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SELF-AERATED FLOW IN OPEN CHANNELS

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SYNOPSIS

Measurements of distribution of air concentration in self-aerated flows are presented. The experiments were made in a rough channel of sand-grain type surface at various slopes and discharges, and the data were used as a basis for study of the mechanism of entrainment of air and to relate the air content and distribution to the characteristics of the flow. The analysis of the data shows that the air distribution can be adequately described by relationships based on a simplified concept of turbulent transport and thus are functions of the flow characteristics. The maximum depth and the mean velocity are both shown to increase above those of a corresponding nonaerated flow.

INTRODUCTION

A characteristic of high-velocity, open-channel flow is the phenomenon of self-aeration in which atmospheric air is insufflated into and mixed with the flow to create the appearance of "white water" with its violently agitated and ill-defined free surface. This condition is frequently observed in flows down steep chutes and spillways.

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It has been reasonably well established by several observers ^{3,4,5,6,7}, that for flows over a spillway from a quiescent reservoir, incipient aeration does not occur on the slope until a point or region is reached at which the boundary layer thickness is equal to the depth. The characteristic roughening of the water surface immediately upstream of the appearance of white water can be readily observed in such occurrences. With increasing initial water depths, the points where the surface roughening and the white water occur move downstream along with the intersection of the boundary layer with the water surface. These considerations suggest that the aeration phenomena are related to the conditions of turbulence in the flow. Examination of such flows by means of high-speed photography indicates that the upper boundary of the flow is rather ill-defined. It consists of a zone that appears white because of entrained air, while above this zone a spray of water droplets occurs moving more or less parallel to the flow and below which is a region of discrete air bubbles suspended in the fluid. Other investigators have described air-entrained flow as being made up of air bubbles entrained in water in the lower levels and water particles in the air in the upper regions. This concept is further emphasized by measurements of the air concentration distribution ^{8,9,10}, which show that the air concentration increases continuously from the bed, exhibiting a smooth transition from a finite value near the bed to a maximum value of 100% at the free surface which value is approached asymptotically.

The results of systematic experiments on distribution of air in self-aerated flows in a channel set at various slopes and operated at various discharges are presented. A provisional analysis is made of the data with respect to air distribution in a vertical section transverse to the direction of flow with a view of arriving at empirical relationships between concentration parameters and the flow characteristics.

EXPERIMENTS AND RESULTS

Apparatus.—The experiments were undertaken in a channel 50 ft long, 1.5 ft wide, and 1 ft deep, and adjustable to any slope from horizontal to nearly vertical. The width was chosen so that for ordinary flows sidewall effects due to growth of the sidewall boundary layer and air entrainment along the sides could be avoided in the central part of the cross section. Earlier measurements of air concentration and velocity in a transverse section at the end of the flume, ⁸ demonstrated that the width was adequate to make the flow two-dimensional in the center part. The flow was obtained by gravity from the laboratory main

³ "Entrainment of Air in Swiftly Flowing Water," by E. W. Lane, Civil Engineering, Vol. 9, No. 2, 1939, pp. 88-91.

⁴ "Air Entrainment on Spillway Faces," by G. H. Hickox, Civil Engineering, Vol. 15, No. 12, 1945, pp. 562-563.

⁵ "Etude de la mise en regime des ecoulements sur les ouvrages a forte pente," by G. Halbronn, La Houille Blanche, No. 1, 1952, pp. 21-40.

⁶ *Ibid.*, No. 3, pp. 347-371.

⁷ *Ibid.*, No. 5, pp. 702-722.

⁸ "Studies of Air Entrainment in Open-Channel Flows," by L. G. Straub and O. P. Lamb, Transactions, ASCE, Vol. 121, 1956, pp. 30-44.

⁹ "Air Entrainment in Steeply Sloping Flumes," by G. Halbronn and G. Cohen de Lara, Proceedings, Minnesota International Hydr. Convention, Minneapolis, Minn., September, 1953, pp. 455-466.

¹⁰ "Flow in a Flume with 1:1 Slope," by M. Viparelli, Proceedings, Minnesota International Hydr. Convention, Minneapolis, Minn., September, 1953, pp. 415-423.

supply flume through two feed lines and controlled by hydraulically-operated valves. The water reached the inlet of the flume through hollow support members and through a rectangular conduit on the underside of the channel. Fig. 1 shows the channel and appurtenances. A control panel separate from the channel contained the actuating components for the hydraulic system and the indicators for the slope, gate opening, and discharge settings.

At the inlet the flow is guided through two 90° vaned bends and a contraction. The inlet gate itself is cantilevered from its mounting at the top of the flume so that there are no slots or guides to disturb the flow. Further guidance is provided by a lip attached to the bottom of the gate and extending inward to further contract the flow. Therefore, the gate opening corresponds to the depth of flow at the upstream end of the channel. The maximum gate opening is 0.5 ft. Pitot

FIG. 1.—VARIABLE SLOPE CHANNEL FOR AIR-ENTRAINMENT EXPERIMENTS

FIG. 2.—PROBES FOR AIR-CONCENTRATION METER

measurements of the flow just downstream of the inlet with a 0.25-ft gate opening showed no deviation greater than 2% from the mean velocity except in the bottom corners where slightly smaller velocities were encountered.

The air-entrainment process was intensified by the installation of artificial roughness on the channel bed that was considerably rougher than the painted steel sidewalls of the channel itself. The material used for the roughness was a commercial nonslip fabric coated with granular particles. The particles, whose mean size was 0.028 in., had a mean spacing of 0.039 in. and were embedded in a mastic.

The discharge rate was measured by two calibrated pitot cylinders mounted in the supply lines and leading to differential gages mounted on the control

panel. Each discharge meter was calibrated separately for flows from 1/2 to 10 cfs with an accuracy of the order of 1%.

The electrical air-concentration measuring instrument has been fully described in an earlier report.¹¹ The method consists basically of a measurement of the difference between the conductivity of a mixture of air and water and the conductivity of the ambient water alone. A strut supporting a pair of probes is combined with the electrical circuit so that the air-concentration measurements may be made in a small region of the flow. The probes were 1/4 in. wide and 1/4 in. apart. The instrument is direct reading, and the relation between conductivity and air concentration can be determined analytically. The strut holding the probes is arranged so that it is possible to traverse the flow cross section both vertically and laterally. The instrument is limited to measurements not closer than 0.02 ft from the walls because of current deflections by the walls at lesser distances. Because the measurements of the air concentration depend on the resistance or specific sensitivity of the suspending water, the meter was zeroed in water continually circulated from the test channel through a deaerating tank, thus providing water of the same character as that in the flume. Oscilloscope measurements show a rapidly fluctuating current between the probes. Because the meter could not follow these rapid fluctuations, the value read on the meter represented an average value. This average compared well with results obtained by means of a mechanical sampler previously developed and which was not subject to the same averaging problems. Fig. 2 shows the probes and strut supports for the air-concentration meter.

Some of the experimental runs also required taking velocity traverses in verticals through the aerated flows. The instrumentation for making these observations had been previously described¹² in some detail. It consists essentially of a device for timing the travel of minute salt-water cloudlets injected into the air-water mixture. The operating principle of the velocity meter consists basically of marking a small element of flowing mixture and then recording the time interval required for this marked element to traverse a fixed distance. The marking is accomplished by making a diminutive part of flowing water more highly conductive to electrical current by injecting a small amount of salt solution into the stream. The passage of this ionized cloudlet is then detected by electrodes at stations fixed 3 in. apart in the flow path. The injections were made at a speed of approximately 20 per sec with individual injections of approximately 0.03 cu cm of 6% saline solution injected in approximately 1/600 sec. The accuracy of the air-concentration meter and the velocity meter was checked by comparing the measured water rate entering the channel with the integrated flow determined by the velocity traverses and the air-concentration traverses over the channel cross section.

