**NONPLANAR SHEAR FLOWS FOR NONALIGNING
NEMATIC LIQUID CRYSTALS**

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NONPLANAR SHEAR FLOWS FOR NONALIGNING NEMATIC LIQUID CRYSTALS*

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ABSTRACT. We investigate the stability of simple planar shear flow between moving parallel plates for nonaligning nematic liquid crystals. We present numerical results for the continuation of the nonplanar solution branch from its bifurcation from the planar solution branch which agree with the experimental results of Pieranski and Guyon [18] that at a critical shear rate the director turns out of the shear plane to an orientation nearly orthogonal to the shear plane.

1. Introduction. The stability of planar shear flow and nonplanar flow instabilities for nonaligning liquid crystals have been the subject of many experimental and theoretical investigations during the past fifteen years [2–4, 9, 16–21, 23–26]. Pieranski and Guyon [18] reported experimental results for flow between moving parallel plates with planar alignment for the director at the walls that at a critical shear rate the director moves from the shear plane to an orientation nearly perpendicular to the shear plane. Zúñiga and Leslie [25, 26] have recently reported the results of numerical calculations for the Ericksen-Leslie equations which confirm that as the shear rate is increased the director moves out of the shear plane at the first instability. In this paper, we use numerical continuation techniques to follow the nonplanar solution path, and our numerical results confirm the experimental results of Pieranski and Guyon [18] that the director should turn out of the plane to an orientation nearly orthogonal to the shear plane.

Cladis and Torza [4] have reported experimental results for Couette flow between rotating cylinders with homeotropic alignment of the director at the walls that as the shear rate is increased the first instability is a “tumbling instability” which is characterized by the exchange of stability of the planar configuration by another planar configuration with a more distorted director field. We have numerically computed the stability of both planar and nonplanar modes along the planar solution curve, and our results demonstrate that as the shear rate is increased the first unstable mode can be either planar or nonplanar, depending on the values of the material constants.

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2. Planar Flows. We consider simple shear flow between parallel plates at a distance $2h$ apart which are parallel to the x - y plane. We assume that the upper plate at $z = h$ is at rest while the lower one at $z = -h$ moves with velocity $\mathcal{V}$ in the y direction. The state of the nematic liquid crystal is described by its velocity $\mathbf{v} = \mathbf{v}(x, y, z, t)$ and its director $\mathbf{n} = \mathbf{n}(x, y, z, t)$ where $|\mathbf{n}| = 1$.

We first investigate simple planar shear flows of the form

$$(2.1) \quad \mathbf{v} = (0, v(z, t), 0), \quad \mathbf{n} = (0, \cos \theta(z, t), \sin \theta(z, t)).$$

For flows of the form (2.1) the Ericksen-Leslie equations [14] are

$$(2.2) \quad \rho \frac{\partial v}{\partial t} = \frac{\partial}{\partial z} \left(g(\theta) \frac{\partial v}{\partial z} + m(\theta) \frac{\partial \theta}{\partial t} \right), \quad -h \leq z \leq h,$$

$$(2.3) \quad 2\gamma_1 \frac{\partial \theta}{\partial t} = 2f(\theta) \frac{\partial^2 \theta}{\partial z^2} + \frac{\partial f(\theta)}{\partial \theta} \left(\frac{\partial \theta}{\partial z} \right)^2 - 2m(\theta) \frac{\partial v}{\partial z},$$

where

$$g(\theta) = \alpha_1 \sin^2 \theta \cos^2 \theta + \frac{\alpha_5 - \alpha_2}{2} \sin^2 \theta + \frac{\alpha_6 + \alpha_3}{2} \cos^2 \theta + \frac{\alpha_4}{2};$$

$$m(\theta) = (\gamma_1 + \gamma_2 \cos 2\theta)/2; \quad f(\theta) = \kappa_1 \cos^2 \theta + \kappa_3 \sin^2 \theta;$$

ρ is the density; $\alpha_1, \dots, \alpha_6$ are the Leslie viscosities; $\kappa_1, \kappa_2, \kappa_3$ are the Frank elastic constants; and $\gamma_1 = \alpha_3 - \alpha_2$, $\gamma_2 = \alpha_6 - \alpha_5$. Thermodynamic inequalities imply that $g(\theta) > 0$ and $f(\theta) > 0$ for all θ and that $\gamma_1 > 0$ [14]. We shall only consider flows in the nonaligning regime $\gamma_1 > |\gamma_2|$, so $m(\theta) > 0$ for all θ .

