REPORT
of
Committee on Thesis

The undersigned, acting as a Committee of
the Graduate School, have read the accompanying
thesis submitted by Paul Darwin Foote
for the degree of Doctor of Philosophy
They approve it as a thesis meeting the require-
ments of the Graduate School of the University of
Minnesota, and recommend that it be accepted in
partial fulfillment of the requirements for the
degree of Doctor of Philosophy.

Anthony Geleny
John F. Tate Chairman
J. H. Derby
E. N. Bauer
W. E. Brooke

May 17 1917

REPORT
of
Committee on Examination

This is to certify that we the undersigned,
as a Committee of the Graduate School, have given
Paul Darwin Foote final oral examination
for the degree of Doctor of Philosophy. We re-
commend that the degree
be conferred upon the candidate.
Minneapolis, Minnesota.

May 19, 1917.

Anthony Jeleny
Chairman
Geo. H. Bauer
John T. Tate
F. P. Leavenworth
C. F. Seiden
H. L. Stobin
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Henry A. Eissner
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Arthur H. Compton

The University of Minnesota
College of Science, Literature, and the Arts
Minneapolis

DEPARTMENT OF PHYSICS

May 16, 1917.

Dean Guy S. Ford,
Graduate School,
University of Minnesota.

Dear Sir:-

This is to certify that Paul D. Foote has passed the final written examination in his major subject for the Degree of Doctor of Philosophy.

Anthony Zeleny

SOME CHARACTERISTICS OF THE MARVIN PYRHELIONETER.

by

Paul D. Foote.

I. Introduction

Pyrheliometers are usually calibrated by comparison observations on the sun with a standard pyrheliometer which has been calibrated by exposing the receiver to a measured quantity of electrical energy. There are no published measurements of a calibration made by exposing the receiver to a known amount of energy of radiation. Such a calibration is highly desirable for always the question may arise as to whether the thermal processes taking place in the pyrheliometer receiver are exactly similar during the electrical calibration and during the radiometric use of the instrument. In the present work both methods were employed for the calibration of a Marvin silver disk pyrheliometer.

II. Description of Instrument.

The Marvin pyrheliometer has been the "working" instrument of the United States Weather Bureau for several years. A complete description of the instrument and methods of use is now in preparation by Professor Marvin, Chief of the Weather Bureau. This pyrheliometer is dynamic in type in that it is necessary to consider the rate at which the receiver gains heat when exposed to radiation, and the rate at which the receiver loses heat when shaded from radiation.¹

1. Kimball, Bull. Mount Weather Obs., (3), Part 2, p.73, 1910.

The essential feature of the instrument is the receiver. In the form used in the present work it consisted of a silver disk about 4.5 cm. in diameter and 0.3 cm. thick inside of which is carefully mounted with the best possible thermal contact a non-inductive spirally wound coil of No.35 silk insulated nickel wire in the form of a 3-lead resistance thermometer, having a total resistance of from 20 to 25 ohms. The coil

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serves both as the thermometer and as the heater in the case of an electrical calibration, the rise in temperature of the thermometer being observed when a known amount of electrical energy is dissipated in the coil. The receiver is mounted within a metal shell which is encased by a wooden shell in order to reduce local temperature variations to a minimum, and the type of suspension of the receiver is such that thermal losses by conduction are negligible. Before the front face of the receiver a limiting diaphragm is placed and leading from this through a hole in the metal and wooden shells is a carefully diaphragmed and blackened tube which serves the purpose of limiting the cone of rays to a convenient solid angle greater than that subtended by the sun. The end of the tube carries a double walled aluminum shutter operated by a magnetic release controlled by a chronograph which may be so set as to open or close the shutter at any desired instant. For solar work the instrument is mounted as an equatorial telescope and is driven by clock work so that the surface of the receiver is always presented normally to the sun. The instrument is shown in Fig. 1.

III. Method of Calibration: General,

The electrical calibration was made by subjecting the nickel coil of the thermometer to a measured current and observing the change in temperature of the silver disk. The radiometric calibration was made in a similar manner except that the heat was supplied radiometrically from an outside source. The source employed was a Lummer-Kurlbaum²

²Waidner & Burgess, This Bull., 3, p.165, 1907.

black body, or a black body of similar type, electrically heated, compensatedly wound to reduce the temperature gradient and to approximate temperature uniformity. The temperature of the inner inclosure, from which the radiation was taken, was measured by standard platinum, platinum-rhodium, thermocouples of the Bureau of Standards, accurately

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calibrated in terms of the melting points of Zn, Sb, and Cu. A water cooled diaphragm was mounted directly in front of the opening to the furnace. This diaphragm acts as the source of radiation. The equation of rate of energy transfer from the furnace to the pyrheliometer receiver is as follows.

$$J = \frac{\sigma}{\pi} (T^4 - T_0^4) \frac{A_1 A_2}{R^2}$$

where A_1 = area of water cooled diaphragm in front of furnace

A_2 = area of inmost or effective diaphragm in the pyrheliometer.

T = absolute temperature of furnace.

T_0 = absolute temperature of pyrheliometer receiver and surroundings.

σ = the Stefan-Boltzmann coefficient of radiation.

R = distance from A_1 to A_2 .

The quantity T_0^4 is negligible, for the present work in comparison with T^4 .

The coefficient σ has been determined in a variety of ways by many investigators, with widely discordant results. Within the past two or three years, however, the agreement has been quite satisfactory, values of different observers showing $\sigma = 5.7 \times 10^{-12}$ watts cm^{-2} deg^{-4} . This value is also the mean of all determinations to the present date. The constant is closely connected with the value of the electronic charge, the constant C_2 of the Planck equation of spectral distribution of the energy of a black body, and with the energy of emission of electrons in photoelectric phenomena. All of these interrelations point to a value of $\sigma = 5.7 \times 10^{-12}$. The most recent work on a direct determination of σ is by Coblentz³ who found as a mean

³ This Bull. 12, p.553, 1916.

of extended experiments $\sigma = 5.72$. Any error in the value of σ assumed, will enter directly into the calibration of the pyrheliometer. It is not likely however that the value of $\sigma = 5.7$ is in error by more than 1 %.