Experimental Data.—For the air-distribution measurements described in this report, the channel slope S and water discharge, Q , were varied independently. Experiments were made for slopes of 7.5, 15, 22.5, 30, 37.5, 45, 60, and 75 degrees. For each slope, measurements of air concentration were made for total-water discharges, Q , from 2.2 to 9.6 cfs and for some slopes up to 15.0 cfs.

The measurements were made at Station 45 (45 ft from the inlet) in a vertical centerline plane. Readings were taken at 0.01-ft intervals on a line normal

11 "An Electrical Method for Measuring Air Concentration in Flowing Air-Water Mixtures," by O. P. Lamb and J. M. Killen, Technical Paper No. 10, Series B, St. Anthony Falls Hydr. Lab., Univ. of Minnesota, March, 1952.

12 "Velocity Measurement of Air-Water Mixtures," by L. G. Straub, J. M. Killen, and O. P. Lamb, Transactions, ASCE, Vol. 119, 1954, pp. 207-220.

to the channel bottom with the lowest point 0.02 ft from the bed and the uppermost point where the probes were completely out of the flow and the meter registered zero water concentration. The concentration profile was obtained for each slope and each water discharge after the channel had been adjusted for equilibrium conditions. For this purpose equilibrium flow was considered to be that condition of the flow of the air-water mixture for which the air-concentration distribution was the same at two sections 10 ft apart along the channel. This condition could be obtained by the adjustment of the initial depth of flow through control of the inlet-gate opening. Repeated measurements of the profiles were made at the two sections for different gate openings until the two air-concentration profiles were similar. Comparisons were made of the concentration at corresponding distances normal to the bed. A typical comparison of the profiles at the two stations for equilibrium flow is shown in Fig. 3.

FIG. 3.—COMPARISON OF AIR-CONCENTRATION PROFILES

The results of the experiments in the form of measured concentrations for each slope and discharge have been tabulated elsewhere.¹³ Table 1 is a sample of these measured concentrations. A sample tabulation of computed parameters is given in Table 2. A more detailed tabulation has been presented elsewhere.¹⁴

The air concentration C is defined as the ratio of the volume of air per unit volume of air and water as measured by the concentration-meter probes. It is assumed that the concentration as measured represents the average value of the concentration in the region described by the probes of the concentration

¹³ "Experiments on Self-Aerated Flow in Open Channels," by L. G. Straub and A. G. Anderson, Proceedings Paper 1890, ASCE, 1958, pp. 28-31.

¹⁴ Ibid., pp. 32-35.

TABLE 1.—AIR CONCENTRATION DISTRIBUTION FOR VARIOUS DISCHARGES AND SLOPES

[illegible]

TABLE 2.—SUMMARY OF EXPERIMENTAL MEASUREMENTS OF ROUGH CHANNEL

Run	Temperature	Q	$\bar{d}$	d_u	d_T	$\bar{d}d_T^{1/2}$	$q/s^{1/2}$	$\bar{C}$	$\bar{C}_T$	C_T	C_1	$\bar{V}$	V_s	$V_s/d_T^{2/3}$	$R \times 10^{-5}$	z	$\bar{V}/V_m$	
(a) 17.5° Slope																		
200	22.3	2.2	0.1209	0.169	0.120	0.0418	4.06	0.285	0.054	0.45	0.008	12.13	0.710	2.92	5.65	1.920	0.022	1.059
202	0	4.2	0.1937	0.260	0.197	0.0860	7.75	0.255	0.070	0.50	0.005	14.45	0.909	2.68	5.91	2.150	0.024	1.017
204	0	6.4	0.2515	0.329	0.257	0.1274	11.80	0.236	0.065	0.50	0.005	16.95	1.039	2.56	9.05	1.900	0.031	1.034
206	0	8.2	0.2943	0.380	0.298	0.1605	15.10	0.225	0.059	0.48	0.004	18.55	1.119	2.51	11.46	1.750	0.035	1.036
(b) 15.0° Slope																		
210	21.0	2.2	0.1010	0.115	0.102	0.0322	2.88	0.348	0.090	0.50	0.031	14.50	0.923	4.41	5.58	1.255	0.023	1.018
212	0	4.2	0.1358	0.234	0.157	0.0600	5.50	0.353	0.114	0.50	0.060	18.50	1.144	3.92	6.03	0.785	0.033	1.036
214	0	6.4	0.1937	0.303	0.200	0.0866	8.37	0.361	0.125	0.46	0.083	22.0	1.292	3.77	9.14	0.527	0.045	1.073
216	0	8.2	0.2275	0.338	0.233	0.1097	10.73	0.327	0.113	0.48	0.067	24.0	1.396	3.63	11.59	0.630	0.049	1.074
(c) 22.5° Slope																		
220	19.6	2.2	0.0886	0.159	0.095	0.0273	2.37	0.443	0.167	0.55	0.107	16.55	1.081	5.19	5.71	0.693	0.028	1.015
222	0	4.2	0.1358	0.243	0.145	0.0517	4.53	0.441	0.172	0.52	0.123	20.6	1.338	4.84	6.21	0.507	0.041	1.015
224	0	6.4	0.1763	0.310	0.190	0.0768	6.90	0.431	0.175	0.50	0.140	24.1	1.530	4.62	9.54	0.388	0.052	1.028
226	0.1	8.2	0.2116	0.351	0.288	0.1009	8.85	0.398	0.157	0.53	0.107	25.8	1.676	4.49	12.20	0.547	0.058	1.014
(d) 30.0° Slope																		
230	21.0	2.2	0.0760	0.165	0.087	0.0224	2.07	0.540	0.261	0.55	0.222	19.30	1.182	6.00	6.33	0.326	0.035	1.078
232	0	4.2	0.1125	0.238	0.139	0.0419	3.96	0.527	0.283	0.61	0.245	24.9	1.494	5.55	7.18	0.345	0.041	1.029
234	1.2	6.4	0.1530	0.303	0.183	0.0655	6.03	0.495	0.255	0.58	0.213	27.9	1.715	5.31	11.04	0.366	0.052	1.094
236	0.2	8.2	0.1909	0.348	0.220	0.0848	7.72	0.480	0.254	0.61	0.200	30.2	1.880	5.16	13.83	0.420	0.058	1.086
(e) 37.5° Slope																		
240 $\frac{1}{2}$	19.6	2.2	0.0605	0.198	0.085	0.0176	1.88	0.695	0.454	0.63	0.431	24.2	1.292	6.66	7.49	0.164	0.048	1.270
242	16.8	4.2	0.0923	0.291	0.148	0.0355	3.59	0.683	0.480	0.68	0.462	30.3	1.706	6.09	15.23	0.142	0.061	1.287
244	5.0	6.4	0.1317	0.361	0.195	0.0582	5.46	0.635	0.417	0.68	0.402	32.3	1.955	5.82	15.40	0.177	0.073	1.188
246	9.3	8.2	0.1638	0.413	0.227	0.0780	7.01	0.603	0.381	0.63	0.363	33.4	2.11	5.67	21.0	0.192	0.081	1.125
(f) 45° Slope																		
250 $\frac{1}{2}$	18.6	2.2	0.0530	0.188	0.090	0.0159	1.74	0.718	0.524	0.71	0.510	27.7	1.430	7.10	8.80	0.140	0.046	1.378
252	12.8	4.2	0.0791	0.303	0.151	0.0307	3.33	0.739	0.560	0.75	0.550	35.4	1.852	6.52	16.31	0.113	0.068	1.427
254	12.9	6.4	0.1065	0.378	0.205	0.0482	5.07	0.718	0.560	0.72	0.555	40.0	2.16	6.21	25.3	0.092	0.079	1.389
256	12.1	8.2	0.1361	0.422	0.240	0.0667	6.50	0.677	0.505	0.72	0.490	40.1	2.34	6.06	29.0	0.122	0.080	1.289
(g) 60° Slope																		
261	19.8	2.2	0.0397	0.190	0.085	0.0116	1.58	0.791	0.628	0.78	0.613	37.0	1.539	7.93	11.50	0.107	0.053	1.737
263	18.0	4.2	0.0648	0.290	0.160	0.0259	3.01	0.777	0.654	0.80	0.645	43.2	2.11	7.15	24.1	0.091	0.061	1.636
265	18.6	6.4	0.0887	0.389	0.220	0.0416	4.59	0.772	0.659	0.81	0.657	48.2	2.49	6.84	37.6	0.0806	0.090	1.570
267	19.2	8.2	0.1163	0.476	0.260	0.0593	5.88	0.756	0.628	0.78	0.610	47.0	2.69	6.59	44.0	0.087	0.098	1.424
(h) 75° Slope																		
270	19.8	2.2	0.0382	0.212	0.100	0.0121	1.49	0.820	0.699	0.83	0.690	38.4	1.767	8.18	14.32	0.093	0.060	1.730
272	22.0	4.2	0.0618	0.333	0.200	0.0275	2.85	0.814	0.733	0.85	0.730	45.4	2.50	7.30	34.6	0.063	0.084	1.654
274	21.3	6.4	0.0874	0.501	0.250	0.0457	4.35	0.823	0.711	0.82	0.715	48.9	2.79	7.03	46.1	0.050	0.125	1.547
276	20.9	8.2	0.1016	0.500	0.270	0.0328	5.56	0.797	0.691	0.81	0.687	53.8	2.90	6.93	54.4	0.0506	0.118	1.555