We utilize the “strong anchoring” condition for $\mathbf{n}$, i.e.,

$$(2.4) \quad \theta(-h, t) = \theta(h, t) = \theta_p,$$

and the “no-slip” boundary condition for $\mathbf{v}$,

$$(2.5) \quad v(-h, t) = \mathcal{V}, \quad v(h, t) = 0.$$

For steady flow, the Ericksen-Leslie equations (2.2)–(2.5) are

$$(2.6) \quad \frac{\partial}{\partial z} \left(g(\theta) \frac{\partial v}{\partial z} \right) = 0, \quad -h \leq z \leq h,$$

$$(2.7) \quad 2f(\theta) \frac{\partial^2 \theta}{\partial z^2} + \frac{\partial f(\theta)}{\partial \theta} \left(\frac{\partial \theta}{\partial z} \right)^2 - 2m(\theta) \frac{\partial v}{\partial z} = 0, \quad -h \leq z \leq h,$$

$$(2.8) \quad \theta(-h) = \theta(h) = \theta_p,$$

$$(2.9) \quad v(-h) = \mathcal{V}, \quad v(h) = 0.$$

Thus,

$$(2.10) \quad g(\theta) \frac{\partial v}{\partial z} = c$$

where c is an integrating constant for (2.6), and (2.10) can be used to eliminate $\partial v / \partial z$ from (2.7) to obtain

$$(2.11) \quad 2f(\theta) \frac{\partial^2 \theta}{\partial z^2} + \frac{\partial f(\theta)}{\partial \theta} \left(\frac{\partial \theta}{\partial z} \right)^2 - 2c \frac{m(\theta)}{g(\theta)} = 0, \quad -h \leq z \leq h,$$

$$\theta(-h) = \theta(h) = \theta_p.$$

We can integrate (2.10) to obtain that

$$(2.12) \quad \mathcal{V} = -c \int_{-h}^h \frac{1}{g(\theta(z))} dz.$$

All of the solutions $\theta(z)$ of (2.6)–(2.9) in the nonaligning case have a single critical point at $z = 0$ with critical value θ_m . So,

$$\theta_m = \theta(0) = \begin{cases} \max_{-h \leq z \leq h} \theta(z) & \text{for } \mathcal{V} \geq 0, \\ \min_{-h \leq z \leq h} \theta(z) & \text{for } \mathcal{V} \leq 0, \end{cases}$$

and all solutions to (2.6)–(2.9) can be parametrized by θ_m . Thus,

$$c = c(\theta_m), \quad \theta(z) = \theta(z, \theta_m).$$

We have obtained numerical solutions for the linearized stability equations for (2.2)–(2.5) given by the following eigenvalue problem for the perturbations $e^{i\lambda t} V(z)$ of $v(z)$ and $e^{i\lambda t} \Theta(z)$ of $\theta(z)$

$$(2.13) \quad \lambda \rho V = \frac{\partial}{\partial z} \left(\frac{\partial g}{\partial \theta} \frac{\partial v}{\partial z} \Theta + g(\theta) \frac{\partial V}{\partial z} + \lambda m(\theta) \Theta \right),$$

$$2\lambda \gamma_1 \Theta = 2 \frac{\partial}{\partial z} \left(f(\theta) \frac{\partial \Theta}{\partial z} \right) + 2 \frac{\partial f}{\partial \theta} \frac{\partial^2 \theta}{\partial z^2} \Theta + \frac{\partial^2 f}{\partial \theta^2} \left(\frac{\partial \theta}{\partial z} \right)^2 \Theta$$

$$(2.14) \quad -2 \frac{\partial m}{\partial \theta} \frac{\partial v}{\partial z} \Theta - 2m(\theta) \frac{\partial V}{\partial z},$$

$$(2.15) \quad V(-h) = V(h) = 0, \quad \Theta(-h) = \Theta(h) = 0.$$

The critical velocity for the planar instability or tumbling instability is characterized as the first turning point on the solution curve for (2.6)–(2.9) with respect to the parameter $\mathcal{V}$. We note that turning points of solutions of (2.6)–(2.9) parametrized by c (where $dc/d\theta_m = 0$) are different from the turning points of solutions of (2.6)–(2.9) parametrized by $\mathcal{V}$ (where $d\mathcal{V}/d\theta_m = 0$) since

$$\frac{d\mathcal{V}}{d\theta_m} = -\frac{dc}{d\theta_m} \int_{-h}^h \frac{1}{g(\theta(z))} dz + c \int_{-h}^h \frac{dg}{d\theta} \frac{1}{g^2} \frac{\partial \theta}{\partial \theta_m} dz.$$

