**NUMERICAL STUDY OF
A SINGULAR SYSTEM OF CONSERVATION LAWS
ARISING IN ENHANCED OIL RESERVOIRS**

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NUMERICAL STUDY OF A SINGULAR SYSTEM OF CONSERVATION LAWS ARISING IN ENHANCED OIL RESERVOIRS*

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Abstract. We present a numerical study of the structure of elementary nonlinear waves and their qualitative behavior, for degenerate hyperbolic system of equations that describe the process of polymer flooding of an oil reservoir.

1. Introduction. We consider the physico-chemical flooding model of a horizontal porous medium describing two phase flow of oil and water, where the water contains additive chemicals (polymers) to increase the oil recovery, which has been considered by many authors [1],[2],[5]. The mass conservation equations modeling the one-dimensional displacement of oil by polymer are:

$$(1.1) \quad \begin{aligned} \phi \partial_t s + U \partial_x F(s, c) + \epsilon \partial_x (\psi \partial_x P_c(s, c)) &= 0 \\ \partial_t (\phi s c + A(c)) + U \partial_x (c F(s, c)) + \epsilon \partial_x (c \psi \partial_x P_c(s, c)) &= \nu \partial_x (D \partial_x c) \end{aligned}$$

where s = saturation of water, $(1 - s)$ = saturation of oil, c = concentration of polymer,

$$F(s, c) = \frac{f_1(s)}{f_1(s) + \frac{\mu_1(c)}{\mu_2(c)} f_2(s)}$$

fractional flow rate of water, f_1, f_2 relative phase permeabilities, μ_1, μ_2 phase viscosities, $\phi(x)$ permeability of the medium, $\psi = \frac{f_2}{\mu_2} F$, $P_c(s, c) = P_2(s, c) - P_1(s, c)$ capillarity pressure, $A(c)$ = amount of polymer sorbed by the porous medium, $U = u_1 + u_2$ characteristic velocity satisfying Darcy's law

$$u_i = -\left(\frac{\phi f_i(s)}{\mu_i(c)}\right) \partial_x P_c(s, c).$$

The diffusion coefficient D comes from Fick's law of molecular diffusion, ϵ is the parameter describing the rate of capillary pressure P_c to the change in the total hydrodynamic pressure across the reservoir, ν is the parameter of convection of the diffusive flow through the reservoir. However, here we study an equivalent but simpler system of equations. We

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assume that $\phi = U = 1$, $\nu = A(c) = 0$, and the capillary effect is restricted to a constant positive value. This will lead to the following viscous system of equations

$$(1.2) \quad \begin{aligned} \partial_t s + \partial_x F &= \epsilon s_{xx} \\ \partial_t(sc) + \partial_x(cF) &= \epsilon(sc)_{xx} \end{aligned}$$

We consider this system of equations over half plane $-\infty < x < \infty$ and $t > 0$, and the functions $s(x, t), c(x, t)$ are restricted to the square $\Omega = (0, 1) \times (0, 1)$. The flux function $F(s, c)$ satisfies the following properties:

1. $F(0, c) = 0, F(1, c) = 1, \forall c \in [0, 1]$.
2. $\frac{\partial F}{\partial c} < 0, (s, c) \in \Omega$.
3. $\frac{\partial F}{\partial s} > 0, (s, c) \in \Omega$.

In Fig. [1] we plot the surface F over Ω .

Figure 1 The function $F(s, c)$

The capillarity parameter ϵ is usually very small. Thus we can set $\epsilon = 0$ to obtain the corresponding inviscid system of equations

$$(1.3) \quad \begin{aligned} \partial_t s + \partial_x F &= 0 \\ \partial_t(sc) + \partial_x(cF) &= 0 \end{aligned}$$

Following Isaacson and Temple [4], we set $G = F/s$ to be the velocity of the aqueous phase, denote (s, sc) by U . Consider the Riemann problem for the quasilinear system of

equations (1.3), that is the initial value with two end states

$$(1.4) \quad U_0(x) = \begin{cases} U_l, & x \leq 0, \\ U_r, & x > 0. \end{cases}$$

Based on the Lax entropy condition, Isaacson [5] has classified the Riemann type solutions for the system (1.3) with (1.4). The characteristic values for the system (1.3) are $\lambda_c = G$ and $\lambda_s = F_s$. These wave speeds are positive for any value of s and c in the unit square Ω . We denote the corresponding elementary waves by C -waves and S -waves. The corresponding integral curves are $G = \text{constant}$, and $c = \text{constant}$. Thus the S -waves will be the solution of the Buckley-Leverett equation consisting of shocks and rarefaction waves for which c is constant. The C -waves are the rarefaction waves along the curve $G = \text{constant}$. These waves will be called contact discontinuities, because the characteristic values are equal at the right and left states of the wave. For a flux function F satisfying (1.3), the eigenvalues λ_c, λ_s are equal along a simple curve T dividing the unit square Ω into two regions: the left region L , and the right region R . Thus, the system (1.3) is linearly degenerate and is not diagonalizable along T . The structure of the Riemann solution connecting two end states will be a sequence of S -waves and C -waves according to the increase of their corresponding characteristic speeds. In this paper, we will investigate the problem numerically. Instead of studying the inviscid system (1.3), we will consider the viscous system of equations (1.3), computing asymptotic behaviour of the solutions for various values of ϵ , and studying some questions that are difficult to answer in a rigorous way, such as the stability and the ϵ zero limit.

