpha$ from the apex of c_i . And we know from Lemma 8 that the innermost detours visit every other cone at distance $[0, 4h_t \tan \alpha]$, which include any cone tilted at an angle θ closer to the apex of c_i than c_j .

10.12 Proof of Lemma 12

On Figure 12, let line $B'A'$ originate from point B' and be parallel to BA . Let the area of $\triangle A'B'D$ be A_3 , the area of $\triangle ABD$ be A_1 and the area of $\triangle EB'D$ be A_2 . The area of triangles $\triangle ABD$ and $\triangle A'B'D$ are related by $\frac{A_3}{A_1} = \frac{|BD|^2}{|B'D|^2} = \frac{\tan^2 \frac{\alpha}{2}}{(1 - \tan \alpha \tan \theta)^2}$. Since $A_3 \leq A_2$ it follows $\frac{A_2}{A_1} \geq \frac{\tan^2 \frac{\alpha}{2}}{(1 - \tan \alpha \tan \theta)^2}$. Finally, note that the ratios of the volume between two cones C_1, C_2 and the areas of their maximum inscribed triangles T_1, T_2 is related by $\frac{vol(C_1)}{vol(C_2)} =$

Figure 12: Computing the volume of intersection between two cones $C_{i'}$, $C_{j'}$ with apex angle α , orientation angle difference $\theta = \epsilon_i \angle \epsilon_j \leq \frac{\alpha}{2}$, height h and radius r . The volume of intersection between $C_{i'}$, $C_{j'}$ exists (grey area) and is always smaller to the volume from Lemma 4 by a constant factor.

$\frac{1}{(1 - \tan \alpha \tan \theta)^3} \frac{\text{area}(T_1)}{\text{area}(T_2)}$. Thus, the volume of intersection of cones $C_{i'}$, $C_{j'}$ compared the volume of intersection of cones C_i , C_j from Lemma 4 is not smaller than $\frac{\tan^2 \frac{\alpha}{2}}{(1 - \tan \alpha \tan \theta)^5}$.

10.13 Proof of Theorem 13

The proof is similar to Theorem 9. The added detours from Lemma 10 result in $k + 1$ circumference paths each with length less than $8\pi h_t \tan \alpha$. These paths can be connected with a double line segment of size less than $4h_t \tan \alpha$.

10.14 Proof of Lemma 14

We will create a number of bins on the surface of a unit sphere such that for any pair of unit vectors $\vec{a}$, $\vec{b}$ on the same bin the condition $\vec{a} \angle \vec{b} \leq \alpha$ is satisfied. If $\vec{a}$ and $\vec{b}$ belong on a unit circle they can be at most α away from each other and we need at least $\frac{2\pi}{\alpha}$ bins to cover all points of the circle. If $\vec{a}$ and $\vec{b}$ belong on a unit sphere then we can divide it into a number of strips of width α , each centered around a circle that is equal to or smaller than the unit circle. Since each strip is α away from each other we can split the surface of the sphere into $\frac{2\pi}{\alpha}$ strips. We can cover each strip in its entirety with $2\frac{2\pi}{\alpha}$ bins and the entirety of the sphere with $8\frac{\pi^2}{\alpha^2}$ bins.