If the “no-slip” boundary conditions are replaced by the boundary conditions

$$(2.16) \quad g(\theta(-h, t)) \frac{\partial v}{\partial z}(-h, t) = \tau, \quad v(h, t) = 0,$$

where τ is the given shear stress on the bottom plate and if the inertial term $\lambda\rho V$ is dropped, then the integrating constant in (2.13) is 0 and so we have that

$$(2.17) \quad \frac{\partial g}{\partial \theta} \frac{\partial v}{\partial z} \Theta + g(\theta) \frac{\partial V}{\partial z} + \lambda m(\theta) \Theta = 0, \quad -h \leq z \leq h.$$

In this case, we can use (2.17) to eliminate $\partial V/\partial z$ in (2.14) and to obtain for the linearized stability equations the Sturm-Liouville eigenvalue problem

$$(2.18) \quad \begin{aligned} 2\lambda \left[\gamma_1 - \frac{m(\theta)^2}{g(\theta)} \right] \Theta &= 2 \frac{\partial}{\partial z} \left(f(\theta) \frac{\partial \Theta}{\partial z} \right) + 2 \frac{\partial f}{\partial \theta} \frac{\partial^2 \theta}{\partial z^2} \Theta \\ &+ \frac{\partial^2 f}{\partial \theta^2} \left(\frac{\partial \theta}{\partial z} \right)^2 \Theta - 2 \frac{\partial m}{\partial \theta} \frac{\partial v}{\partial z} \Theta + 2 \frac{m(\theta)}{g(\theta)} \frac{\partial g}{\partial \theta} \frac{\partial v}{\partial z} \Theta, \\ \Theta(-h) &= \Theta(h) = 0. \end{aligned}$$

For $\lambda = 0$ the equations (2.13)–(2.15) are the equations defining a critical point for solutions of (2.6)–(2.9) parametrized by the plate velocity $\mathcal{V}$ whereas for $\lambda = 0$ the equations (2.18) are the equations defining a critical point for solutions of (2.6)–(2.8), (2.16) parametrized by the boundary shear stress τ .

3. Nonplanar flows. We have also computed the flow of nematic liquid crystals of the nonplanar form

$$(3.1) \quad \begin{aligned} \mathbf{v}(z, t) &= (u(z, t), v(z, t), w(z, t)), \\ \mathbf{n}(z, t) &= (\cos \phi(z, t) \cos \theta(z, t), \sin \phi(z, t) \cos \theta(z, t), \sin \theta(z, t)). \end{aligned}$$

Since the flow is incompressible, we have that $w = 0$. The “no-slip” boundary conditions for $\mathbf{v}$ and the “strong anchoring” boundary conditions for $\mathbf{n}$ are now given by

$$(3.2) \quad \begin{aligned} u(-h, t) &= 0, & u(h, t) &= 0, \\ v(-h, t) &= \mathcal{V}, & v(h, t) &= 0, \\ \theta(-h, t) &= \theta_p, & \theta(h, t) &= \theta_p, \\ \phi(-h, t) &= \frac{\pi}{2}, & \phi(h, t) &= \frac{\pi}{2}. \end{aligned}$$

The linear momentum equations in the Ericksen-Leslie model for flows of the form (3.1) are [14,15]

$$(3.3) \quad \rho \frac{\partial u}{\partial t} = \tau_{13,3},$$

$$(3.4) \quad \rho \frac{\partial v}{\partial t} = \tau_{23,3},$$

$$(3.5) \quad 0 = -\frac{\partial p}{\partial z} - \left(\frac{\partial \mathcal{W}}{\partial n_{k,3}} n_{k,3} \right)_{,3} + \tau_{33,3},$$

where $p = p(z, t)$ is the pressure, $\mathcal{W} = \mathcal{W}(\mathbf{n}, \nabla \mathbf{n})$ is the Oseen-Frank free energy [14], and the viscous stresses $\tau_{13,3}$ and $\tau_{23,3}$ are given by

$$\begin{aligned}\tau_{13} &= (M(\theta) + N(\theta) \cos^2 \phi) \frac{\partial u}{\partial z} + N(\theta) \sin \phi \cos \phi \frac{\partial v}{\partial z} \\ &\quad + 2K(\theta) \cos \phi \frac{\partial \theta}{\partial t} - 2L(\theta) \sin \phi \frac{\partial \phi}{\partial t}, \\ \tau_{23} &= N(\theta) \sin \phi \cos \phi \frac{\partial u}{\partial z} + (M(\theta) + N(\theta) \sin^2 \phi) \frac{\partial v}{\partial z} \\ &\quad + 2K(\theta) \sin \phi \frac{\partial \theta}{\partial t} + 2L(\theta) \cos \phi \frac{\partial \phi}{\partial t},\end{aligned}$$

