LAPLACIAN UNIMODULAR EQUIVALENCE OF GRAPHS

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Abstract. Let G be a graph. Let $L(G)$ be the difference of the diagonal matrix of vertex degrees and the adjacency matrix. This note addresses the number of ones in the Smith Normal Form of the integer matrix $L(G)$.

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Let $G = (V, E)$ be a graph with vertex set $V = V(G) = \{v_1, v_2, \dots, v_n\}$ and edge set $E = E(G)$. Denote by $D(G)$ the diagonal matrix of vertex degrees and by $A(G)$ the $(0, 1)$ -adjacency matrix. The *Laplacian matrix* is $L(G) = D(G) - A(G)$. Of course, G_1 and G_2 are isomorphic graphs if and only if there is a permutation matrix P such that $P^t L(G_1) P = L(G_2)$. One typically deduces that isomorphic graphs have similar Laplacian matrices, partially explaining interest in the Laplacian spectrum. (See, e.g., [3].)

Recall that an n -by- n integer matrix U is **unimodular** if $\det U = \pm 1$. So, the unimodular matrices are precisely those integer matrices with integer inverses. Two integer matrices A and B are said to be **congruent** if there is a unimodular matrix U such that $U^t A U = B$. Consequently, isomorphic graphs have congruent Laplacian matrices. Unlike its spectral counterpart, this observation has a partial converse: If G_1 is 3-connected, then G_1 is isomorphic to G_2 if and only if $L(G_1)$ and $L(G_2)$ are congruent [6]. The *computational* significance of this partial converse is problematic since there is no canonical form for congruence [2]. On the other hand, integer matrices cannot be congruent if they are not equivalent, and the question of unimodular equivalence is easily settled by the Smith Normal Form.

Recall that integer matrices A and B are **equivalent** if there exist unimodular matrices U_1 and U_2 such that $U_1 A U_2 = B$. Denote by $d_k(G)$ the k -th *determinantal divisor* of $L(G)$, i.e., the greatest common divisor (GCD) of all the k -by- k determinantal minors of $L(G)$. (It follows, e.g. from the Matrix-Tree Theorem, that $d_{n-1}(G) = t(G)$, the number of *spanning trees* in G ; and $d_n(G) = 0$, because $L(G)$ is singular.) Of course, $d_k(G) | d_{k+1}(G)$, $1 \leq k < n$. The **invariant factors** of G are defined by $s_{k+1}(G) = d_{k+1}(G)/d_k(G)$, $0 \leq k < n$, where $d_0(G) = 1$. The Smith Normal Form of $L(G)$ (*Smith Normal Form of G*) is

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$$S(G) = \text{diag}(s_1(G), s_2(G), \dots, s_n(G)).$$

(Previous work on $S(G)$ was reported in [4]. An application of $S(G)$ to graph “bicycles” appears in [1].)

If G_1 and G_2 are isomorphic, then $L(G_1)$ and $L(G_2)$ are equivalent, i.e., $S(G_1) = S(G_2)$. If this observation had a partial converse (e.g., for 3-connected graphs), it would have great computational significance because $S(G)$ can be obtained from $L(G)$ by a sequence of elementary row and column operations.

Figure 1. Two graphs with the same SNF.

Example 1. The graphs in Figure 1 share the Smith Normal Form, $\text{diag}(1, 1, 1, 5, 15, 0)$. Note that G_1 is 3-connected.

In spite of this discouraging example, the Smith Normal Form of G yields several bona fide graph theoretic invariants. In this note we are interested in one of them, namely the multiplicity of 1 in $S(G)$. We denote this multiplicity by $b(G)$; thus $b(G)$ is the largest value of k such that (either $d_k(G)$ or) $s_k(G) = 1$.

Suppose G is a connected graph with at least two vertices. Because G must have an edge, $b(G) \geq 1$. It is easy to see that $d_2(K_n) = n$, so equality holds for $G = K_n$. At the other extreme, $b(G) \leq n - 1$, with equality if and only if $t(G) = 1$, i.e., if and only if G is a tree.