meter and is applicable to the midpoint of the probes. Essentially continuous curves of air concentration with respect to distance normal to the bed were obtained for each experiment. The concentration increases gradually from the bed, more rapidly in the central region and then more slowly in the upper region, apparently asymptotically approaching the limit of 100% air concentration, as is shown in Fig. 3, and subsequently in Fig. 7. It appears that the complete curve is comprised of two parts that have basically different characteristics. Although there is no sharp demarcation between the two parts, the curves tend to support the description that in the lower regions of the stream air bubbles are suspended in water, while the upper regions consist of water droplets in air. The existence of these water droplets can readily be detected by holding one's hand just above the main flow.

Because there is no definite upper boundary to the air-entrained flow in an open channel, questions of the definition of the linear dimensions of the flow arise. A number of different depth parameters related to the air concentration and distribution may be defined, each having particular applications in describing the bulk-flow conditions. These depth parameters may be described as follows:

1. A depth $\bar{d}$ is used to represent a mean depth of flow that would exist if all of the entrained air were removed up to the highest point where water is found. It therefore corresponds to the depth of a nonaerated flow of a given discharge with a velocity equal to that of the entrained flow and is defined as

$$\bar{d} = \int_0^{\infty} (1-C) dy \quad \dots \dots \dots (1)$$

in which y is the normal distance from the bed and C is the concentration of air as a function of y . The value of $\bar{d}$ can be approximated with sufficient accuracy by a summation of the products of the water concentrations and their corresponding incremental depths.

2. An upper boundary of the air-entrained flow or an upper depth d_u may be defined as the value of y where the air concentration has some arbitrarily prescribed value. For some purposes, such as determining the mean depth, and because of the asymptotic nature of the curve, it is convenient to define d_u as that value of y where the air concentration C is 0.99. Other depths may be similarly defined. One such depth has previously been defined⁶ as that corresponding to the value of y where the air concentration is $C = 0.95$. This arbitrarily was chosen as a depth where the concentration of air could be reliably measured and included from 98% to 99% of all the water and consequently represented an upper limit for the water.

3. A third depth parameter may be defined on the basis of the air-concentration distribution as that depth d_T representing the value of y where the transition from the distribution in the lower regions to that in the upper regions occurs. Its value would then depend on the characteristics of the channel as well as the air distribution. This depth parameter will be discussed in detail subsequently.

The mean air concentration in the vertical can also be defined in different ways depending upon the particular application. The mean concentration over the whole range of concentrations measured is defined as

$$\bar{C} = \frac{1}{d_u} \int_0^{d_u} C dy \quad \dots \dots \dots (2)$$

in which $\bar{C}$ is the mean concentration of the entire flow and d_u is the so-called upper limit where the concentration is 0.99. Another mean concentration, $\bar{C}_T$, is defined as the mean concentration in the region below the transition depth $y = d_T$ and applies to that air being transported by the flow.

$$\bar{C}_T = \frac{1}{d_T} \int_0^{d_T} C dy \dots \dots \dots (3)$$

Dimensional plots of $\bar{d}$, d_u , and d_T for the various discharges and for each slope are presented in Fig. 4 to indicate the range of values as determined from the concentration data. In Fig. 4 (c) the mean depth $\bar{d}$ varies with discharge and slope in much the same manner as nonaerated flow. The mean depth increases regularly with the discharge and decreases with the slope. The data for the upper limit d_u (Fig. 4 (a)) indicate that for a given discharge, d_u tends to decrease at first and then to increase as the slope increases. Consequently, the maximum depth tends to be nearly the same for corresponding discharge rates on the various lower slopes. It appears that the increased depth due to aeration increases with both slope and discharge. However, in combination with the mean depth, which decreases with slope and increases with discharge, the upper limit tends to decrease and then increase as the slope increases. As a result, over a considerable range of slopes (15° to 30°), the upper limit is essentially the same and depends only on discharge. A similar situation exists in regard to d_T (Fig. 4 (b)) in that it also shows a tendency to decrease gradually and then to increase gradually as the slope increases. Consequently, the range of variation of d_T for any given discharge is relatively little over a wide range in slopes.

The mean concentration $\bar{C}$ for the entire flow and $\bar{C}_T$, the mean concentration in the region below d_T , have been plotted in Figs. 5 (c) and 5 (b) in terms of the discharge and for each slope. It is interesting to note that the mean concentration for the entire flow is nearly constant for a quite wide range of discharges but increases considerably with the slope. The mean concentration $\bar{C}_T$ for the region below d_T has similar characteristics. In Fig. 5 (a) the concentration C_T at the transition depth is plotted. This concentration varies but little with the depth but increases with the channel slope.