IV. SIMPLE THEORY OF THE MARVIN PYRHELIOMETER.

Pursuing the general treatment given by Kimball⁴ and others for various pyrliometers, the following describes the action of the

4. Bull. Mount. Weather Obs. 3, Part 2, 1910.

Marvin instrument, omitting the consideration of certain corrections which are discussed later. The case considered is for alternate periods of heating and cooling of 60 seconds duration.

- Let q = solar intensity at the earth's surface
 S = entire surface of receiver
 s = area receiving radiation
 c = water equivalent of receiver
 h = mean coeff. of heat loss from receiver, assuming Newton's law of cooling
 $E = \frac{hS}{c}$ a constant of any particular instrument
 T = temp. of receiver.
 T' = temp. of surroundings
 T_1 = temp. at beginning of cooling
 T_2 = temp. at beginning of heating
 t = time in seconds.

The variable thermometric state is expressed by

$$(1) \quad cdT = qsd t - h S (T - T') dt$$

(A) Cooling: When shutter is closed $q = 0$, whence

$$\frac{dT}{T - T'} = \frac{d(T - T')}{T - T'} = -\frac{hS}{c} dt \quad \text{since } T' \text{ is constant}$$

$$T - T' = A e^{-\frac{hS}{c} t} = A e^{-Et}$$

when $t = 0 \quad T = T_1$

(2) whence $T = T' + (T_1 - T') e^{-Et}$ the equation for cooling

(B) Heating from (1) $\frac{dT}{(T - T') - \frac{qs}{Ec}} = -E dt$

and since $T = T_2$ when $t = 0$

(3) $T = T' + \frac{qs}{Ec} + (T_2 - T' - \frac{qs}{Ec}) e^{-Et}$ the equation for heating.

The amount of cooling in 60 seconds is given by eq. (2) when 0 and 60 are substituted for t , whence:

(4) Cooling in 60 seconds = $\Delta T_c = T_1 - T' - T_1 e^{-60E} + T' e^{-60E}$

By similarly using eq.(3)

$$(5) \text{ Heating in 60 seconds } = \Delta T_h = T' + \frac{qs}{EC} + (T_2 - T' - \frac{qs}{EC}) e^{-60E} - T_2$$

In the use of this instrument the temperature of the receiver is so adjusted that the amount of cooling ΔT_c equals the amount of heating ΔT_h so that after a two minute period consisting of one heating and one cooling the receiver returns to its original temperature. Whence on equating (4) and (5) one obtains

$$(6) T_1 - T' = -T_2 + \frac{qs}{EC} + T'$$

But $T_1 - T_2 = \Delta T$ = the total change in temperature during a 60 second period. Whence from (6) and (2) it is readily seen that;

$$(7) \Delta T_{60} = \frac{qs}{EC} \frac{1 - e^{-60E}}{1 + e^{-60E}} = \text{const.} \times q = \frac{1}{F} \Delta Q$$

Where F is a constant and ΔQ the heat incident upon the receiver in 60 seconds.

Equation (7) states that the change in temperature in 60 seconds equals a constant of the instrument times the solar intensity, and is independent of the room temperature. This is derived on the assumption of Newton's law of cooling, but since all the temperature differences generally observed are fairly small there is probably no serious question in regard to the applicability of this law. Questions of greater moment are those regarding the types of temperature gradient established in the receiver under the four different conditions of cooling and heating, electrically and radiometrically, and the lag of the thermometer. These are considered later.

An experimental condition which is of interest, as will appear below, is that in which the readings over the first 10 second period immediately following a shift of the shutter are discarded, and the change in temperature of the receiver is observed for the 50 second interval immediately following. It is readily shown that the fall in temperature during such

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a 50 sec. interval of cooling is equal to the rise in temperature during the corresponding 50 sec. interval of heating.

$$(8) \text{ cooling } T_{10} - T_{60} = (T_1 - T') (e^{-10E} + e^{-60E}) \text{ from eq. (2)}$$

$$(9) \text{ heating } T_{60} - T_{10} = -(T_2 - T' - \frac{qs}{EC}) (e^{-10E} - e^{-60E}) \text{ from eq. (3)}$$

But from (2) & (3) evaluated over a 60 sec. interval we have eq. (10)

$$\text{and (11): (10) } T_1 - T' = \frac{\Delta T_{60}}{2} + \frac{1}{2} \frac{qs}{EC}$$

$$(11) \quad T' - T_2 = \frac{\Delta T_{60}}{2} - \frac{1}{2} \frac{qs}{EC}$$

Substituting (10) and (11) in (8) and (9) it is seen that the cooling in 50 seconds equals the heating in 50 seconds.

If we denote the amount of heating or cooling in the 50 sec. interval defined above, by ΔT_{50} , the relation between ΔT_{50} and ΔT_{60} is given by equation (4) and (8) as follows:

$$(12) \quad \Delta T_{60} = \frac{1 - e^{-60E}}{e^{-10E} - e^{-60E}} \Delta T_{50}$$

Equation (12) is discussed later.

V. Method of Calibration. Detailed Description.

In the radiometric calibration the pyrhelimeter was exposed to and shaded from the radiation emitted through an opening in a blackened, water cooled diaphragm placed in front of the black body. The opening through the water cooled compartment was conical and of greater angle than that demanded by the pyrhelimeter. Care was always taken to operate the pyrhelimeter at such a distance from this diaphragm that the radiation striking the receiver came from the portion of the black body of which the temperature was measured by the thermocouples. The temperature gradient throughout this section of the black body is easily made practically negligible.⁵

⁵ Waidner and Burgess, This Bull. 3, p.167, 1907.