with

$$\begin{aligned}M(\theta) &= \alpha_4 + (\alpha_5 - \alpha_2) \sin^2 \theta, \\ N(\theta) &= (2\alpha_1 \sin^2 \theta + \alpha_3 + \alpha_6) \cos^2 \theta, \\ K(\theta) &= \alpha_3 \cos^2 \theta - \alpha_2 \sin^2 \theta, \\ L(\theta) &= \alpha_2 \sin \theta \cos \theta.\end{aligned}$$

The equation (3.5) simply yields an expression for the pressure and can therefore be neglected. From the Ericksen-Leslie angular momentum equations we obtain

$$\begin{aligned}(3.6) \quad &2f(\theta) \frac{\partial^2 \theta}{\partial z^2} + f'(\theta) \left(\frac{\partial \theta}{\partial z} \right)^2 - G'(\theta) \left(\frac{\partial \phi}{\partial z} \right)^2 \\ &- 2m(\theta) \left(\frac{\partial u}{\partial z} \cos \phi + \frac{\partial v}{\partial z} \sin \phi \right) = 2\gamma_1 \frac{\partial \theta}{\partial t},\end{aligned}$$

$$\begin{aligned}(3.7) \quad &2G(\theta) \frac{\partial^2 \phi}{\partial z^2} + 2G'(\theta) \frac{\partial \theta}{\partial z} \frac{\partial \phi}{\partial z} \\ &+ \alpha_2 \sin 2\theta \left(\frac{\partial u}{\partial z} \sin \phi - \frac{\partial v}{\partial z} \cos \phi \right) = 2\gamma_1 \cos^2 \theta \frac{\partial \phi}{\partial t},\end{aligned}$$

where $f(\theta) = \kappa_1 \cos^2 \theta + \kappa_3 \sin^2 \theta$ and $G(\theta) = (\kappa_2 \cos^2 \theta + \kappa_3 \sin^2 \theta) \cos^2 \theta$.

In the steady-state case, equations (3.2)–(3.4), (3.6), (3.7) become

$$\begin{aligned}(3.8) \quad &\frac{d}{dz} \left((M(\theta) + N(\theta) \cos^2 \phi) \frac{du}{dz} + N(\theta) \sin \phi \cos \phi \frac{dv}{dz} \right) = 0, \\ &\frac{d}{dz} \left(N(\theta) \sin \phi \cos \phi \frac{du}{dz} + (M(\theta) + N(\theta) \sin^2 \phi) \frac{dv}{dz} \right) = 0, \\ &2f(\theta) \frac{d^2 \theta}{dz^2} + f'(\theta) \left(\frac{d\theta}{dz} \right)^2 - G'(\theta) \left(\frac{d\phi}{dz} \right)^2 \\ &\quad - 2m(\theta) \left(\frac{du}{dz} \cos \phi + \frac{dv}{dz} \sin \phi \right) = 0, \\ &2G(\theta) \frac{d^2 \phi}{dz^2} + 2G'(\theta) \frac{d\theta}{dz} \frac{d\phi}{dz} \\ &\quad + \alpha_2 \sin 2\theta \left(\frac{du}{dz} \sin \phi - \frac{dv}{dz} \cos \phi \right) = 0,\end{aligned}$$

with the boundary conditions

$$(3.9) \quad \begin{aligned} u(-h) &= 0, & u(h) &= 0, \\ v(-h) &= \mathcal{V}, & v(h) &= 0, \\ \theta(-h) &= \theta_p, & \theta(h) &= \theta_p, \\ \phi(-h) &= \frac{\pi}{2}, & \phi(h) &= \frac{\pi}{2}. \end{aligned}$$