ANALYSIS OF THE DATA

Distribution of Air in Aerated Flow.—A qualitative concept of air-entrainment phenomena has been established with the aid of the air-concentration determinations made transverse to the direction of self-aerated flows, high-speed photographs and motion pictures of the flow, and other similar information. Since it has been shown that aeration of flows on a steep slope begins in a region where the boundary layer generated by the channel bed has reached the water surface, it is to be presumed that aeration depends primarily on the turbulence intensity of the flow. Aeration appears to occur when the transverse velocities of turbulence are sufficiently strong near the air-water interface to cause clumps of water to break through the surface into the air and then fall back by gravity into the flowing stream. This breaking away of clumps of water from the main stream and the falling back into the stream with attendant splashing and breaking into a heterogeneous spray of globules and droplets is associated with the insufflation of air into the stream. It occurs particularly with water flow at high

FIG. 4.—DEPTHS OF AERATED FLOW AS A FUNCTION OF DISCHARGE

FIG. 5.—AIR CONCENTRATIONS AS FUNCTIONS OF SLOPE AND DISCHARGE

velocities on steep slopes. The air carried back is then distributed throughout the flow by turbulent transfer.

Observations by means of instruments indicate that two regions on self-aerated flow develop: (1) an upper region of heterogeneous clumps, globules, and droplets of water ejected from the flowing liquid stream into the atmosphere at more or less arbitrary velocities, and (2) a lower region consisting of air bubbles distributed through the flow by turbulent transport fluctuations, that can be described by some boundary-layer equation. Between the two regions is a transition zone defined by a transition depth that is a fluctuating surface necessarily at a statistical mean elevation above the channel bottom.

Distribution of Air in Upper Region.—If it is assumed that the components normal to the bed of the turbulent velocity fluctuations are randomly distributed so that the mean value is zero, those in the outward direction will constitute one-half of a Gaussian distribution. Those in or near the surface that penetrate the surface will carry large clumps and smaller globules of water into the atmosphere against the force of gravity a distance proportional to the square of the individual transverse velocities. The maximum height of the trajectories that they follow will also be randomly distributed about zero. The path will be a long, flat trajectory because of the large, longitudinal component of the local velocity. If the distances from the surface from which the particles are projected are represented by one-half of a Gaussian distribution, the frequency $f(y')$ of particles projected a distance y' above the transition depth may be expressed as

$$f(y') = \frac{2}{h\sqrt{\pi}} e^{-\left(\frac{y'}{h}\right)^2} \dots\dots\dots (4)$$

in which h is a measure of the mean distance the particles are projected above d_T and e is the base of the natural logarithms. The factor 2 is used to indicate that only particles being projected in the outward direction are being considered. That is, half of the complete Gaussian distribution is being taken so that $\int_0^{\infty} f(y') dy' = 1$. Furthermore, since it is presumed that the flow is uniform on the average, particles are equally likely to be projected at all points of the transitional surface. Then the proportion of all particles leaving a unit area of the transitional surface that reach or pass through a corresponding area at elevation y' will be

$$P_{y'} = \frac{2}{h\sqrt{\pi}} \int_0^{\infty} e^{-\left(\frac{y'}{h}\right)^2} dy' \dots\dots\dots (5)$$

and the number of particles reaching or passing through this area in both directions during the sampling period will be

$$N_{y'} = N_T P_{y'} \dots\dots\dots (6)$$

in which N_T is the total number of particles that leave or return to the transition surface per unit area during the sampling period. The average number of particles of water that reach or pass through any horizontal area per unit time may be assumed proportional to the average concentration of water particles. Therefore, using Eqs. 5 and 6, the air concentration at any distance y'

above the transition level in terms of the concentration at the transition level, C_T , and the representative height, h , is

$$\frac{1-C}{1-C_T} = \frac{2}{h\sqrt{\pi}} \int_{y'}^{\infty} e^{-\left(\frac{y'}{h}\right)^2} dy' \dots\dots\dots (7)$$

Eq. 7 is, of course, the cumulative Gaussian probability curve and is applicable to the aeration phenomenon only for $y' > 0$. The gradient of the concentration from Eq. 7 is

$$\frac{dC}{dy} = \frac{2(1-C_T)}{h\sqrt{\pi}} e^{-\left(\frac{y'}{h}\right)^2} \dots\dots\dots (8)$$

and when $y' = 0$, $\frac{dC}{dy}$ is a maximum whose value is

$$\left(\frac{dC}{dy}\right)_{\max} = \frac{2(1-C_T)}{h\sqrt{\pi}} \dots\dots\dots (9)$$

and occurs where the concentration is C_T and the depth is d_T .

The constants C_T and h are properties of the flow related to the intensity of the turbulence in the neighborhood of d_T . This, in turn, is probably dependent on the velocity and roughness of the channel.

Distribution of Air in Lower Region.—The air entrained at the transition surface by the turbulent fluctuations is then distributed throughout the flow in the region below d_T by the turbulence in the stream resulting in a statistical equilibrium between the bouyancy of the air and the concentration gradient. Such equilibrium is described by

$$-C V_b + \epsilon_b \frac{dC}{dy} = 0 \dots\dots\dots (10)$$

in which C is the air concentration at a normal distance y from the bed. The rising velocity of the air bubbles, V_b , is considered as being negative, and ϵ_b is a mixing parameter for the air-bubble transfer. In aerated flows the value of ϵ_b as a function of y unknown, and a number of assumptions as to the form of the function can be made. However, it may be presumed that the air bubbles are transported in the same manner as momentum, and it appears reasonable as an approximation to assume ϵ_b to be proportional to ϵ_m , the momentum mixing parameter, as is done in the case of transport of other substances such as sediment. In open channels in the non-aerated condition where the shear is linearly distributed from zero at the surface to a maximum at the bed and the velocity profile is considered to be logarithmic throughout the depth of flow, it has been shown¹⁵ that ϵ_m has a parabolic form being zero at the surface and at the bed with a maximum at the centerline and can be expressed in the form

$$\epsilon_b = \beta k \sqrt{\tau_0/\rho} \left(\frac{dT-y}{dT} \right) y \dots\dots\dots (11)$$

¹⁵ "Transportation of Suspended Sediment by Water," by V. A. Vanoni, Transactions, ASCE, Vol. 111, 1946, pp. 67-113.

in which $\sqrt{\tau_0/\rho} = V_s$, the shear velocity, τ_0 is the boundary shear at the bed, ρ is the density of the fluid, β is a proportionality factor in $\epsilon_b = \beta \epsilon_m$, and k is the Karman universal constant. In the case of transport in a closed channel, however, the shear at the upper boundary is not zero and the maximum velocity occurs at a point between the upper and lower boundary, at which point the shear is also zero. This requires that ϵ_m also be zero at this point and that the distribution of ϵ_m consists of two parabolas having zero at the bed, the point where the maximum velocity occurs, and at the upper boundary, and maximum values at intermediate points above and below the point of maximum velocity. It has been shown,¹⁶ however, that suspended matter is actually transported transversely across the level of zero mixing coefficient, and it has been suggested that the mixing coefficient be considered constant in this region. Because it is likely that the vertical transport of suspended matter depends only on the transverse velocity fluctuations in the vertical direction, such transport may well occur even if the correlation between the vertical and longitudinal fluctuations approaches zero.

Measurements of velocity distribution in aerated flows show^{8,9}, that the velocity tends to decrease as the upper limit of the flow is approached and that the maximum occurs well down in the flow proper. This implies that a shear is developed in the transition zone. A plausible explanation for such a shear is the change in momentum engendered by the return of water clumps and drop-lets to the flow at the end of their trajectory through the atmosphere above d_T . In the course of that trajectory, they suffer a loss of velocity due to atmospheric drag on the mass. The shear developed at d_T is analogous to an upper boundary and a distribution of the type found for closed channels might be expected.