The emfs of the two couples were measured by a Leeds and Northrup potentiometer, shown in Fig.1, and in converting the emf readings to temperature

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the usual consideration was given to the correction of the "thermoelectric" scale to the thermodynamic scale.⁶

6. This Bull. 2, p.11, 1913.

The resistance of the thermometer of the pyrheliometer was measured by the precision thermostated bridge described in detail by Mueller.⁷

7. Bur.Stds. Sci. P. No.288.

This dial type bridge reads directly to 0.0001 ohm and by interpolating the readings of the galvanometer the sensibility of which was determined at all points of the scale used, at least one more figure in the resistance measurement could be relied upon. The scale deflections were always maintained as nearly zero as possible.

The small bridge current passing through the thermometer coil in general had an appreciable heating effect. The current was maintained constant by using rather high voltage and swamping resistance so that the small variations in resistance of the thermometer were negligible. When necessary the current was measured by the potentiometric method and controlled by an adjustable low resistance slide wire. The constant current was allowed to flow during the entire radiometric calibration, during both heating and cooling of the receiver, so that it produced no net effect upon the measurements. Fig.2 shows the experimental arrangement for a radiometric calibration.

The main difficulty with the electrical calibration was to devise a suitable means for measuring the resistance during the period of heating. The wiring diagram is given in Fig.2. The maximum current required for heating was still small enough to permit the use of the bridge in the manner illustrated. The galvanometer sensibility was maintained approximately the same for both the "heating" and "cooling" bridge currents and the galvanometer was critically *damped* in each case by suitable series and shunt resistances. Both thermometer currents were measured potentiometrically and were maintained constant by

adjusting a slide wire rheostat. The heating of each current was computed, using the "mean" resistance of the thermometer coil throughout the periods of heating and of cooling. The net heating effect is equal to the difference between the heat developed during the period of heating and ^{that} developed during the period of cooling.

In a large portion of the work, for both the radiometric and electrical calibration, readings were taken every 10 seconds in order to determine accurately the form of the heating and cooling curves. For enabling the observer to obtain readings exactly upon the 10 second period the chronograph was arranged to tap a bell at the proper instant. One observer operated the bridge and another the potentiometer. 7 The relation between the temperature of the thermometer and its resistance requires an independent experiment in which the receiver is removed from the pyrliometer and mounted in a constant temperature bath, the temperature of which may be varied over the range required. The temperature relation so found may be accurately expressed by a parabolic equation. Thus for silver block No. II $R = 23.041 + 0.10033t + 0.001093t^2$ and for silver block No. III $R = 19.521 + 0.08394t + 0.00010127t^2$ where t is the temperature centigrade. These experiments were kindly performed by Prof. H.H. Kimball of the ^{U.S.} Weather Bureau.

The simple theory derived in Sec. IV requires certain modifications to fit the true experimental conditions. The room temperature T' does not remain constant during a run of an hour required for any one set of measurements. The result is that there is a gradual creep back and forth in the equilibrium position of the thermometer temperature so that the thermometer does not return exactly to its initial temperature after a complete cycle of one heating and one cooling. Each observed heating and cooling curve may be readily corrected to the ideal curve by assuming a linear change in temperature over the complete cycle in question. Thus, for example, if, after a cycle of two minutes, the

thermometer coil had risen $.001^\circ$ the measurements at every 10 sec. interval were corrected by adding $1/12$, $2/12$, $3/12$, etc., of $.001^\circ$ to the 1st, 2nd, 3rd, etc., reading of the cycle. A correction method developed by Prof. Marvin which is more convenient and accurate, and which will be described in his paper referred to above, was frequently employed. The corrected curves were always used in the computation of the results. In general variations of the above description tend to eliminate themselves over a long period, the temperature rising for part of the series and falling for the rest of the time. However runs were obtained for all possible conditions, viz., continually increasing temperature, continually decreasing temperature, and practically constant temperature. No dependence of the results upon such conditions could be observed.

VI. Detection of the Presence of Lag.

A second modification of the simple theory discussed in Sec. IV is required by the presence of a "lag", in the temperature or resistance measurements. In order to obtain electrical insulation of the thermometer coil from the silver receiving disk the wire is silk covered. This silk covering of course acts also as a thermal insulator. The result is that the temperature of the thermometer lags behind that of the disk when the heating is radiometric, and leads the temperature of the disk when the heating is electrical. In the case of an entire cooling following a radiometric heating and for a part of the cooling following an electrical heating, the thermometer temperature again lags behind that of the disk. If the lag effect is not corrected for, the "constant" F (Sec. IV, eq. (7)) increased for radiometric exposures and decreases for electrical exposures as the time of exposure is shortened. In the case of electrical heating where the heat supplied is put directly into the thermometer coil, the lag effect due to thermal insulation increases the observed ΔT over the value of ΔT for the disk, and thus the measured F is too low. The reverse is true when the heating is radiometric. In order to detect the presence of any lag the receiver was exposed to

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radiation or to the heating current and then cooled, the periods of heating and cooling being equal and of the following durations, 10, 20, 30, 60, and 120 seconds. Observations were made at the beginning and end of the coolings and heatings, i.e., at the instant of shifting from a cooling to a heating and vice versa. The results of several experiments with Marvin pyrheliometer No.7, Disk No.3, are given in Fig.3. Here the ordinate are values of F' obtained by multiplying the value of F , Sec.IV, Eq.7, by $\frac{(5.5)(1.20)}{1.217}$ for Instrument No.7. The constant F' is more convenient than F in reducing solar measurements to calories/cm.² min. and will be used exclusively hereafter. If there were no lag effect whatever the two curves in Fig. 3 would coincide exactly and would pass through the point $F' = 2.135$ at $t = 60$ sec. as will be shown later. The curve F' versus t then would differ slightly from a horizontal straight line, because of the fact that the cooling of the disk follows Newton's exponential law instead of a linear relation with time. The above experiment demonstrates clearly the presence of a lag effect. The magnitude of this lag may be calculated in the manner shown below.

