**GLOBAL SOLUTIONS OF MAXWELL'S EQUATIONS
IN AN ELECTROMAGNETIC FIELD
WITH THE TEMPERATURE-DEPENDENT
ELECTRICAL CONDUCTIVITY**

By

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Global Solutions of Maxwell's Equations in an Electromagnetic Field with the Temperature-Dependent Electrical Conductivity ¹

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Abstract: This paper deals with Maxwell's equations in a quasi-stationary electromagnetic field subject to the effects of the temperature field. This model is encountered in the penetration of a magnetic field in substances where the electrical conductivity depends on the temperature. Similar phenomena also occur in some industrial problems such as the thermister. Taking the effect of the Joule heat into the consideration, we obtain a strongly coupled nonlinear system. The global solvability is established for this system.

Key Words and Phrases: Maxwell's Equations, coupled nonlinear system, global solvability.

AMS(MOS)Subject Classifications: 35Q20, 35K55, 78A97.

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1. Introduction

In a quasi-stationary electromagnetic field the magnetic induction $\vec{B}$, the magnetic field $\vec{H}$ and the electric field $\vec{E}$ satisfy the approximate Maxwell equations (cf. Landau and Lifshitz [4], page 186-187):

$$\operatorname{div} \vec{B} = 0, \quad (1.1)$$

$$\operatorname{curl} \vec{H} = \frac{4\pi\sigma}{c} \vec{E}, \quad (1.2)$$

$$\operatorname{curl} \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \quad (1.3)$$

where σ is the electrical conductivity and c is a physical constant. With non-ferromagnetic bodies, if the strength of magnetic field is weak, the magnetic induction is proportional to the strength of the magnetic field (cf. [4], page 120), that is

$$\vec{B} = \mu \vec{H}, \quad (1.4)$$

where μ is the magnetic permeability.

In the study of the penetration of a magnetic field in a medium the electrical resistance strongly depends on the temperature. On the other hand, the electric current $\vec{J}$ satisfies Ohm's law (cf. [4], [7]):

$$\vec{J} = \sigma(u) \vec{E}. \quad (1.5)$$

It follows that the heat source produced by the Joule heat is

$$\begin{aligned} \vec{J} \cdot \vec{E} &= \sigma(u) \vec{E} \cdot \vec{E} \\ &= \frac{c}{4\pi\sigma(u)} |\nabla \times \vec{H}|^2, \end{aligned}$$

where the equation (1.2) has been used in the above derivation.

With the assumption of the constant permeability, we have, after normalizing the constants, the following form of Maxwell's equations and the heat equation:

$$\operatorname{div} \vec{H} = 0, \quad (1.6)$$

$$\frac{\partial \vec{H}}{\partial t} = -\nabla \times [k(u) \nabla \times \vec{H}], \quad (1.7)$$

$$u_t - \Delta u = k(u) |\nabla \times \vec{H}|^2, \quad (1.8)$$

where $k(u) = \frac{1}{\sigma(u)}$ is so-called the resistivity of the material.

Obviously, the equation (1.6) is true as a result of (1.7) if the initial state satisfies this equation. It is our objective in the present paper to study the equations (1.7)-(1.8) subject to appropriate initial and boundary conditions.

It is easy to see that if we take $\vec{H} = \{w, 0, 0\}$, the system (1.6)-(1.8) is reduced into a system which is similar to the thermister problem in an electric field (steady-state case):

$$\nabla[k(u) \nabla w] = 0, \quad (1.9)$$

$$-\Delta u = k(u) |\nabla w|^2. \quad (1.10)$$

This problem has been studied by many authors (cf. [1], [2], [6], for example, and the references listed therein). In these previous studies, the maximum principle plays an essential role in the proof of the existence. However, our system is much more complicated. The maximum principle can no longer be applied for the magnetic field $\vec{H}(x, t)$. We shall seek the weak solvability. With the help of Simon's compactness results (cf. [5]), we are able to establish the global existence.

In Section 2, we will state the problem in a weak form and define the weak solution. In Section 3, we construct an approximate problem and show that the

approximate problem admits a unique classical solution; then the compactness argument is used to prove the existence of a weak solution.