The complex distribution found for closed channels may be approximated by a parabolic distribution over the entire region below d_T , with the maximum value incorporated in the factor β . This approximation is equivalent to assuming a distribution for ϵ_b as given by Eq. 11. Substituting the value of ϵ_b , given in Eq. 11 into Eq. 10,

$$CV_b = \beta k \sqrt{\tau_0/\rho} \left(\frac{d_T - y}{d_T} \right) y \frac{dC}{dy} \dots \dots \dots (12)$$

which upon integration, assuming that V_b is independent of y , is

$$C = C_1 \left(\frac{y}{d_T - y} \right)^z \dots \dots \dots (13)$$

in which

$$z = \frac{V_b}{\beta k \sqrt{\tau_0/\rho}} = \frac{V_b}{\beta k V_s} \dots \dots \dots (14)$$

and C_1 is a constant whose value is the concentration at $y = \frac{d_T}{2}$.

Application to Experimental Data.—The use of Eqs. 7 and 13 as a description of the air-concentration distribution characteristics depends on the degree to

16 "Turbulent Transfer Mechanism and Suspended Sediment in Closed Channels," by H. M. Ismail, Transactions, ASCE, Vol. 117, 1952, pp. 409-446.

which they fit the experimental data. Two factors having a bearing on this are the form of the equation and the magnitude of the constants. Whether or not the form of the equations is reasonably correct depends on the extent that the assumptions made approximate the actual mechanism in their development. The constants involved depend on the intensity of the turbulence and at the present cannot be determined analytically, but must be evaluated empirically from systematic experimentation.

Eq. 9 permits the determination of the transition depth d_T and the concentration C_T at that depth. From a plot of the concentrations in terms of the normal distance from the bed, the point where the gradient dC/dy is a maximum was graphically located, as shown in Fig. 6 (a). The corresponding values of d_T and C_T are noted. Using this value of C_T , the ratio $\frac{(1-C)}{2(1-C_T)}$ was plotted as a function of y on probability paper, on which a cumulative Gaussian distribution plots as a straight line. The value of h , or rather the value of σ , the standard deviation of the distribution, wherein $h = \sigma\sqrt{2}$, is obtained from the slope of this straight line. Alternately, h can be determined directly from Eq. 9 by evaluating the gradient at the point where dC/dy is a maximum and inserting the value of C_T obtained for that point. Fig. 6 (b) shows a plot of the concentrations in the upper region of the flow plotted in this manner with the standard deviation indicated. It is apparent from this plot that the concentrations possess a cumulative Gaussian distribution.

In the region of flow below d_T , Eq. 13 indicated that the concentration C is a function of $\frac{y}{d_T - y}$ which, when plotted logarithmically, results in a straight line whose slope is equal to the exponent z , and C_1 is the value of C at $\frac{y}{d_T - y} = 1$. The concentration data for the experiment have been plotted in this manner in Fig. 6 (c) to show the degree of agreement with the form of Eq. 13 and to indicate the evaluation of C_1 and z . Inasmuch as C approaches infinity as y approaches d_T , the experimental data must depart from the curve for the larger values of $\frac{y}{d_T - y}$, but in this case the data agree with the curve up to $\frac{y}{d_T} \approx 0.9$.

The straight lines drawn through the data in Figs. 6 (b) and (c) have been transposed to the original plot in Fig. 6 (a) to show the characteristics of the curves on the arithmetic scales. Eqs. 7 and 13 have also been plotted with the data for a number of typical experiments using various discharges and slopes shown in Fig. 7 for purposes of comparison. Because of the character of Eq. 13 in the neighborhood of $y = d_T$, the two curves are connected by a transition to pass from one curve to the other.

Relation of Parameters of Flow Conditions.—Based on the foregoing analysis of the aeration phenomenon, a number of parameters were developed as being descriptive of both the magnitude and the distribution of the air. These parameters varied with the flow conditions such as the channel slope and discharge, which in turn, along with the roughness characteristics of the channel, govern intensity and scale of the generated turbulence. The flow turbulence is created initially at the bed by the wakes and eddies formed by the flow over the roughness elements and the turbulent eddies are then diffused upward into the flow stream. The intensity of turbulence at the transitional surface that appears to be paramount in the aeration process should then depend on both the initial generation and the depth of the mixture to the transitional surface. The intensity of the turbulence is related to the boundary shear τ_0 on the bed, which can

FIG. 6.—DETERMINATION OF AIR-CONCENTRATION DISTRIBUTION PARAMETERS

FIG. 7.—COMPARISON OF MEASURED CONCENTRATIONS WITH THEORETICAL EQUATIONS FOR VARIOUS SLOPES

be expressed by a so-called shear velocity $V_{sb} = \sqrt{\tau_0/\rho}$. Because of the shear that is presumed to exist at the transitional surface, it is not possible in the present experiments to differentiate the bed shear from the total shear. As a first approximation, the total shear or a shear velocity based on the total boundary shear may be used as a measure of turbulence intensity. To test this hypothesis, the mean concentration was plotted in Fig. 8 (a) in terms of $V_s/dT^{2/3}$ in which V_s is defined as

$$V_s = \sqrt{g d_T \sin \alpha} \dots \dots \dots (15)$$

and g is the acceleration due to gravity. The expression $V_s/dT^{2/3}$ is empirical and was chosen as that form which best correlated the data. The measured values of the mean concentration fall reasonably well on a single curve with a tendency for the points corresponding to the smaller discharges to depart from the mean curve. When CT , the mean concentration in the area below dT is considered, the correlation is even better, as shown in Fig. 8 (b). For the higher discharges for each slope the points all fall on a smooth curve, while there is a clear departure from the curve for the low discharges on the corresponding slopes. It appears that as the discharge decreases below a certain value for a given slope, an additional factor becomes important in reducing the concentration. The nature of this factor is not readily apparent, but it should be noted that a decrease in discharge corresponds to a decrease in dT , and it is quite possible that an instability in flow pattern arises when the depth becomes small. Considering the data as a whole, it is rather significant that when plotted in this manner, mean air concentrations ranging from approximately 0.05 to 0.75 corresponding to discharges ranging from 4 to 15 cfs on slopes with ranges from 7.5° to 75° are correlated along a single curve.

The parameter $V_s/dT^{2/3}$ includes the depth dT to the transitional surface. This depth is a function of the air-concentration distribution and is difficult to determine explicitly. It is, however, related to the discharge and slope and in general to the channel roughness. In these experiments the channel roughness is a constant so that for these data, dT depends only on the slope and discharge. In terms of these measured characteristics, $V_s/dT^{2/3}$ is related to $S/q^{1/5}$ where the slope $S = \sin \alpha$ and α is the angle of inclination of the bed and q is the unit discharge of water. Consequently, the air concentration that may be expected in a high-velocity open channel can be related to the flow characteristics. This relationship is shown in Fig. 9 (a), in which the mean concentration as measured is plotted as a function of $S/q^{1/5}$ for the particular channel in which the experiments were performed. For channels of different roughness characteristics, the curve will probably be similar but shifted an amount to correspond to the different roughness. A similar plot is given in Fig. 9 (b) showing the mean concentration between the bed and the transitional surface where the depth is dT . Here again, as in Fig. 8 (b), correlation between $\bar{C}_T$ and $S/q^{1/5}$ appears to be better than that between $\bar{C}$ and $S/q^{1/5}$.