VII/ Analysis of the Heating and Cooling Curves.

Heatings and coolings were made alternately every minute for a period of 1/2 to 1 hour during which time bridge readings of the thermometer temperature were taken every 10 seconds. The heatings and coolings for the entire period were used to form a typical heating and cooling curve. This curve was found to be an exponential as would follow from Newton's law of cooling. Thus in the case of cooling according to Newton's law we have as in Sec, IV:

$$(1) \quad \frac{dT}{dt} = -E (T - T')$$

Since resistance is measured directly, instead of temperature, it is more convenient to express equation (1) in terms of resistance R . Over small ranges of temperature, change in temperature is proportional to

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change in resistance, i.e.

$$R = a + bT$$

$$\text{and } \frac{dR}{dt} = b \frac{dT}{dt}$$

Hence

$$(2) \quad \frac{dR}{dt} = -E (R - R')$$

$$(3) \quad R = R' + Ae^{-Et}$$

$$(4) \quad \frac{dR}{dt} = -AEe^{-Et}$$

$$(5) \quad \log_{10} \left(\frac{-\Delta R}{\Delta t} \right) = \log AE - .4343Et$$

If the observations, accordingly, satisfy Newton's law of cooling the plot of $\log_{10} \left(\frac{-\Delta R}{\Delta t} \right)$ versus t will be a straight line. The slope of this straight line determines E the coefficient of cooling. As an illustration of this method the following examples may be cited.

Table I

Cooling and heating curve data, electrical calibration.

	Time	$(R_t - R_0)$ observed.
heating	0	.00000 ohms
	10	.00690
	20	.01241
	30	.01780
	40	.02296
	50	.02805
	60	.03292
cooling	10	.02612
	20	.02055
	30	.01521
	40	.01004
	50	.00495
	60	.00000

Table 2

Analysis of heating and cooling curve, electrical calibration.

mean	$-(\Delta R / \Delta t)_{10 \text{ sec}}$		$\log_{10} \left(\frac{-\Delta R}{\Delta t} \right)$		$-\Delta R / \Delta t$	
time	heating	cooling	mean	observed	curve	computed
5	.000690	.000680	.000685	6.836-10	6.748-10	.000560
15	531	557	544	.736	.737	546
25	539	534	536	.729	.726	532
35	516	517	516	.713	.714	518
45	509	509	509	.707	.703	505
55	487	495	491	.691	.691	491
1	2	3	4	5	6	7

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The plot of $\log \left(\frac{-\Delta R}{\Delta t} \right)$ versus t is given by circles in Fig.4. Columns 6 and 7 of the above table are obtained from the values given by this plot. On account of the lag (in this case a lead) the observed value of $\Delta R/\Delta t$ in the interval 0 to 10 sec. is far too great. After the first 10 seconds however the observations lie very satisfactorily on the straight line of Fig. 4. The coefficient E determined by the slope of the straight line is $E = 0.00269$. From columns 2 and 3 it is seen that the heating curve is similar in type to the cooling curve as demanded by the theory developed in Sec. IV. This series is picked at random from a large number. Some of the series obtained showed exact agreement of the heating and cooling curves, others showed somewhat poorer agreement than the example cited. Since every point on the cooling curve except the point taken at time 0 to 10, mean $t = 5$, lies well upon the straight line of Fig.4, it is clear that the error due to lag is negligible (see below) after 10 seconds. Hence in the general use of the instrument it is sufficient to take readings 10 seconds and 60 seconds after the starting of a heating and of a cooling thus giving a heating or cooling over a 50 second interval. To obtain the correction factor, f , to convert a 50 second heating or cooling into a 60 sec. interval use is made of Eq.12, Sec.IV. $f = \frac{1 - e^{-60E}}{e^{-10E} - e^{-60E}} = 1.217$ for $E = .00269$

Table 3.

Cooling and heating curve data; radiometric calibration.

	Time	$(R_t - R_0)$ observed.
Heating	0	.00000
	10	112
	20	247
	30	381
	40	508
	50	634
	60	759
Cooling	10	646
	20	511
	30	377
	40	250
	50	124
	60	000

Table 4.

Analysis of heating and cooling curve, radiometric calibration.

t	$-\Delta R/\Delta t$		$\log(-\Delta R/\Delta t)$		$-\Delta R/\Delta t$	
	heating	cooling	mean	obs.	curve	computed.
5	.000112	.000113	.000112	6.049	6.144	.000139
15	135	135	135	.130	.133	136
25	134	134	134	.127	.122	132
35	127	127	127	.104	.111	129
45	126	126	126	.100	.100	126
55	125	124	124	.093	.089	123
1	2	3	4	5	6	7

The plot of $\log \left(-\frac{\Delta R}{\Delta t} \right)$ versus t is given by dots in Fig. 4.

Columns 6 and 7 are obtained from the values given by this plot.

On account of the lag effect the observed value of $\Delta R/\Delta t$ in the interval 0 to 10 seconds is too small, (in the case of electrical heating it is too great). After the first 10 seconds however the observations lie satisfactorily on the straight line of Fig. 4. The coefficient E was found to be .00349. Columns 2 and 3 show that the heating and cooling curves are similar. The correction factor to convert 50 second intervals into 60 second intervals is $f = 1.215$.

Several series of experiments were made to determine the form of the heating and cooling curves as illustrated above, for various amounts of energy expended both electrically and radiometrically in the receiver. These are summarized in the following table. Column 1 gives the total energy supplied to the receiver in a one minute interval.

Table 5.
Summary of Preliminary Calibrations.
Electrical Calibration.

Calories	Correct ΔR_{60}	$R_{60} - R_{10}$ $= \Delta R_{50}$	f	$E \cdot 10^5$	observed $\Delta R'_{60}$	$\Delta R_{60}/\Delta R'_{60}$
.830	.01756	.01443	1.217	365	.01956	.898
.825	.01765	.01450	1.217	284	1961	.900
1.475	.03152	.02592	1.216	269	3292	.958
0.365	.00782	.00642	1.218	288	827	.946
						Mean .93

Radiometric calibration.