The linearized stability equations for the Ericksen-Leslie equations for steady flows of the form (2.1) with respect to flows of the form (3.1) are given by the eigenvalue problem (2.13)–(2.15) for planar perturbations $e^{i\lambda t}V(z)$ of $v(z)$ and $e^{i\lambda t}\Theta(z)$ of $\theta(z)$, and by the eigenvalue problem for the nonplanar perturbations $e^{i\lambda t}U(z)$ of $u(z) = 0$ and $e^{i\lambda t}\Phi(z)$ of $\phi(z) = \pi/2$ given by

$$(3.10) \quad \begin{aligned} \lambda \rho U &= \frac{d}{dz} \left(g_1(\theta) \frac{dU}{dz} + g_2(\theta) \frac{dv}{dz} \Phi - \lambda \alpha_2 \sin \theta \Phi \right), \\ \lambda \gamma_1 \Phi &= \frac{d}{dz} \left(f_1(\theta) \frac{d\Phi}{dz} \right) + q_1 \left(\theta, \frac{d\theta}{dz}, \frac{d^2\theta}{dz^2}, \frac{dv}{dz} \right) \Phi - \alpha_2 \sin \theta \frac{dU}{dz}, \\ U(-h) &= U(h) = 0, & \Phi(-h) &= \Phi(h) = 0, \end{aligned}$$

where

$$\begin{aligned} f_1(\theta) &= \kappa_2 \cos^2 \theta + \kappa_3 \sin^2 \theta, \\ f_2 \left(\theta, \frac{d\theta}{dz}, \frac{d^2\theta}{dz^2} \right) &= \frac{\kappa_2 - \kappa_1}{2} \sin 2\theta \frac{d^2\theta}{dz^2} + (\kappa_2 - (3\kappa_3 - \kappa_1 - 2\kappa_3) \sin^2 \theta) \left(\frac{d\theta}{dz} \right)^2, \\ q_1 \left(\theta, \frac{d\theta}{dz}, \frac{d^2\theta}{dz^2}, \frac{dv}{dz} \right) &= f_2 \left(\theta, \frac{d\theta}{dz}, \frac{d^2\theta}{dz^2} \right) + \gamma_2 \sin \theta \cos \theta \frac{dv}{dz}, \\ 2g_1(\theta) &= \alpha_4 + (\alpha_5 - \alpha_2) \sin^2 \theta, \\ 2g_2(\theta) &= (\alpha_6 + \alpha_3 + 2\alpha_1 \sin^2 \theta) \cos \theta. \end{aligned}$$

4. Computational Results. We approximated steady-state solutions to the differential equations for planar shear flow by reformulating the second-order system of ordinary differential system (2.6)–(2.9) as a system of first-order ordinary differential equations and by using the method of collocation at Gauss points with continuous piecewise polynomials of degree four to discretize the system of ordinary differential equations [1,8] to obtain a set of nonlinear equations parametrized by the plate velocity $\mathcal{V}$. We then used implemented the method of pseudo-arclength continuation [11] by using the software package AUTO [7,8], to compute the solution branch of steady, planar solutions. The starting point for the solution branch was taken to be the trivial solution to (2.6)–(2.9) given by $v(z) = 0$ and $\theta(z) = \theta_p$ at the plate velocity $\mathcal{V} = 0$. A simple calculation shows that this starting point is a regular point for the solution branch.

The stability of nonplanar perturbations of the planar solutions on the planar solution branch is determined by the numerical solution of the eigenvalue problem for the numerical approximation of (3.10) which is obtained by using the method of

finite differences with the mesh points taken to be the Gauss quadrature points at which the planar solutions have been obtained. We solved the resulting generalized matrix eigenvalue problem by using stabilized elementary similarity transformations to reduce the the generalized eigenproblem to Hessenberg form and by using the QR method implemented in the EISPACK software package [22].

We first present computational results for solution branches followed by pseudo-arclength continuation for solutions to (2.6)–(2.9) parametrized by $\mathcal{V}$ which show that the first instability can be planar or nonplanar, depending on the material constants. We model the behavior of 8CB just above the smectic A-nematic transition temperature by the following material constants for $\epsilon > 0$:

$$(4.1) \quad \begin{aligned} \alpha_1 &= 12\epsilon|\alpha_2|, & \alpha_2 &= -.7, & \alpha_3 &= \epsilon|\alpha_2|, \\ \alpha_4 &= .58, & \alpha_5 &= .7, & \alpha_6 &= \epsilon|\alpha_2|, \\ \kappa_1 &= 1.41 \times 10^6, & \kappa_2 &= 1.023\epsilon\kappa_1, & \kappa_3 &= 2.605\epsilon\kappa_1, \end{aligned}$$

where the viscosities are in poise, the elastic constants are in dyne, and $h = 1$ cm. The boundary conditions for these computations were given by

$$\theta_p = \pi/2.$$

In Figure 1, we observe that the nonplanar instability occurs at a lower plate velocity $\mathcal{V}$ than the planar instability for $.5 \leq \epsilon < .75$. At $\epsilon = .75$ a new pair of limit points has developed, and we observe in Figures 1–3 that for $.75 < \epsilon < 1.1$ the planar instability occurs at a lower plate velocity $\mathcal{V}$ than that for any of the nonplanar modes. At $\epsilon = 1.1$ we observe that the upper solution curve is no longer stable for velocities larger than the critical velocity. Thus, for $\epsilon > 1.1$ at plate velocities $\mathcal{V}$ larger than the critical plate velocity there is not a stable steady planar solution.