The results shown in Fig. 9 (b) provide a means of estimating the mean air concentration that may be expected in an equally rough, wide open channel of known unit discharge and slope. The depth of flow or the magnitude of the various previously described depth parameters, which are important in design, is a function of the air concentration.

Effect of Entrained Air on the Flow.—It was suggested previously that the air insufflated into the flow is distributed throughout the flow by turbulent transport. The entrained air has the effect of changing the flow from one of water alone to

FIG. 8.—MEAN CONCENTRATION AS A FUNCTION OF $V_s/d_t^{2/3}$

the flow of a mixture of air and water. Several characteristic depths were defined, such as d_u to represent the uppermost extent of the water particles projected from the flow, $\bar{d}$ to represent the depth of flow of the water alone, and d_T to represent a new equilibrium depth brought about by the presence of air in the flow. For design purposes, it is of considerable importance to know how

FIG. 9.—MEAN CONCENTRATION AS A FUNCTION OF $S/q^{1/5}$

these parameters and the actual flow velocity are related to the bulk flow properties or what differences might be expected in these parameters in relation to a corresponding nonaerated flow for which the depth may be computed by means of well-established flow formulas such as the Manning or Chezy formula.

In order to investigate such a relationship, tests were performed in the experimental channel to establish the resistance of the channel to nonaerated flow,—that is, flow of water alone. The slope was decreased so that for comparable discharges the flow would be nonaerated. These slopes ranged from approximately 1° to a maximum of approximately 6.5°, above which aeration was observed to occur. The discharges were in the same range as those used in the aeration experiments. Normal flow was established for each combination of slope and discharge, and the mean depth was determined by averaging the depths measured at various points along the centerline.

The relationship between the depth of flow for the nonaerated condition and the discharge and slope is shown in Fig. 10, in which the measured depth d_m is

FIG. 10.—RELATIONSHIP OF DEPTH OF DISCHARGE AND SLOPE FOR NONAERATED FLOW

plotted as a function of $q/S^{1/2}$ on logarithmic scales. The equation of the line drawn through the points is

$$q = 90.5 d_m^{3/2} (\sin \alpha)^{1/2} \dots \dots \dots (16)$$

For the range of the variables encountered in the nonaerated flow, it appears that the Chezy formula best describes the data and the coefficient 90.5 characterizes this particular roughness. The mean velocity is then

$$V_m = 90.5 d_m^{1/2} (\sin \alpha)^{1/2} \dots \dots \dots (17)$$

If it were assumed that the same type of flow equation applied to aerated flow, the mean velocity of the air-water mixture flowing at a depth d_T is

$$\bar{V} = C d_T^{1/2} (\sin \alpha)^{1/2} \dots \dots \dots (18)$$

in which $\bar{V}$ is the mean velocity of the air-water mixture and C is the Chezy coefficient. Assuming that the velocity of the air and water components are equal, then Eq. 18 can be written in terms of the water discharge, as

$$q = C (\bar{d}) (d_T)^{1/2} (\sin \alpha)^{1/2} \dots \dots \dots (19)$$

in which $\bar{d}$ is the mean depth computed by Eq. 1 and represents the cross-sectional area of the water. Eq. 19 was tested by plotting the experimental data for aerated flow in the form $\bar{d} d_T^{1/2}$ as a function of $q/(\sin \alpha)^{1/2}$ in Fig. 11. Superimposed on the plot is the line for nonaerated flow in the experimental channel taken from Fig. 10. The data cluster about the straight line for nonaerated flow, indicating that an equation similar to Eq. 19 will correlate the data, and in addition, when the parameters $\bar{d}$ and d_T are used, the Chezy resistance coefficient is essentially the same as that for nonaerated flow. Fig. 11 shows that

FIG. 11.—RELATIONSHIP OF DEPTH TO DISCHARGE AND SLOPE FOR AERATED FLOW

at least for the conditions of these experiments, air-entrained flow, when considered to be a flow of a mixture, follows the same laws as for nonaerated flow when depth parameters related to the properties of the mixture are used to define the linear flow dimensions. The agreement of the data for aerated and nonaerated flow shown in Fig. 11 substantiate somewhat the deductions made in regard to the air-entrainment process.

An evaluation of $\bar{d}$, d_T , and d_u for specific flow conditions is not possible from Fig. 11. A clearer idea of the changes in these parameters will be obtained if they are compared with the depth, d_m , computed in the usual manner, using Eq. 16 which includes for this channel the Chezy coefficient of 90.5. It is the depth at which the water would flow in this channel at the given discharge and slope assuming no air entrainment. The ratios d_u/d_m , d_T/d_m , and $\bar{d}/d_m$ as functions of mean air concentration are plotted in Fig. 12. In all cases, of course, the ratios approach unity as the mean concentration approaches zero.

FIG. 12.—INFLUENCE OF AIR CONCENTRATION OF DEPTHS OF AERATED FLOW
RELATIVE TO CORRESPONDING NONAERATED FLOW

That is, for water flow only, $d_u = \bar{d} = d_T = d_m$. As the mean air concentration increases, the ratios depart from unity in such a way that as $\bar{C}$ approaches 1.0, $\bar{d}$ becomes small, while d_u and d_T necessarily increase.

The upper limit of the spray d_u increases considerably with increased air concentration. This in turn means an increase in the turbulence intensity to project water particles farther into the atmosphere. For the highest concentrations measured, the value of d_u is nearly four times as large as the non-aerated depth for a corresponding discharge and slope. The transition depth, as might be expected, also increases relative to the computed depth, because increasing concentration bulks the flow and makes the effective depth greater. On the other hand, the mean depth decreases with increasing concentration. It appears from Fig. 12 that concentration has relatively little effect on d_T and $\bar{d}$ until the air concentrations are greater than approximately 50%. For the

FIG. 13.—INFLUENCE OF AIR CONCENTRATION OF VELOCITY OF AERATED FLOW RELATIVE TO CORRESPONDING NONAERATED FLOW

slopes where the air concentrations were the greatest (being approximately 85%), $\bar{d}$ was reduced to one-half of d_m , while d_T was nearly twice as large as d_m .

The decrease in $\bar{d}$ with increase in mean air concentration implies a corresponding increase in the mean water velocity above that computed on the basis of d_m , because $\bar{d}$ is the cross-sectional area of the water flow. The trend of the mean water velocity in terms of the nonaerated velocity in this channel is shown in Fig. 13. Again, the velocity of aerated flow is about the same as that of a corresponding nonaerated flow until the mean concentration is in the neighborhood of 50%. It then increases rather rapidly.

CONCLUSIONS

1. Self-aerated, high-velocity, open-channel flow appears to consist of two distinct phases: an upper region consisting primarily of independent droplets and larger agglomerations of water that move independently of the stream proper, and a lower region in which discrete air bubbles are suspended in a turbulent stream and are distributed by the mechanism of turbulence.

2. In the upper region, the distribution of water-air agglomerate and droplets can be adequately described by the Gaussian cumulative probability equation, the constants of which are properties of the flow conditions. In the lower region, the distribution of the air agrees closely with an equation for turbulent mixing based on an approximation for the distribution of the mixing parameter.

3. The magnitude of the entrained air concentrations apparently depends on the intensity of turbulent fluctuations generated at the rigid bed and the depth of flow. The mean concentration could be correlated with the parameter $V_s/d_T^{2/3}$ or in terms of the flow characteristics. The mean concentration $\bar{C}$ is a function of $S/q^{1/5}$, in which S is the sine function of the slope angle, and q is the unit discharge.