.135	1.219	272	1.052
.197	1.208	154	1.040
.154	1.218	269	1.042
.253	1.220	299	1.040
.320	1.215	249	1.036
		<hr/>	<hr/>
<i>final</i> Mean		1.216 ₄ 261	Mean 1.04

The above values were taken with the receiver in different conditions as to the blackening upon its surface. Alterations in the surface affect the values of f and E to some extent. In the later work $f = 1.217$ was employed since this value has been used by the Weather Bureau for some time and is practically identical with the above determination. However the exact value of f is of no material significance since it simply cancels out in the final computations. The important point is whether f is the same for an electrical calibration as for a radiometric calibration. That the values are identical in the two cases is amply shown by the above table. On account of the lag, with readings taken over a 60 second interval instead of a 50 second interval the true change in resistance, ΔR_{60} , is less than the observed change, R'_{60} for the electrical calibration, in the ratio $\Delta R_{60}/\Delta R'_{60} = 0.93$. For the radiometric calibration the true change ΔR_{60} is greater than the observed change, $\Delta R'_{60}$, in the ratio 1.04. Hence for 1 minute exposures with observations taken over 60 second intervals the electrical and radiometric calibrations will differ by about 12%/. This again illustrates the necessity of correcting for the lag.

As shown above the observed cooling curve follows exactly Newton's law of cooling after the first 10 seconds. This is what would be expected if the lag is small. The change in temperature of the receiver in the 50 second period is exactly that of the thermometer although

the thermometer is actually at a higher temperature, during cooling, than the disk. The lag consists of two parts, that due to the insulation around the thermometer coil and that due to the galvanometer. It will be shown later that in the case of the radiometric calibration, the measured temperature of the coil lags from 1 to 2 seconds behind the effective temperature of the disk. On the assumption that the temperature of the disk is uniform throughout, the effect of the lag can be estimated in the manner shown by Harper.⁸

This bull. 8, p.666; 1912.

u = temperature of disk at time t

v = temperature of thermometer at time t

λ = lag in seconds.

$$(1) \quad \frac{\partial v}{\partial t} = \frac{1}{\lambda} (u - v)$$

$$(2) \quad u = A + Be^{-Et} \quad \text{Newton's law for period of cooling.}$$

Suppose at time $t = 0$, $u = u_0 = v = v_0$

Substituting (2) in (1) and integrating.

$$(3) \quad v - u = \left(v_0 + \frac{E\lambda A - u_0}{1 - E\lambda} \right) e^{-t/\lambda} + \frac{E\lambda}{1 - E\lambda} (u - A)$$

For an actual case where the pyrheliometer was sighted on the sun the following values of v_0 , u_0 , and A were obtained by converting the resistances observed into equivalent temperatures.

$$\lambda = 2$$

$$E = .0026$$

$$u = 10.0139 + 23.8763e^{-.0026t}$$

$$A = 10.0139$$

$$u_0 = v_0 = 33.8902$$

Substituting these values in (3) one finds:

$$(4) \quad v - u = -.125e^{-.5t} + .00523 (u - 10.0139)$$

and computing equation (4) for various times, the following table is obtained.

Table 6.

Computation of Thermometric Lag

t	u	$.125e^{-5t}$	$.00523 (4 - A)$	$v - u$
0	33.8902	.125	.1249	0°
5	33.5822	.01026	.1233	.113°
10	33.2766	.00084	.1217	.121
20	32.6797	.000006	.1185	.118
30	32.0995		.1155	.116
60	30.4425		.1068	.107
1	2	3	4	5

Thus as shown by column 5 of the above table, the temperature of the thermometer coil after the first 10 seconds of cooling is higher than that of a disk by practically a constant amount, 0.11° to 0.12°, for the series of observations made by sighting on the sun. The change in temperature of the thermometer during cooling is identical with the change in the disk over the 50 second interval. If similar computations are made for a heating curve it will be found that the temperature of the thermometer is less than that of the disk by essentially a constant amount over the 50 second interval and that the change in temperature of the thermometer is again identical with the change in temperature of the disk.

A typical heating and cooling curve for a radiometric calibration is shown in Fig.5. The small circles represent observed points. The line ACE represents the corrected heating and cooling curve. The total change in temperature of the disk in terms of the coil resistance is given by the distance CD. This is equal to the observed change in 50 seconds PQ or P'Q' multiplied by the factor 1.217. It is also equal to the computed change RE in 50 seconds multiplied by the same factor. Another way to obtain the true change in 60 seconds is to extrapolate the observed curve PQ back to 0 time (or to the time of the beginning of a cooling) and take the difference in the ordinates of the points

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S and Q.

The graphical method of obtaining the lag for a radiometric calibration is shown in Fig. 6.

The observed points are represented by circles. Through points, 2,3,4,5,6,7, and 8,9,10,11,12,13, pass the best computed exponential curves. Extrapolate the cooling curve to the point b and move the heating curve up the the position l b. The distance xy measured along the time axis is twice the lag of the thermometer.

In the case of electrical heating the lag effect is somewhat more complicated. During the heating the thermometer coil is at a higher temperature than the disk. The thermometer "leads" the disk on this account. But the galvanometer on the bridge still lags. The net result may be effectively either a lag or a lead. It might be possible also, although this was not observed, that the readings of the bridge represent the true temperature of the disk, the galvanometer lag exactly balancing the thermometer lead. In cooling, the thermometer rapidly falls behind the disk, and the galvanometer and thermal insulation of the coil act together to produce a lag. The lag λ was taken as half the distance xy Fig. 6 even for the electrical heatings. This is not exactly correct but the main object of the following table is to show that the effective lag is small. The data can not be considered quantitatively.