2. Numerical Scheme. We will use finite difference approximations to the following simplified system of equations

$$(2.1) \quad \begin{aligned} \partial_t s + (sG)_x &= \epsilon s_{xx} \\ \partial_t c + Gc_x &= \epsilon c_{xx} + \epsilon \left(\frac{2c_x s_x}{s} \right) \end{aligned}$$

Let $\Delta t, \Delta x$ denote the time and space steps. Define $s_j^n = s(n\Delta t, j\Delta x)$, and $c_j^n = c(n\Delta t, j\Delta x)$ for positive integers n, j . We denote D_+, D_-, D_0 to be the standard forward, backward and centered finite difference operators. We restrict our computations to a finite interval by introducing artificial boundary conditions so the computations will be stopped before reaching the boundaries, or by introducing moving system of coordinates with a fixed speed as we have done in this analysis. For the system (2.1) we introduce the following implicit, unconditionally stable scheme of second order in time and space $O(\Delta t^2 + \Delta x^2)$. The scheme is

$$(2.2) \quad \begin{aligned} \frac{3}{2} \frac{s_j^{n+1} - s_j^n}{\Delta t} - \frac{1}{2} \frac{s_j^n - s_j^{n-1}}{\Delta t} &= +\epsilon \frac{D_+ D_- s^{n+1}}{2} + R_s(s^n, c^n, s^{n-1}) \\ \frac{3}{2} \frac{c_j^{n+1} - c_j^n}{\Delta t} - \frac{1}{2} \frac{c_j^n - c_j^{n-1}}{\Delta t} &= +\epsilon \frac{D_+ D_- c^{n+1}}{2} + R_c(s^n, c^n, c^{n-1}) \end{aligned}$$

where the functions R_s, R_c are defined as follows

$$\begin{aligned} R_s &= -D_0(s^n G(s^n, c^n)) + \epsilon \frac{D_+ D_- s^{n-1}}{2} \\ R_c &= -G(s^n, c^n) D_0 c^n + \epsilon \frac{D_+ D_- c^{n-1}}{2} + \epsilon \frac{2D_0 c^n D_0 s^n}{s^n}. \end{aligned}$$

Since this finite difference scheme approximates a system of equations of hyperbolic character for small ϵ , the discretization should satisfy the additional stability condition of Courant-Friedrichs-Lewy (CFL), that is

$$(2.3) \quad \Delta t \leq \frac{\Delta x}{M}$$

where M is a positive constant defined as follows

$$M = \max_{\Omega}(\lambda_s, \lambda_c)$$

In fact in our computations, we take $M = 2$.

The consistency and linearized stability of this scheme is a standard analysis. Tveito [10] has shown similar analysis of an explicit scheme of first order in time for a similar system of equations. However, such an explicit scheme is not efficient for the study of asymptotic (large time) behavior of the solutions and their stability due to restrictions on time steps and round off errors. Therefore, it is essential to work with high order schemes such as the proposed scheme (2.2).

3. Numerical results. We begin this section by specifying the flux function F to be of the form

$$F(s, c) = \frac{s^2}{s^2 + (.5 + c)(1 - s)^2}.$$

Then the characteristic field for fixed value of $\bar{G}$ will be given by the following explicit formula

$$c = -.5 + \frac{s - \bar{G}s^2}{\bar{G}(1 - s)^2}$$

and the transition curve T will be given by

$$c = -.5 + \frac{s^2}{1 - s^2}.$$

Figure [2] depicts the curve T and characteristic fields for various values $\bar{G} = 1.08, 1.12, 1.16, 1.2, 1, 24$. We choose $U_l = (s_l, c_l) = (.55, .2695473) = U_1$; then we have $\bar{G}(s_l, c_l) = 1.2$. The characteristic curve defined by G_l will intersect the transition curve T at the point $U_4 = (.7142857, .5416664)$. The intersection points of the horizontal line passing through U_l

Figure 2

with the transition curve and the curve $\bar{G}$ are $U_2 = (.6594571, c_l)$ and $U_3 = (.7906975, c_l)$. These four points are shown in Fig [2].

3.1 Riemann structure. We demonstrate that the structure of the Riemann type solutions of the inviscid system (1.3) as done in [5] and [6] is the limit of the corresponding profiles of the viscous system (1.2) as ϵ approaches 0, and for finite time level. We provide the following example to illustrate the validity of this assertion. Let $U_l = U_1$ and take $U_r = (.2, .7) = U_5$ which means that $U_l \in L$ and $U_r \in L$ with c_r greater than the maximum of the characteristic curve corresponding to U_l , that is $c_5 > c_4$. See Fig [2]. Let $\Delta x = 1/300, \Delta t = \Delta x/2, \epsilon = .0006$. Isaacson has shown that the connection $U_l \mapsto U_r$ will be the sequence S -wave, C -wave, S -wave. Figure [3] shows that the solution of (1.3) with this initial data will also evolve into a profile connecting the left state to the right state by $S - C - S$ wave profiles. The variable s is plotted at time $T = 0, 3, 9, 15$ on Figure [3], and in Fig [4] we plot the profile of the variable c at the same successive time. As we decrease the value of ϵ , the width of all profiles decreases to zero for a finite time.