4. The maximum height of water (or spray) above the rigid bed relative to a corresponding nonaerated flow increases rapidly with mean air concentration in the flow. The transition depth is greater than the depth of a corresponding nonaerated flow because of the bulking effect of the entrained air.

5. The mean velocity of an air-entrained flow is greater than that of a corresponding nonaerated flow by an amount that increases with the air concentration and corresponds to a decrease in the mean depth.

6. When the effective depth of aerated flow is taken to be the transition depth and the mean depth is the cross-sectional area per unit width of channel, the flow is described by a formula of the Chezy type involving approximately the same roughness coefficient as applies for nonaerated flow in the same channel.

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APPENDIX. NOTATION

- C = Concentration, ratio of volume of air to volume of air plus water;
 $\bar{C}$ = mean concentration in vertical section;
 C_1 = concentration at point $y = d_T/2$;
 $\bar{C}_T$ = mean air concentration in region between lower boundary and d_T ;
 C_T = concentration at transition depth $y = d_T$;
 $\bar{d}$ = mean depth of flow;
 d_u = upper limit of flow or value of y when $C = 0.99$;
 d_T = transition depth between upper and lower regions of flow;

- d_m = mean measured depth in nonaerated flow;
 e = base of the natural logarithms;
 g = acceleration due to gravity;
 h = mean height to which water particles are projected above d_T ;
 k = Von Karman universal constant for velocity distribution;
 $N_{y'}$ = number of particles reaching or passing through a horizontal area a distance y' above transition depth d_T ;
 N_T = number of particles that leave or return to an area at the transition depth;
 $P_{y'}$ = proportion of all particles leaving area at transition depth that reach or pass through horizontal area at y' above transition depth;
 Q = total-water discharge;
 q = water discharge per unit width of channel;
 R = Reynolds number = $\frac{4\bar{v}d_T}{\nu}$;
 S = channel slope;
 V = local velocity;
 V_m = mean velocity for nonaerated flow;
 $\bar{V}$ = mean velocity in vertical section;
 V_s = shear velocity = $\sqrt{g(d_T) \sin \alpha}$;
 V_b = rising velocity of air bubbles;
 y = normal distance from channel bottom;
 y' = outward normal distance above transition depth;
 z = exponent in air-concentration equation;
 α = angle of inclination of channel;
 β = constant;
 ϵ_b = mixing coefficient for air bubbles in turbulent flow;
 ϵ_m = mixing coefficient for momentum transfer;
 ν = kinematic viscosity of water;
 ρ = density of water;
 σ = standard deviation of air distribution above d_T ;
 τ = shear force; and
 τ_0 = boundary shear force.

DISCUSSION

MICHELE VIPARELLI.¹⁷—In 1943, many data collected on big hydraulic structures were published in an article on steep channels, and it was stated¹⁸ that laboratory research on models would have been of small interest only because the laws of similitude are not valid for air entrainment. Now it can be seen that the laboratory research in Minnesota, the results of which are presented by the authors, and research in Naples, the results of which were given by the writer,^{19,20} not only succeeded in realizing new methods of measurements, but made clear the nature of the phenomenon. General conclusions for prototype design were also deduced.

In Minnesota, recourse has been made to an electric apparatus for measuring air concentration, C , and to an electric and chemical apparatus for recording instantaneous velocities in a given point of the air and water flow. The local average velocity, V , was obtained through a statistical analysis of recorded data.

In Naples the quantities measured were the dynamic pressure δ on the tip of a pitot tube, the discharge, Q_s , of the water only, through a vertical strip of unit width, and the discharges of air and water siphoned from a point of the current. For this aim, three simple hydraulic instruments were used: the common pitot tube, the open sampler, and the closed sampler.

The open sampler, disposed at distance y from the bottom, collects the water discharge $Q_s(y)$ of the current above y . The ratio $q = \frac{\Delta Q_s}{\Delta y}$ gives the water discharge at elevation y , through an elementary unit area, whose normal is in the direction of the flow. The unit water discharge, q , has the dimension of a velocity and agrees with the average local velocity, V , in a water flow.

The quantity $\sqrt{2g\delta}$ was found to depend on the values of $\sqrt{Vq}$ according to a linear law. In a current of water only, $v = q = \sqrt{2g\delta}$.

In the closed sampler, air and water siphoned from the current became separated and their volumes were measured. The ratio of air volume to the volume of air plus water clearly depends on the air concentration in the point of measurement. However, that ratio depends also on the shape of the tip of pipe through which air and water are sucked from the current, and on the depression in the closed sampler.

After many trials using the closed sampler, a tip was adopted with a hole opened toward the direction of the movement, similar to the tip of a simple pitot tube. The depression in the sampler was made to assume different values, in order to obtain that value for which the water discharge through the hole of the tip was equal to qw , in which w is the area of the hole and q the value of the corresponding unit discharge. The ratio of air volume to the volume of air plus water so collected in the closed sampler was interpreted as

¹⁷ Prof. Ing., Inst. of Hydr. and Hydr. Constr., Naples Univ., Naples, Italy.

¹⁸ "Open Channel Flow at High Velocity," by L. Standish Hall, Transactions, ASCE, Vol. 108, 1943, pp. 1394-1434.

¹⁹ "Fast Water Flow in Steep Channel," by M. Viparelli, I.A.H.R. Convention, Lisbon, 1957.

²⁰ "Correnti Rapide," by M. Viparelli, L'Energia Elettrica, Milan, July, 1958.

the ratio C_t of air discharge to that of air plus water through the unit area. In effect, when the water discharge through the hole is equal to $q w$, the flow in the point of measurement is not, or almost not, disturbed.

Hence for a flow of air and water it can be written as

$$q = \frac{1}{1 + C_t} V \dots\dots\dots (20)$$

When C_t is not large, air concentrations, C , of the authors, and C_t are absolutely different. Values of C , obtained by means of an electrical apparatus, depend on air concentration in a plain parallel to the flow, while those of C_t depend on the air discharge through unit area of cross section. In effect, the writer tried to measure air concentrations by means of an electric apparatus, and obtained much larger values than those measured by means of the closed sampler, according to the method explained above.

The authors and the writer have both admitted that particles of water are projected randomly from the transition surface in the overlying region. The measured air concentration, C , depends on the average number of particles of water that reach or pass through any area parallel to the bottom per unit time at elevation $y' = y - d_T$ (the authors define d_T as the transition depth). The parallel water concentration is measured by $1 - C$, at least for values of C that are not too large. The authors have found that $1 - C$ depends on y' , according to the cumulative Gaussian probability curve.

The writer admitted that the water particles above the transition preserve a constant average velocity. In this case, $Q_s(y')$ resulting by the product of average velocity for the number of water particles that pass through a unit normal strip above y' must be distributed according to the cumulative Gaussian probability curve. Experimental data confirm that $Q_s(y')$ is distributed according to the cumulative Gaussian curve.

As Q_s and C depend on y' according to the same rule, the conclusions of the authors on water distribution give new proof of the hypothesis made by the writer that the average velocity of water particles above the transition surface assumes a constant value along a normal to the bottom. The other proof was deduced by the writer from the law by which Pitot's data depend on the unit discharge q .

The authors indicate that the velocity of flow, measured by means of the chemical-electrical apparatus already mentioned, decreases above the transition depth. As the values of measured velocities are not given, the writer thinks that the observed decrease of velocity should be very small. In this case, data referring to upper layers collected in both researches may be considered thoroughly compatible among them.