THEOREM 1. *Let $S = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$. Suppose the induced subgraph $G[S]$ is a forest each of whose w components is a path on at least two vertices. Then $d_{k-w}(G) = 1$, i.e., $b(G) \geq k - w$.*

Proof. We may assume the numbering is chosen so that the vertices of S comprise the first k vertices of G , and that the numbering of the vertices of S respects

its components, i.e., we may assume the leading k -by- k principal submatrix of $L(G)$ is a direct sum of w tridiagonal matrices of the form

$$A = \begin{pmatrix} a_1 & -1 & 0 & 0 & \dots & 0 & 0 \\ -1 & a_2 & -1 & 0 & \dots & 0 & 0 \\ 0 & -1 & a_3 & -1 & \dots & 0 & 0 \\ & \ddots & & & & & \\ 0 & 0 & 0 & \dots & & -1 & a_r \end{pmatrix}$$

where a_i is the degree in G of the i -th vertex in the component of $G[S]$ to which A corresponds. The $(r-1)$ -square submatrix of A obtained by striking out its first row and last column is upper triangular with -1 's down the main diagonal. Hence, its determinant is ± 1 . Evidently, then, the leading k -by- k principal submatrix of $L(G)$ contains a $(k-w)$ -square submatrix of determinant ± 1 . $\square$

COROLLARY 1. *Let G be a connected graph of diameter $\delta(G)$. Then $b(G) \geq \delta(G)$.*

Proof. Let $u, w \in V(G)$ be a distance $\delta = \delta(G)$ apart. Let $u = v_1, v_2, \dots, v_{\delta+1} = w$ be a shortest path from u to w . If $\{v_i, v_j\} \in E$ for $i+1 < j$, there would be a shorter path from u to w in G . Let $S = \{v_1, \dots, v_{\delta+1}\}$. Then $G[S]$ is the path $P_{\delta+1}$. The result follows from Theorem 1. $\square$

It has been shown that two connected graphs with the same blocks (maximal 2-connected subgraphs) have the same Smith Normal Form [4,5]. Therefore, Corollary 1 can be improved to the extent that $b(G)$ is an upper bound for the sum of the diameters of the blocks of G . Similarly, a certain "twist operation" (described in [6]) can be used to improve the result even further.

COROLLARY 2. *Let G be a connected graph on $n \geq 2$ vertices. Then $b(G) \geq 1$ with equality if and only if $G = K_n$.*

Proof. We have already seen that $b(K_n) = 1$. If $G \neq K_n$, then $\delta(G) \geq 2$. The result follows from Corollary 1. $\square$

THEOREM 2. *Let $H = G - e$ be the edge subgraph of G obtained by deleting the (single) edge e . For $1 \leq k < n$,*

$$d_k(G) | d_{k+1}(H) \text{ and } d_k(H) | d_{k+1}(G).$$

Proof. Suppose edge $e = \{v_i, v_j\}$ is deleted from G to produce H . Let A be the matrix obtained from $L(G)$ by adding rows (and columns) $1, 2, \dots, j-1, j+1, \dots, n$ to row (and column) j (respectively). Note: (1) A is the matrix obtained from $L(G)$ by replacing each entry in its j -th row and column by 0; (2) there is a unimodular matrix U such that $A = U^t L(G) U$; (3) the matrix $B = U^t L(H) U$ is obtained from $L(H)$ by zeroing out its j -th row and column. Because equivalent matrices have

the same sequence of determinantal divisors, $d_k(G) = d_k(A)$, the GCD of all the k -by- k determinantal minors of A , and $d_k(H) = d_k(B)$, $1 \leq k \leq n$. So, it suffices to prove that $d_k(A)|d_{k+1}(B)$ and $d_k(B)|d_{k+1}(A)$. Let $S, T \subseteq V(G) = V(H)$ be $(k+1)$ -element sets of vertices. Denote by $A[S : T]$ and $B[S : T]$ the submatrices of A and B , respectively, lying in rows corresponding to S and columns corresponding to T . If $v_j \in S \cup T$, then $\det A[S : T] = \det B[S : T] = 0$. If $v_i \notin S \cup T$, then $A[S : T] = B[S : T]$. In either of these cases, $d_k(A)|\det B[S : T]$ and $d_k(B)|\det A[S : T]$. The only remaining possibility is that $v_i \in S \cup T$ but $v_j \notin S \cup T$. In this case, $A[S : T] = B[S : T] + F$, where F is the $(k+1)$ -by- $(k+1)$ matrix whose only nonzero entry is a 1 in the row corresponding to vertex $v_i \in S$ and the column corresponding to vertex $v_j \in T$. Thus,