Next, we present computational results for 8CB at 35° C. for the nonplanar solution branch of the form (3.1) which we have numerically continued from the bifurcation point on the planar solution branch. The bifurcation point on the planar solution branch is determined by the planar solution $\theta(z)$, $v(z)$ with plate velocity $\mathcal{V}$ for which $\lambda = 0$ is an eigenvalue for the nonplanar eigenvalue problem (3.10). We then used the software package AUTO to switch to the nonplanar solution branch defined by the differential system (3.8)–(3.9).

At 35° C., the material constants for 8CB have been measured to be [10,12,13]

$$(4.2) \quad \begin{aligned} \alpha_1 &= 1.3420, & \alpha_2 &= -0.6965, & \alpha_3 &= 0.1395, \\ \alpha_4 &= 0.5600, & \alpha_5 &= 0.5275, & \alpha_6 &= -0.0295, \\ \kappa_1 &= 1.2800 \times 10^6, & \kappa_2 &= 0.6000 \times 10^6, & \kappa_3 &= 1.4000 \times 10^6, \end{aligned}$$

where the viscosities are in poise, the elastic constants are in dyne, and $h = 1$ cm. These computations used the boundary data $\theta_p = 0$ in (3.9). The position of the bifurcation point in Figure 4 is marked by “+”. Profiles of the solution $u(z)$, $v(z)$, $\phi(z)$, and $\theta(z)$ are given in Figures 5–8 at the points on the solution branch marked by A–W.

We observe that for plate velocities $\mathcal{V} > .00025$ cm/sec the director is nearly orthogonal to the shear plane in agreement with the experimental observations of Pieranski and Guyon [18].

FIG. 1. Solution branches for $\epsilon = .5, .6, .7$, and $.75$ for the material constants given in (4.1) and $\theta_p = \pi/2$. For $\epsilon = .5, .6$, and $.7$ the nonplanar instability occurs at a lower plate velocity $\mathcal{V}$ than the planar instability, but a new pair of limit points has developed for $\epsilon = .75$.

FIG. 2. Solution branches for $\epsilon = .77, .79, .81$, and $.86$ for the material constants given in (4.1) and $\theta_p = \pi/2$. For $\epsilon = .77, .79, .81$, and $.86$ the planar instability occurs at a lower plate velocity V than that for any of the nonplanar modes.

FIG. 3. Solution branches for $\epsilon = .9, 1.0, 1.1$, and 1.5 for the material constants given in (4.1) and $\theta_p = \pi/2$. For $\epsilon = .9$ and $\epsilon = 1.0$ the planar instability occurs at a lower plate velocity $\mathcal{V}$ than that for any of the nonplanar modes. For $\epsilon = 1.1$ and $\epsilon = 1.5$ the upper branch of the solution curve is not stable for plate velocities $\mathcal{V}$ larger than the critical plate velocity. Thus, for $\epsilon = 1.1$ and $\epsilon = 1.5$ there does not exist a stable steady planar solution.

FIG. 4. Nonplanar solution branch for 8CB at 35° C. The position of the bifurcation point is marked by "+".

FIG. 5. Solution profile of $u(z)$ for the nonplanar branch shown in Figure 4. The solution profiles are labeled A–W to correspond to points similarly labeled on the nonplanar solution branch in Figure 4.

FIG. 6. Solution profile of $\phi(z)$ for the planar branch shown in Figure 4. The solution profiles are labeled A-W to correspond to points similarly labeled on the nonplanar solution branch in Figure 4.

FIG. 7. Solution profile of $v(z)$ for the planar branch shown in Figure 4. The solution profiles are labeled A–W to correspond to points similarly labeled on the nonplanar solution branch in Figure 4.

FIG. 8. Solution profile of $\theta(z)$ for the planar branch shown in Figure 4. The solution profiles are labeled A-W to correspond to points similarly labeled on the nonplanar solution branch in Figure 4.

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