Table 7. Thermometric lag.			
Method	λ (lag)	Method	λ (lag)
radiometric	1.3	electrical	3.4
"	1.3		3.2
"	1.7		3.0
"	1.3		1.4
"	1.0		1.7
"	1.2		
"	1.1	Mean of all = 1.7 sec.	
"	0.9		

That the lag can not be large is also clearly demonstrated every time a pyr heliometer is used. If the lag were large the resistance

would continue to increase, after an exposure, for some time after the shutter was closed. Actually the galvanometer reverses direction showing decreasing resistance almost at the instant the shutter is closed. In the derivation above showing that the lag effect is damped out after a few seconds, it was assumed that both the coil and the disk were at the same temperature at the instant of changing the shutter, i.e. when $t = 0$. Actually since the previous heating or cooling have an effect, the two are at a slightly different temperature at $t = 0$ but at some later instant, as is seen from Fig.5 the cooling curve of the disk crosses the cooling curve of the thermometer and thus the two show the same temperature at that point. Since the time that the disk and thermometer show the same temperature is so nearly $t = 0$ the computations in the above discussion are not materially altered. Accordingly, it has been proven that the actual lag is small and that its effect is negligible after 10 seconds. Cooling and heating curves were made for exposures of 120 seconds but in the actual use of the instrument it does not appear that anything is gained by doubling the time of exposure.

Blackening of the receiver: The choice of a proper method of blackening the receiver is of considerable importance. An improperly blackened receiver may show a reflection coefficient as high as 10%/. One pyrliometer examined had a poorly blackened receiver. The result was that the radiometric calibration (observed value of $F = \frac{\Delta Q}{\Delta T}$) was far too high and electrical calibration too low. The most satisfactory black surface is prepared in the manner specified by Coblentz.⁹

⁹ Coblentz, This Bull. 11, p.140; 1915.

The receiver is first given a thin coat of a mixture of fine lamp black and platinum black in alcohol with sufficient turpentine to make it adhesive. The disk is then smoked with soot from a sperm candle. The smoke is produced by holding a small iron funnel over the candle. When properly made such a surface absorbs about 98.8% of the radiation

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falling upon it. The loss by reflection, $1.2^\circ/\circ$,¹⁰ can be corrected for in working up the calibration data.

LQ. Coblentz. This Bull. 9, p.322, 1913.

VIII. Final Calibration of Marvin Pyrheliometer. Receiver No.3.

Carefully blackened. The following table summarizes the results of separate experiments with this instrument. Each experiment represents a series of observations which in most cases extended over an hour. The first column gives the total energy supplied to the disk during each minute of heating. It is clearly shown that the calibration constant F' is independent of the amount of energy supplied although this latter extended over a considerable range.

Table 8.

Final Calibration of Receiver No.3.
Electrical Calibration.

Cal/minute	F'	
.2704	2.104	
.2713	2.176	
.4871	2.089	
.7659	2.098	
.7697	2.100	
3.136	2.100	Data taken in 1915. November.
7.211	2.098	
13.181	2.115	
21.321	2.101	
	2.109	

	Mean 2.110	
.2755	2.141	
.4925	2.116	
.4945	2.099	Supposedly more accurate data taken March 1916
.7772	2.089	

	2.111	

Final mean of electrical calibrations $F = 2.110$

Final Radiometric Calibration

Cal/min	F'	temperature	distance
.1983	2.200	1601°abs	94.0 cm.
.3351	2.173	1602	72.4
.4631	2.186	1731	71.9
.3138	2.214	1649	79.2

	Mean 2.200		

The above calibrations were made under all possible different conditions of room temperature, viz., warm, cold, rising, falling, and steady temperature. Rising and falling room temperatures were corrected for by the methods described above.

The constant F' showed no systematic changes with systematic changes in the room temperature or rate of change of room temperature. The final uncorrected values of F' , the means of the above data, are Electrical: $F' = 2.110$ Radiometric $F' = 2.200$. Corrections must be applied to these means as follows.

- (1) A correction of $+1.3^\circ/\circ$ must be subtracted from the radiometric calibration to take account of the loss of radiation by reflection at the surface of the disk. This correction is known accurately for the type of surface used.
 - (2) The black body furnace is known to depart somewhat from the ideal condition of "blackness". The radiator employed was of white Marquardt porcelain. From work of Coblentz it is believed that such a radiator is over $99^\circ/\circ$ black. Coblentz suggests that a correction of 0.5 to $0.8^\circ/\circ$ should be used. This correction should be subtracted from the observed value of F' .
 - (3) The most uncertain correction, which is also subtractive, is due to the absorption of the radiation by the water vapor in the air. For distances used, Coblentz's¹¹ experiments would indicate that this
-
11. This Bull. 12, p.529, 1916.
-
- correction may amount to from 1 to $3^\circ/\circ$ depending upon the humidity.
- (4) A further correction which is small and also subtractive is due to the use of water cooled diaphragms at a lower temperature than that of the room. This correction is discussed below.

Neglecting the 4th correction the total correction to be subtracted from the radiometric determination of F' is from 3 to $5^\circ/\circ$. If these corrections are applied the following values of F' are obtained.

Table 9.Values of the Constant F' , Radiometric Calibration.

correction	<i>Corrected</i> F'
0°/°	2.200
3	2.134 ← electrical cal. =
4	2.112 $2.110 + 1.2^\circ/\circ = 2.135$
5	2.090

The electrical calibration accordingly agrees with the radiometric calibration within the accuracy with which the necessary corrections are known. Since the controlling error lies in the variable corrections which while known to exist are not accurately known in magnitude, it is better for the present to choose the electrically determined value of F' and correct this constant for the known loss by reflection at the surface of the disk when the pyrheliometer is used for radiometric solar work. This correction of $1.2^\circ/\circ$ is then additive and the final value of the constant F' is as follows:

$$F' = 2.110 + 1.2^\circ/\circ = 2.135$$

IX. Solar observations with Marvin Pyrheliometer Receiver No. 3
and Smithsonian Standard Pyrheliometer No.1.

Using the above value of $F' = 2.135$ for the Marvin pyrheliometer, solar measurements were made and comparison observations were taken at the same place and time with the Smithsonian standard. The following table summarizes the results. Each value represents a half hour series of readings and the solar intensity was remarkably constant during every run here recorded. The observations were taken at the Solar Radiation Laboratory of the Weather Bureau by Prof. Kimball, Mr. Hand and the writer.