$$(1) \quad \det A[S : T] = \det B[S : T] + \det B[S - v_i : T - v_i].$$

Since $d_k(B)$ divides both terms on the right-hand side of (1), $d_k(B)|\det A[S : T]$. But, $B[S - v_i : T - v_i] = A[S - v_i : T - v_i]$. It follows that $d_k(A)$ divides two of the terms in (1) so it must divide $\det B[S : T]$ as well. We have shown that $d_k(A)$ (respectively $d_k(B)$) divides every $(k+1)$ -by- $(k+1)$ determinantal minor of B (respectively, A). Thus, it divides their GCD. $\square$

Question 1. Let $H = G - e$. Does $s_k(G)|s_{k+1}(H)$, and/or does $s_k(H)|s_{k+1}(G)$?

COROLLARY 3. Let $H = G - e$. Then $|b(G) - b(H)| \leq 1$.

Proof. By definition, $1 = d_{b(G)}(G)$. By Theorem 2, $d_{b(G)-1}(H) = 1$, so $b(H) \geq b(G) - 1$. Similarly, $b(G) \geq b(H) - 1$. $\square$

We now explore the relationship between $b(G)$ and the number of edges in G . For $n > k \geq 1$, let $F(n, k)$ denote the family of connected graphs G on n vertices satisfying $b(G) = k$. Then, for example, $F(n, 1) = \{K_n\}$ and $F(n, n-1)$ is the set of trees on n vertices. Let $m(n, k)$ and $M(n, k)$ denote, respectively, the minimum and maximum of the number of edges in G , as G ranges over $F(n, k)$. Then, $m(n, k) \leq M(n, k) \leq C(n, 2)$, *binomial coefficient n -choose-2*, with equality throughout for $k = 1$. On the other hand, $M(n, k) \geq m(n, k) \geq n - 1$, with equality throughout for $k = n - 1$.

COROLLARY 4. For $n > k > 1$, $m(n, k) = 2(n - 1) - k$.

Proof. Let $G \in F(n, k)$ be a graph with $m(n, k)$ edges. Since G is connected, it contains a spanning tree T . We may obtain T from G by successively removing the $m(n, k) - (n - 1)$ edges of G not belonging to T . By Corollary 3, $b(T) - b(G) \leq m(n, k) - (n - 1)$. Since $b(T) = n - 1$ and $b(G) = k$, we conclude that $m(n, k) \geq 2(n - 1) - k$. It remains to produce a graph $G_{n,k} \in F(n, k)$ having $2(n - 1) - k$ edges. For this, we need some additional notation.

If $G = (V(G), E(G))$ and $H = (V(H), E(H))$ are graphs on disjoint sets of vertices, let $G + H$ be their *union*, i.e., $V(G + H) = V(G) \cup V(H)$, and $E(G + H) =$

$E(G) \cup E(H)$. Suppose G has n_1 vertices and H has n_2 vertices. A *coalescence* of G and H is any of the $n_1 n_2$ graphs on $n_1 + n_2 - 1$ vertices that can be obtained from $G + H$ by identifying (i.e., coalescing into a single vertex) a vertex of G and a vertex of H . We denote any of these coalescences by $G * H$ (with the understanding that “ $*$ ” is not a well defined binary operation).

For $n > k > 1$, let $G_{n,k} = K_{2,n-k} * P_{k-1}$, where $K_{2,n-k}$ is the complete bipartite graph and P_{k-1} is the path on $k-1$ vertices. Then $G_{n,k}$ has n vertices and $m = 2(n-k) + (k-2) = 2(n-1) - k$ edges. By a direct computation (or by [5, Thm 1] or [4, Prop 1]), one obtains $b(G_{n,k}) = k$. $\square$

There is no 2-connected graph G with $b(G) = 3$ having $n = 6$ vertices and $m = m(6, 3) = 7$ edges.