2. Weak Formulation

For reader's convenience, we list some notions below. Let B be a Banach space and $1 \leq p < \infty$,

$$L^p(0, T; B) = \{f | f : [0, T] \rightarrow B \text{ with the norm } \|f\|_{L^p(0, T; B)} < \infty, \}$$

where

$$\|f\|_{L^p(0, T; B)} = \left[\int_0^T \|f\|_B^p dt \right]^{\frac{1}{p}}.$$

The spaces $W^{m,k}(\Omega)$, $H_0^1(\Omega)$, *etc.* are the common Banach spaces with the usual norms, and B^m denotes the product space $B \times B \cdots \times B$. Let Ω be a bounded domain in R^3 and $Q_T = \Omega \times (0, T]$. The initial and boundary conditions are given by

$$\vec{H}(x, t) = \vec{F}(x, t), \quad (x, t) \in S_T = \partial\Omega \times (0, T]; \quad (2.1)$$

$$u(x, t) = g(x, t), \quad (x, t) \in S_T; \quad (2.2)$$

$$\vec{H}(x, 0) = \vec{H}_0(x), \quad x \in \Omega; \quad (2.3)$$

$$u(x, 0) = u_0(x), \quad x \in \Omega. \quad (2.4)$$

Without loss of generality, we may assume that $g(x, t) = 0$. A weak solution of the problem (1.7)-(1.8), (2.1)-(2.4) is defined as follows:

Definition: A pair of functions $(\vec{H}(x, t), u(x, t))$ is said to be a weak solution of the problem (1.7)-(1.8), (2.1)-(2.4) if $\vec{H}(x, t) - \vec{F}(x, t) \in L^2(0, T; H_0^1(\Omega)^3)$ with

$\operatorname{div} \cdot \vec{H}(x, t) = 0$ a.e. in Q_T , and $u(x, t) \in L^\infty(0, T; L^1(\Omega)) \cap L^1(0, T; W_0^{1,1}(\Omega))$, which satisfy the following integral identities:

$$\int \int_{Q_T} [-\vec{H} \cdot \vec{\psi}_t + k(u) \nabla \times \vec{H} \cdot \nabla \times \vec{\psi}] dx dt - \int_{\Omega} \vec{H}_0(x) \cdot \vec{\psi}(x, 0) dx = 0; \quad (2.5)$$

$$\int \int_{\Omega} u[\phi_t + \Delta \phi] dx dt = - \int \int_{\Omega} k(u) |\nabla \times \vec{H}|^2 \phi dx dt \quad (2.6)$$

for any $\vec{\psi}(x, t) \in H^1(0, T; H_0^1(\Omega)^3)$ with $\vec{\psi}(x, T) = 0$ and for any $\phi(x, t) \in C_0^\infty(Q_T)$. Moreover, the limit of $u(x, t)$ as t tends to 0 is $u_0(x)$ in the sense of Lebesgue measure.

The following lemma plays a fundamental role in proving the existence. The reader can consult J. Simon's paper [5] (Corollary 4, on page 85) for its proof.

Lemma 2.1: Let X, B and Y be Banach spaces with $X \subset B \subset Y$. Moreover, the imbedding operator from $X \rightarrow B$ is compact. Let F be a set of functions from $[0, T] \rightarrow X$. If F is bounded in $L^p(0, T; X)$, where $1 \leq p < \infty$, and

$$\frac{\partial F}{\partial t} = \left\{ \frac{\partial f}{\partial t} : f \in F \right\} \text{ is bounded in } L^1(0, T; Y).$$

Then F is relatively compact in $L^p(0, T; B)$.

Throughout this paper, we assume:

H(1): The function $k(u)$ is continuous in R^1 , and there exist two constants k_1 and k_2 such that

$$0 < k_1 \leq k(u) \leq k_2.$$

H(2): The function $\vec{F}(x, t)$ can be extended into Q_T such that the extended function $\vec{F}(x, t) \in W_p^{2,1}(Q_T)^3$, the initial data $\vec{H}_0(x) \in W_p^2(\Omega)^3$ with $\operatorname{div} \vec{H}_0(x) = 0$ and $p > \frac{5}{2}$ and $u_0(x) \in L^2(\Omega)$. Moreover, the following consistency condition is satisfied:

$$\vec{F}(x, 0) = \vec{H}_0(x), \quad x \in \partial\Omega.$$

Our main result is

Theorem: Under the assumptions H(1)-H(2), the problem (1.6)-(1.8), (2.1)-(2.4) admits a weak solution globally in time.