3.2 Width of the profiles. In this section we explore the effect of capillarity parameter ϵ (viscosity) on the thickness of the profiles of the elementary waves. The S -profiles are the solution of the Buckley-Leverett equation. We substitute solutions of the form $U = U(\frac{x-Vt}{\epsilon})$ in (1.2), where V is the speed of the shock profile. We can see that the width of the shock profile is of order ϵ and inverse to the strength of the shock which

leads to the following formula

$$W = \frac{\epsilon}{|U_l - U_r|}.$$

Here W denotes the width . The rarefaction waves of the Buckley-Leverret equation expand according to the following formula

$$(3.2) \quad W = (\lambda_r - \lambda_l)t.$$

Next we consider the profile of the C -waves which are contact discontinuities with $\lambda_r = \lambda_l = G(s_l, c_l)$. One might expect that the width of the profile of contact discontinuities is of finite thickness. It is not clear how to derive an approximate formula for the width of these waves from theoretical considerations. We obtain the following formula based on computational data

$$(3.3) \quad W \approx \frac{1}{\min(|s_r - s_l|, |c_r - c_l|)} \epsilon^\alpha t^\beta$$

where $\alpha \approx \frac{1}{3}, \beta \approx \frac{1}{2}$. Thus we conclude that the width of the profiles of C -waves increases with time.

THE S-C-S STRUCTURE At Time T=0,3,9,15

Figure 3 The function $s(x,t)$

Figure 4 The function $c(x,t)$

3.3 Uniqueness. The solution of the Riemann problem of the inviscid system (1.3), corresponding to two end states is unique. However, different connections may correspond to the same end states. This fact is consistent with the entropy admissibility criteria. For example, let $U_l = U_1$ and $U_r = U_4$, then we may have the following stable connections: C -Wave, $S - C$ -Wave, or $C \dots C \dots C$ waves. However, the one-to-one correspondence between the two end states and the connections will be restored in the viscous system (1.2). This will occur due to the expanding width of the C -waves.

3.4 Stability. In this section, we first show the validity of the following important claim: the maximum value of the initial data $c(0, x)$ is constant in time. The importance of this stems from the fact that the final configuration of the initial value problem is defined not only by the end states but also by the maximum value of c . In order to demonstrate the validity of this claim we choose the following example. Take $s(0, x) = s_1$ and $c(0, x) = c_1 + \chi$ where χ is a characteristic function such that

$$\chi = \begin{cases} \bar{c}, & x \in [a, b], \\ 0, & x \notin [a, b]. \end{cases}$$

This initial data is a constant state with a simple perturbation see Fig [5]. We find the numerical solution of the viscous system (1.2) with these initial data, where ϵ is taken to be small enough. In general the solution will consist of two waves moving along the characteristic fields corresponding to C - and S -waves. This is precisely what we observe in Fig [6], where this nonlinear wave breaks into a C -wave along the integral curve $\bar{G}$, and S -waves along $c = \text{const} = c_l$. The S -wave along its characteristic is the solution of the Buckley-Leverett equation, which is a scalar conservation law, so it will decay as an N -wave to zero due to nonlinearity and viscosity. However the C -wave moves as a stable wave with constant upper value equal to $\bar{c}$. This value $\bar{c}$ remains constant, because the nonlinear interaction along the $\bar{G}$ curve has the same speed. However the profile of the C -wave increases in time following the width formula (3.3), and this diffusive process leads to the decay of these waves to a constant state. vanish. This also can be seen from the linearized dispersion relation around a point (c_1, s_1) in the C -direction

$$(1.9) \quad \lambda = -\epsilon k^2 - i\bar{G}k$$

where k is the wave number and λ is the growth factor. The negative real part $-\epsilon k^2$ of λ leads to the decaying of any perturbation to zero; that is we have a stability in the case $\epsilon \neq 0$. We also may conclude the important claim that there is no stability of any waves of the inviscid system (1.3). This observation can be seen in Fig [7]. It is interesting to note that if we perturb a constant state on the transition curve then the perturbations will move along the C -characteristics only. This fact is demonstrated in Fig [8]. We conclude that the viscosity matrix is a stabilizing factor in the solutions with respect to perturbations, and the absence of capillarity from the equations results in the instability of all elementary solutions. This fact is different from the stability results for the strictly hyperbolic system of equations.

THE STATE U At T=0,4,6

Figure 5

THE DEGENERATE STATE U At T=0

Figure 6

THE DEGENERATE STATE U At T=0,4

Figure 7

THE STATE U At T=0,4,6

Figure 8

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