The different meanings of C and C_t should merit an investigation. The dependence of the one on the other may illuminate the relation between the average local velocities and the random velocity components of turbulence.

In the present case it can be stated that values of parallel concentration C are scarcely useful in computations referring to water discharge and mean velocity. They can be misleading, because in lower layers the transverse concentration C_t is always much smaller than C . In the Naples research, C_t was found to vary below the transition depth from less than 0.001 for a 11° slope to $0.2\% + 0.3\%$ for a 23° slope and $2\% + 3\%$ for a 45° slope. According to these values of C_t , the measured values of $\sqrt{2 g \delta}$, q , and V are practically equal in lower layers, at least for a slope of less than 45° (in Naples the largest slope given to the experimental flume was 45°).

Other results of interest obtained in Naples are the following:

In the lower layers the velocity distribution follows the logarithmic law valid for smooth walls if the channel walls are smooth, and the law valid for rough walls in other cases.

The depth of transition d_T can be assumed as the height of the part of current that contributes to overcome the resistances.

The unit water discharge, q , at elevation y' above the transition surface is given by the product: number by velocity of water particles, reaching and not surpassing that elevation. The velocity which does not vary with y' is equal to the maximum velocity U of the flow along y' . Consequently, q , like the number of water particles, is distributed according to a Gaussian law. Precisely, it is

$$q = U e^{-1/2 \left(\frac{2g y'}{\delta U^2} \right)^2} \dots \dots \dots (21)$$

in which the standard deviation δ was found to depend on the roughness of walls. In effect, δ varies from 0.0020 for smooth walls to 0.0040 for very rough walls.

For $y' = y - d_T = 0$, the total water discharge $Q_S(0)$ through a transverse strip in the upper layers is

$$Q_S(0) = 0.5 \sqrt{2\pi} \delta \frac{U^3}{2g} \dots \dots \dots (22)$$

From the foregoing results, the writer was encouraged to draw more general conclusions than the authors did, also because the examination of data gathered by Nitchiporovich²¹ on the Gizeldons chute presented evidence that results of laboratory research were confirmed by tests on a prototype.

In the lower layers, where water flows with air bubbles in suspension, the average velocity can be computed with the same equations used in computations referring to currents of pure water, at least for a slope equal to or less than 45° . In these equations, cross section, wetted perimeter, and hydraulic radius are to be computed below the transition surface, because the weight of water particles dispersed in the air does not contribute to overcome the resistance at the walls.

To contain the flow above transition surface the wall of steep channels must be raised adding to d_T a height $\frac{c\delta U^2}{2g}$, in which U is the velocity of flow at the transition surface, δ represents the standard deviation, and c a constant of proportionality. It is convenient to give U the value of the maximum velocity in the section, computed according to the knowledge on uniform flow in open channels.

The constant c cannot be given once and for all, but must be assigned for each case. In effect, because water drops are distributed according to the Gaussian law, they can reach increasing heights but in decreasing numbers. Therefore, the opportunity of elevating less or more the wall depends on the inconveniences that water drops can cause.

The total water discharge is

$$Q = rv + b 0.5 \sqrt{2\pi} \delta \frac{U^3}{2g} \dots \dots \dots (23)$$

²¹ "Rezultati ispitivanyi bistrotoka Gizeldonskoy gidrostanciye," by A. A. Nitchipovich, Gidrotehnicheskoe Stroitel'stvo, No. 5, 1934.

in which r is the area of the hydraulic section below d_T , and b is the breadth of the channel at transition surface.

These conclusions are acceptable if the flow in the steep channel is regular. However, it is necessary to not forget that transversal waves and irregularities in the flow can easily appear and make not enough high the training walls.

LORENZ G. STRAUB,²⁴ F. ASCE and ALVIN G. ANDERSON,²⁵ M. ASCE.—All of the data and conclusions as to the mechanism of transport of entrained air were based on measurements of air concentration for various flows and slopes. The mean depth $\bar{d}$ and transition depth d_T were defined in terms of the air concentration distribution. Knowing the water discharge and the mean depth which implies the area per unit width of water flow, the mean velocity may be computed as was shown in Table 2.

The concept of the transition depth, d_T , defined from the shape of the air concentration distribution curve provided a means of establishing the depth at which the mixture of air and water flowed as an open channel flow subject to the usual laws of open channel flow. If the channel is hydraulically rough so that the friction coefficient is independent of the mixture Reynolds number, the resistance coefficient of the aerated flow should be the same as that for nonaerated flow. This assumption was tested by making experiments in the same channel under nonaerated conditions and comparing the results. The comparison showed that for rough channel used in the present experiments the resistance coefficient was essentially the same for both aerated and nonaerated flow when the depth of the aerated flow is defined as d_T . The graphical comparison is shown in Fig. 11.

With this agreement in resistance coefficients it was possible to compute the velocity and depth of nonaerated flow having the same water discharges and slopes as the aerated flows used in the experiments. Figs. 12 and 13 show the ratios of the aerated depths $\bar{d}$, d , and d_u to the computed depth for an equivalent nonaerated flow d_m , as well as the ratio of the mean velocity for aerated flow to the mean velocity of an equivalent nonaerated flow.

Because the mean velocity of the aerated flow was computed from the water discharge divided by the mean depth $\bar{d}$ as determined from the air concentration distribution curve, it appeared desirable to make an independent measurement of the velocity in the aerated flow to compare with the computed value. For this purpose velocity and air concentration measurements were made for several aerated flows, from which the mean velocity of the water could be determined. The mean water velocity was expressed as

$$\bar{V} = \frac{\sum_0^{d_u} (1 - C) V \Delta y}{\sum_0^{d_u} (1 - C) \Delta y} \dots \dots \dots (24)$$

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in which V is the local velocity and C is the air concentration at elevation y normal to the bed.

The mean velocity for an open channel flow for the same discharge and slope was computed in accordance with Eqs. 16 and 17 just as though no air were entrained. The ratio of this velocity V_m and the mean velocity for the equivalent air entrained flow was determined and plotted in Fig. 13, which is reproduced with the new points added to it as Fig. 14. The agreement of velocity ratio based on velocity measurements and that based on air concentration measurements is remarkably good.

The computation of the normal nonaerated elements of open channel flow in a particular application forms a base from which the elements pertaining to aerated flow depart as a consequence of the entrained air. The data and curves given in Figs. 9(a), 12 and 13 form the basis for estimating the depths and mean

FIG. 14.—COMPARISON OF MEAN VELOCITY OF AIR ENTRAINED FLOW FROM VELOCITY MEASUREMENTS AND FROM AIR CONCENTRATION DISTRIBUTION

velocity in a steep channel of equal roughness on which air entrainment will occur. The value of the Manning n for the experimental channel varies from approximately 0.012 to 0.016, depending on the depth of flow, so that the channel roughness corresponds approximately to a surface roughness such as may be found in practice. In a particular instance of flow on a steep channel, the water discharge and slope are known. Thus, $S/q^{1/5}$ can be computed, in which S is the sine of the slope angle. The application of this value to the curve in Fig. 9(2) will supply an estimate of the expected mean air concentration. Entering Figs. 12 and 13 with this mean air concentration will yield the various depths and the mean velocity of the stabilized air entrained flow in terms of the equivalent depth and mean velocity computed in the usual way as though the flow were not aerated.