3. Proof

We start with the following approximate problem:

$$\vec{H}_t = -\nabla \times [k(u)\nabla \times \vec{H}], \quad (x, t) \in Q_T = \Omega \times (0, T], \quad (3.1)$$

$$u_t = \Delta u + \frac{k(u)|\nabla \times \vec{H}|^2}{1 + \varepsilon k(u)|\nabla \times \vec{H}|^2}, \quad (x, t) \in Q_T, \quad (3.2)$$

subject to the initial and boundary conditions (2.1)-(2.4), where $\vec{H}_0(x)$ and $u_0(x)$ are replaced by the smooth approximation functions $\vec{H}_{0\varepsilon}(x)$ and $u_{0\varepsilon}(x)$ such that $\operatorname{div} \vec{H}_{0\varepsilon} = 0$ and the consistency conditions hold up to the second order.

Lemma 3.1: The approximate problem admits a unique classical solution:

$$\vec{H}_\varepsilon(x, t) \in C^{2,1}(\bar{Q}_T)^3; \quad u_\varepsilon(x, t) \in C^{2,1}(\bar{Q}_T).$$

Moreover, the following estimates hold:

$$\begin{aligned} \operatorname{div} \vec{H}_\varepsilon(x, t) &= 0, \quad \text{in } Q_T, \\ \sup_{0 \leq t \leq T} \int_\Omega |\vec{H}_\varepsilon|^2 dx + \int \int_{Q_T} k(u) |\nabla \times \vec{H}_\varepsilon|^2 dx dt &\leq C_1, \end{aligned} \quad (3.3)$$

$$\sup_{0 \leq t \leq T} \int_\Omega |u| dx + \int \int_{Q_T} |\nabla u| dx dt \leq C_2, \quad (3.4)$$

where the constants C_1 and C_2 depends only on the known data, but not on ε .

Proof: The existence can be shown easily by the Schauder fixed point theorem since the nonlinear term in (3.2) is bounded. The proof of uniqueness is also standard,

and so it only remains to derive the estimates (3.3)-(3.4). First of all, we take the divergence in the equation (3.1) to see

$$(\operatorname{div} \vec{H}_\epsilon)_t = 0,$$

which implies

$$\operatorname{div} \vec{H}_\epsilon(x, t) = \operatorname{div} \vec{H}_{0\epsilon}(x) = 0.$$

Next, we note that for $u(x) \in H_0^1(\Omega)$,

$$\int_{\Omega} (\nabla \times \vec{u}) \cdot \vec{v} dx = \int_{\Omega} \vec{u} \cdot (\nabla \times \vec{v}) dx.$$

If we take the inner product to (3.1) with $\vec{H}_\epsilon - \vec{F}$, we have

$$\begin{aligned} & \frac{1}{2} \int_{\Omega} [|\vec{H}_\epsilon - \vec{F}|^2 - |\vec{H}_{0\epsilon}(x) - \vec{F}(x, 0)|^2] dx + \int \int_{Q_T} (\vec{H}_\epsilon - \vec{F}) \cdot \vec{F} dx dt \\ &= - \int \int_{Q_T} k(u) |\nabla \times \vec{H}_\epsilon|^2 dx dt + \int \int_{Q_T} k(u) (\nabla \times \vec{H}_\epsilon) \cdot (\nabla \times \vec{F}) dx dt. \end{aligned}$$

By Cauchy-Schwartz's inequality,

$$\int \int_{Q_T} (\vec{H}_\epsilon - \vec{F}) \cdot \vec{F}_t dx dt \leq \delta \int \int_{Q_T} |\vec{H}_\epsilon - \vec{F}|^2 dx dt + C(\delta) \int \int_{Q_T} |\vec{F}_t|^2 dx dt.$$

Now taking δ sufficiently small and using Gronwall's inequality, we have the desired estimate (3.3).