Question 2. What is the minimum number of edges in G , as G ranges over the 2-connected graphs in $F(n, k)$, $k \geq 3$?

COROLLARY 5. For $n - 1 > k \geq 1$, $M(n, k) = C(n, 2) - k + 1$.

Proof. Let $G \in F(n, k)$ be a graph with $M(n, k)$ edges. We may obtain G by removing the $C(n, 2) - M(n, k)$ edges of K_n not belonging to G . By Corollary 3, $b(G) - b(K_n) \leq C(n, 2) - M(n, k)$. Since $b(G) = k$ and $b(K_n) = 1$, we see $M(n, k) \leq C(n, 2) - k + 1$. As in the proof of Corollary 4, it remains to describe a graph $G^{n,k} \in F(n, k)$ with $C(n, 2) - k + 1$ edges. For $k = 1$, there is only one possibility, namely, $G^{n,1} = K_n$. For $n - 1 > k > 1$, let $G^{n,k} = (P_k + K_{n-k}^c)^c$, the complement of the union of P_k with the complement of K_{n-k} . Note that $G^{n,k}$ has $m = C(n, 2) - (k - 1)$ edges.

It remains to show that $b(G^{n,k}) = k$. A graph with $m = C(n, 2) - k + 1$ edges cannot have *more* than k ones in its Smith Normal Form because if it did, the addition of more edges would eventually lead to K_n . By Corollary 3, the addition of a single edge can change the number of ones in the Smith Normal Form by at most 1. So, $b(G^{n,k}) - b(K_n) \leq k - 1$ and, since $b(K_n) = 1$, we have $b(G^{n,k}) \leq k$.

Next we show that $b(G^{n,k}) \geq k$ by exhibiting a k -by- k submatrix of $L(G^{n,k})$ whose determinant is ± 1 . We may suppose the vertices of $G^{n,k}$ have been numbered so that the k vertices associated with P_k come first. Thus, the leading $(k+2)$ -by- $(k+2)$ principal submatrix of $L(G^{n,k})$ is

$$A = \begin{pmatrix} n-2 & 0 & -1 & -1 & \dots & -1 & -1 & -1 & -1 \\ 0 & n-3 & 0 & -1 & \dots & -1 & -1 & -1 & -1 \\ -1 & 0 & n-3 & 0 & \dots & -1 & -1 & -1 & -1 \\ & & & \dots & & & & & \\ -1 & -1 & -1 & -1 & \dots & n-3 & 0 & -1 & -1 \\ -1 & -1 & -1 & -1 & \dots & 0 & n-2 & -1 & -1 \\ -1 & -1 & -1 & -1 & \dots & -1 & -1 & n-1 & -1 \\ -1 & -1 & -1 & -1 & \dots & -1 & -1 & -1 & n-1 \end{pmatrix}.$$

Let B be the k -by- k submatrix of $L(G^{n,k})$ obtained by striking out rows k and $k + 1$, and columns 1 and $k + 2$ from A . We show that $\det B = \pm 1$ by a sequence of elementary row and column operations. Subtract row 1 (of B) from row k . If $k = 2$, the resulting matrix is

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

and the proof is complete. Otherwise, the only nonzero entry in the k -th row of the resulting matrix is -1 in the $(k, 1)$ position. We may use it to annihilate the other nonzero entries in column 1. In the resulting matrix, subtract row 2 from row 1. The only nonzero entry left in row 1 is -1 in column 2. Use it to annihilate any other nonzero entries in column 2. Then subtract row 3 from row 2. The only nonzero entry in row 2 is -1 in column 3. Use it to zero out any other nonzero entries in column 3. Continuing in this way, we eventually reduce B to

$$C = \begin{pmatrix} 0 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & -1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & -1 & \dots & 0 & 0 \\ 0 & 0 & \dots & & & & \\ 0 & 0 & 0 & 0 & \dots & 0 & -1 \\ -1 & 0 & 0 & 0 & \dots & 0 & 0 \end{pmatrix}.$$

Since $\det B = \det C = \pm 1$, the proof is complete. $\square$

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