Table 10.

Data on Solar Observations.

date	Marvin	Smithsonian	Marvin/Smithsonian
Nov.10,1915	1.138 cal/cm ² min.	1.189 cal/cm ² min.	.949
"	1.312	1.388	.945
"	1.225	1.302	.941
Nov.26,1915	1.124	1.169	.962
Nov.27,1915	1.159	1.253	.925

			Mean .944

There is thus a discrepancy of 5.6°/° between the readings of the two instruments. In order to make the readings agree the calibration constant of the Marvin pyrheliometer would have to be increased as follows:

$$F' = \frac{1}{.944} \times 2.135 = 2.262$$

The uncorrected radiometric calibration which gives the highest possible value of F' , was 2.200 and to this must be applied subtractive corrections greater than 3°/°. Even if no correction were applied the disagreement of the two instruments is nearly 3°/°. As stated at the beginning of this paper, any error in the Stefan-Boltzmann constant would be carried directly into the constant F' . A value of $F' = 2.262$ would require that $\sigma = 6.0 \times 10^{-12}$ instead of 5.7×10^{-12} watts cm⁻² deg⁻⁴. In view of the recent accurate determinations of σ by various observers the value $\sigma = 5.70$ appears good to 1°/°. Furthermore the furnace used in the radiometric calibration is exactly the same design as that which has been used for the determination of σ so that any possible but unlikely inherent error in the furnace design would not enter into consideration. But finally admitting that the radiometric calibration were incorrect because of certain unknown errors in the method and possible uncertainty in the value of σ , what is to be said of the electrical calibration? After all question of lag has

been eliminated, and after a maximum additive correction for reflection loss is applied, the Marvin pyrliometer, electrically calibrated still gives readings lower by 5.6% than the Smithsonian instrument which is calibrated by comparison with a pyrliometer which had been calibrated electrically.

In connection with the present work two questions arose which should be considered more in detail. One is in regard to the loss of heat from the receiver by conduction along the lead wires. The lead wires were of No. 28 copper and about 15 cm. of these wires were inside the metal shell encasing the receiver, the object being to reduce the temperature gradient along each wire from the receiver to the outside room temperature. The heating of the lead wires developed by the measuring current is negligible and is extremely small for all the heating currents used in the electrical calibration. Since however the type of gradient in the lead wires may be slightly different in the radiometric and electrical calibrations, and in the case of the electrical calibration may be different for heating and cooling there is a possibility of some small error on this account. However, several preliminary experiments devised to detect an error from this source gave negative results and a rough computation of the magnitude of the effect to be expected indicated that it was very small relative to the total energy expended in the receiver.

The second question concerns the type of temperature gradient established in the receiver during the various stages of the calibrations. It was assumed that the disk itself is of uniform temperature, while the thermometer led or lagged behind the uniform temperature of the disk. Actually, however, in the case of a radiometric heating the front face of the receiver is at a higher temperature than the back face, and on this account the total loss of heat by air convection is greater than if the same quantity of heat entering the receiver were so distributed as to produce a uniform temperature. During an electrical heating the maximum temperature is at the center of the disk. The average surface

temperature of the disk is lower than the mean temperature of the disk and hence the loss by convection is less than it would be if the same quantity of energy expended in the disk were so distributed as to produce a uniform temperature. Whence the constant h of Newton's law of cooling, cf. Sec. IV is larger during a radiometric heating than during an electrical heating. Since F , cf. Sec. IV eq. (7), is proportional to h , the value of F would be slightly greater for the radiometric heating than for the electrical heating. In the final work however, when the character of the surface of the receiver was unaltered, the constant $E \propto h$ was found to be about the same for the radiometric calibration as for the electrical calibration. Furthermore it would appear that the gradients established in the receiver must be different in the two cases of heating and cooling. Since E was observed to have about the same value within experimental errors, for heating as for cooling, it can be concluded that whatever be the types of gradient in the four cases of electrical and radiometrical heating and cooling, the possible error due to this source must be very small.

X. Suggestions for further work.

With the present experimental arrangement it is not feasible to extend the work further. Certain improvements must be made in the apparatus if higher accuracy is to be obtained.

(a) The entire system, furnace, diaphragms, and pyrliometer, must be mounted in a chamber in which the humidity can be carefully controlled and reduced to a minimum when so desired. In designing this chamber attention must be given to the elimination of any possible means for radiation to be reflected, by the side walls, from the furnace into the pyrliometer.

(b) The shutter on the pyrliometer should be replaced by a shutter at the furnace.¹² This new shutter must be water cooled, electromagneti-

The importance of this has been noted by Coblentz c.f. This Bull. 12, p. 513
1916.

cally operated, and must be located between the furnace and the water cooled diaphragm in front of the furnace. In this way all external conditions affecting the pyrheliometer remain the same during a heating as during a cooling. The only difference then radiometrically between a heating and a cooling is the exchange of the opening of a definite size, given by the water cooled diaphragm, into the furnace with a black water cooled diaphragm of the same size at a measured temperature. Fig. 7 illustrates the conditions which were effective in the present calibration but which could be remedied by the use of a properly located shutter.

The temperature of the pyrheliometer receiver of area A_0 is T_0 .

The temperature of the walls and shutter of the pyrheliometer is T_1 and the area of the shutter, when closed, exposed to the receiver is A_1 . When the shutter is open the pyrheliometer receives radiation from the large area A_2 of the water cooled diaphragm at a temperature T_2 and from the small area A_3 opening into the furnace at a temperature T_3 .