To prove the estimate (3.4), we need the following lemma:

Lemma 3.2: Assume that $u(x, t)$ is a distribution solution of the following problem:

$$u(x, t) \in L^1(0, T; W_0^{1,1}(\Omega)),$$

$$u_t - \Delta u = f(x, t), \quad (x, t) \in Q_T,$$

$$u(x, 0) = u_0(x), \quad x \in \Omega,$$

where $f(x, t) \in L^1(Q_T)$. Then the following estimate holds

$$\sup_{0 \leq t \leq T} \int_{\Omega} |u| dx + \int \int_{Q_T} |Du| dx dt \leq C.$$

This result can be shown by using Green's representation. For completeness, we sketch the proof. We may assume that the initial value and the inhomogenous term are smooth. Then Green's representation gives

$$u(x, t) = \int_{\Omega} G(x, y; t, 0) u_0(y) dy + \int \int_{Q_T} G f dy d\tau.$$

Observe that

$$\int_{\Omega} G dx \leq C;$$

and also that

$$\int_{\Omega} |DG(x, y, t, \tau)| dx \leq \frac{C}{\sqrt{t - \tau}}, \quad t > \tau,$$

it follows that

$$\int_{\Omega} |u| dx \leq C[||u_0||_{L^1(\Omega)} + ||f||_{L^1(Q_T)}].$$

The second estimate can be proved in a similar manner.

Now we are ready to prove the main theorem.

The proof of the Theorem: First of all, we have the following identity:

$$\int_{\Omega} \sum_{i=1}^3 (u_{ix}^2 + u_{iy}^2 + u_{iz}^2) dx = \int_{\Omega} (\operatorname{div} \cdot \vec{u})^2 dx + \int_{\Omega} |\nabla \times \vec{u}|^2 dx$$

for any $\vec{u} \in H_0^1(\Omega)^3$. This can be proved by direct calculations.

By Lemma 3.1 and the above property we know

$$\{\vec{H}_\varepsilon\} \in L^2(0, T; H_0^1(\Omega)^3).$$

From the equation (3.1), we also have

$$\frac{\partial \vec{H}_\varepsilon}{\partial t} \in L^2(0, T; H^{-1}(\Omega)^3).$$

Lemma 2.1 implies that there exists a subsequence of $\{\vec{H}_\varepsilon(x, t)\}$, still denoted by itself, which converges to a function almost everywhere in Q_T . We denote the limit function by $\vec{H}(x, t)$. Moreover, the weak compactness of $L^2(Q_T)$ implies that

$$\nabla \times \vec{H}_\varepsilon(x, t) \rightharpoonup \nabla \times \vec{H}(x, t)$$

in $L^2(Q_T)^3$ weakly.

Now $\{u_\varepsilon(x, t)\} \in L^1(0, T; W_0^{1,1}(\Omega))$, hence, $\nabla u \in L^1(0, T; L^1(\Omega)^3)$, and so

$$\Delta u \in L^1(0, T; (W_0^{1,\infty}(\Omega))'),$$

where $(W_0^{1,\infty}(\Omega))'$ denotes the conjugate space of $W_0^{1,\infty}(\Omega)$.

As $L^1(\Omega) \subset (W_0^{1,\infty}(\Omega))'$, it follows by the equation (3.2) that

$$\left\{ \frac{\partial u_\varepsilon}{\partial t} \right\} \in L^1(0, T; (W^{1,\infty}(\Omega))').$$

Moreover, since the imbedding operator from $W_0^{1,1}(\Omega)$ into $BV(\Omega)$ is compact (cf.[3]), where $BV(\Omega)$ denotes the Banach space which is composed of all of functions with finite total variations. It follows by Lemma 2.1 that $\{u_\varepsilon(x, t)\}$ is relatively compact in $L^1(0, T; BV(\Omega))$. Consequently, there exists a subsequence of $\{u_\varepsilon\}$ (denoted by itself) which converges to a function denoted by $u(x, t)$ a.e. in Q_T .

Now we are ready to show that $\{\vec{H}(x, t), u(x, t)\}$ is a weak solution. The difficulty lies in the nonlinear term $|\nabla \times \vec{H}_\varepsilon|^2$ since we only know that $\nabla \times \vec{H}_\varepsilon$ converges weakly. However, this difficulty can be overcome by using the first equation (2.5).

The integral identity (2.5) can be easily derived. Indeed, by multiplying the equation (3.1) by an arbitrary test function $\vec{\psi}(x, t)$ and integrating over Q_T , we have after the integration by parts that

$$\int \int_{Q_T} [-\vec{H}_\varepsilon \cdot \vec{\psi}_t + k(u_\varepsilon) \nabla \times \vec{H}_\varepsilon \cdot \nabla \times \vec{\psi}] dx dt = \int_\Omega \vec{H}_{0\varepsilon} \cdot \vec{\psi}(x, 0) dx,$$

which concludes the equation (2.5) as $\varepsilon \rightarrow 0$.

In order to show that $u(x, t)$ is a distribution solution, noting that for any vector fields $\vec{G}$ and $\vec{F}$, we have

$$\nabla \cdot [\vec{G} \times \vec{F}] = (\nabla \times \vec{G}) \cdot \vec{F} - \vec{F} \cdot (\nabla \times \vec{G}).$$

we use the above formula to rewrite the right-hand term in (3.2) in the following form

$$k(u_\varepsilon) |\nabla \times \vec{H}_\varepsilon|^2 = \nabla \cdot [k(u_\varepsilon) \vec{H}_\varepsilon \times (\nabla \times \vec{H}_\varepsilon)] - \vec{H}_\varepsilon \cdot \vec{H}_{\varepsilon t}.$$

Let $\phi(x, t) \in C_0^\infty(Q_T)$. We multiply the equation (3.2) by $\phi(x, t)$ to obtain

$$\int \int_{Q_T} [(u_\varepsilon + \frac{1}{2} |\vec{H}_\varepsilon|^2) \phi_t + u \Delta \phi] dx dt = \int \int_{Q_T} [k(u_\varepsilon) \vec{H}_\varepsilon \times (\nabla \times \vec{H}_\varepsilon)] \cdot \nabla \phi dx dt.$$

As $u_\varepsilon(x, t)$ and $\vec{H}_\varepsilon(x, t)$ converge to $u(x, t)$ and $\vec{H}(x, t)$ almost everywhere in Q_T , we can take the limit under the integral and obtain

$$\int \int_{Q_T} [u \phi_t + u \Delta \phi] dx dt = \int \int_{Q_T} [-\frac{1}{2} |\vec{H}|^2 \phi_t + (k(u) \vec{H} \times (\nabla \times \vec{H})) \cdot \nabla \phi] dx dt.$$

To obtain the integral identity (2.6), we choose $\vec{\psi}(x, t) = \vec{H}(x, t) \phi(x, t)$ in the identity (2.5) to see

$$\begin{aligned} & \int \int_{Q_T} \frac{1}{2} |\vec{H}|^2 \phi_t dx dt \\ &= \int \int_{Q_T} k(u) |\nabla \times \vec{H}|^2 dx dt + \int \int_{Q_T} k(u) (\nabla \times \vec{H}) \cdot (\nabla \phi \times \vec{H}) dx dt \\ &= \int \int_{Q_T} k(u) |\nabla \times \vec{H}|^2 dx dt + \int \int_{Q_T} [k(u) \vec{H} \times (\nabla \times \vec{H})] \cdot \nabla \phi dx dt, \end{aligned}$$

where the second equality comes from the following identity:

$$\vec{A} \cdot [\vec{B} \times \vec{C}] = \vec{B} \cdot [\vec{C} \times \vec{A}]$$

for any vector fields $\vec{A}$, $\vec{B}$ and $\vec{C}$. Consequently, we have the desired identity (2.6). This completes our proof of the theorem.

Remark 3.1: For the steady-state system, the similar existence theorem can be proved.

Remark 3.2: The uniqueness is an open question. It would also be interesting to study the regularity of solutions.

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