Let J_1 = radiation received by the disk from the closed shutter.

J_2 = radiation received by the disk from the large water cooled diaphragm, when the shutter is open

J_3 = radiation received by the disk, shutter open, from the furnace.

The net radiation measured is $J = J_2 + J_3 - J_1$

$$J_1 = \frac{\sigma}{\pi} (T_1^4 - T_0^4) \frac{A_1 A_0}{d^2}$$

$$J_2 = \frac{\sigma}{\pi} (T_2^4 - T_0^4) \frac{A_2 A_0}{D^2}$$

$$J_3 = \frac{\sigma}{\pi} (T_3^4 - T_0^4) \frac{A_3 A_0}{D^2}$$

hence since T_0^4 is always negligible compared with T_3^4 , and A_3 compared with A_2

$$(1) \quad J = \frac{\sigma}{\pi} A_0 \left[\left(\frac{T_3}{100} \right)^4 \frac{A_3}{D^2} + \left\{ \left(\frac{T_2}{100} \right)^4 - \left(\frac{T_1}{100} \right)^4 \right\} \frac{A_1}{d^2} \right] 10^8$$

In the calibration of the pyr heliometer the water cooled diaphragm was always at a slightly lower temperature than that of the shutter.

If we take a very extreme case as an example, suppose:

$$T_3 = 1600^\circ \text{ abs.}$$

$$T_2 = 290$$

$$T_1 = 300$$

$$T_0 = 300$$

$$D = 70 \text{ cm.}$$

$$A_3 = .93 \text{ cm}^2$$

$$A_1 = 16 \text{ cm}^2$$

$$d = 30 \text{ cm.}$$

Then equation (1) above becomes:

$$(2) \quad J = \frac{\sigma}{\pi} A_0 10^8 (12.44 - .18)$$

Thus for this extreme example where the temperature of the diaphragm is 10° lower than that of the pyr heliometer shutter the 2nd term of equations (1) and (2) amounts to $1.4^\circ/\circ$ of the 1st term. The 2nd term was always neglected in the calibration. In general it would only amount to a few tenths of one per cent. Moreover it is always a subtractive correction to J and hence would reduce the observed value of F' , like all the other corrections discussed above.

(c) As a further check upon the behavior of the dynamic type of pyr heliometer of which the Marvin is an example, it would be highly interesting to study at the same time a pyr heliometer of the static type. The Marvin dynamic pyr heliometer readily lends itself to this modification. The entire tube and case could be jacketed and accurately maintained at a constant temperature. An ice bath while furnishing the desired constancy of temperature could not be easily used on account of its being usually below the dew point, allowing water vapor to condense upon the receiver, when the instrument was used for solar comparisons. But it would not be

difficult to secure sufficient constancy of temperature by use of flowing water thermostated to, say, 30°C. Such a pyrheliometer would not be very satisfactory for ordinary field work on account of the inconvenience of operation but as a laboratory instrument its behavior so different in principle from the dynamic type would permit interesting comparisons of solar measurements by two distinct methods.

XI. Summary

For the first time, we believe, a pyrheliometer has been calibrated by two methods, the usual electrical method, and a radiometric method. In the radiometric method, a known quantity of radiation from a black body was allowed to fall upon the pyrheliometer receiver in exactly the same manner employed for solar measurements. The calibrations by the two methods agreed if the Stefan Boltzmann co-efficient were chosen as $\sigma = 5.7 \times 10^{12} \text{ watts cm}^{-2} \text{ deg}^{-4}$ the latest and most accurate determination of this coefficient of total radiation. Or conversely, the coefficient has been observed as 5.7 with an accuracy of probably about 3%.

The behavior of the Marvin pyrheliometer has been carefully investigated. A lag, part of which is due to the galvanometer of the bridge, has been found to exist, and, for the silver disk No. 3, was experimentally shown to be less than 2 sec. Both theoretically and experimentally it was shown that effect of this lag is negligible after 5 to 10 seconds. The cooling and heating of the receiver follows Newton's law of cooling. Cooling curves and heating curves are identical in form for both the case of electrical heating and the case of radiometrical heating. This fact is of especial interest for it is very difficult to obtain measurements during the electrical heating. Measurements upon the cooling following an electrical heating are sufficient.

In order to completely eliminate all errors due to a lag effect, readings should be made at 10 seconds and 60 seconds following the begin-

ning of a heating or cooling. The factor for converting readings of temperature or resistance change over this 50 second interval to corresponding changes over a complete 60 second period is 1.217. This factor is the same for both electrical and radiometric heating and is known with an accuracy of 0.1%/. There is no advantage in making the periods of heating and cooling 120 seconds in duration. Periods of 60 seconds are sufficient. The method of blackening the receiver is of great importance. The best method of blackening is that used by Coblentz and is described above. The calibration constant F' of Sec. IV is independent of the rate at which energy is supplied to the receiver. This is of importance because a radiometric calibration must necessarily be made with an energy supply considerably less than that received in solar determinations.

A Marvin pyrheliometer calibrated by two methods, electrical and radiometric, was compared by solar observations with the U.S. Weather Bureau Smithsonian standard pyrheliometer No. 1 calibrated by comparison with the Smithsonian primary standard water flow pyrheliometer. The Marvin pyrheliometer thus calibrated gave 5.6% lower values of the solar radiation than the Smithsonian pyrheliometer. The explanation of this large discrepancy requires further investigation. Suggestions are given for continuing this investigation.

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experimental work was done at the Bureau of Standards under the direction of Dr. Stratton.

Physical Laboratory,

University of Minnesota,

April 1917.

Fig 4

During the process of obtaining observations during both heating and cooling of a standard calibration, the time of heating and cooling is controlled by rheostat C and B . Galvanometer G is used to determine the resistance of the pyrheliometer during the process of heating and cooling.