VORTEX RINGS OF ONE FLUID IN ANOTHER IN FREE FALL

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Abstract

We present and interpret experiments in which vortex rings of one immiscible liquid are created in another from drops falling from rest under gravity. These rings are associated with circulations generated by viscosity and, unlike classical vortex rings which occur in miscible liquids at high Reynolds numbers, they can exist even at very low Reynolds numbers. Since the rings do not diffuse, they are well-defined. Nonetheless, there are many similarities in the dynamics of formation and flow of miscible and immiscible rings. We identify parameters which appear to correlate our observations and show photographs of some of the more interesting things we have seen.

I. CLASSICAL VORTEX RINGS

By way of comparison, it is instructive to recall that in classical hydrodynamics, it is usual to consider vortical regions embedded in an otherwise irrotational flow. In the case of the ring, a cross-section (Fig. 1(a)) is like the “potential vortex” (Fig. 1 (b)): the flow outside a cylinder which rotates rigidly. This is the Taylor problem (flow between two concentric cylinders with the inner one rotating and the outer one fixed) with the outer cylinder moved to infinity. The streamfunction ψ of the flow, with $(\psi_y, -\psi_x)$ for the velocity in the $x - y$ -plane, and $r = (x^2 + y^2)^{1/2}$, is then $\psi = c \ln r$. Thus, $\Delta\psi = 0$ and the flow is irrotational. The potential vortex satisfies the no slip boundary condition at the cylindrical boundary, and it is one of only a few potential flow solutions of the Navier-Stokes equations.

In the classical theory of vortex rings, viscosity is absent, diffusion is absent and surface tension is absent. For the classical case, we think of a smoke ring. Many beautiful photographs of vortex rings in miscible liquids can be found in the photograph album of Van Dyke¹.

Vortex rings can be generated in a number of ways. One way is to impulsively eject a puff of fluid from a circular opening into a bath of the same fluid, as in the smoke ring². Another is to let a drop of liquid fall into a pool of the same liquid³. A third method is to force a buoyant fluid into a tank of water (see section 6.3.2 of Ref. 4)⁵. These experiments do not involve immiscible liquids. Rings are more easily created in miscible rather than immiscible liquid. Thomson and Newall⁶ did an interesting study of ring formation and their stability in miscible and immiscible liquids. They say that

It is not every liquid, however, which, when dropped into water, gives rise to rings, for if we drop into water any liquid which does not mix with it, such as chloroform, the drop in consequence of the surface tension remains spherical as it descends. In fact, we may say that, with some few exceptions to be noticed later, rings are formed only when a liquid is dropped into one with which it can mix. This is important, because surface tension has been supposed to play an important part in the formation of these rings; it is difficult, however, to see how any appreciable surface tension can exist between liquids that can mix, and as far as our experiments go they tend to show that it is only the absence of surface tension which is necessary for their production. There are, as we shall show later, many cases where rings are formed under circumstances in which there is no possibility of capillary action, such as when the liquid into which the drop falls is the same as the drop itself.

Most of the experimental studies of vortex rings are for rings generated impulsively or otherwise in a single fluid or between miscible liquids. Most of the theoretical studies proceed from the inviscid equations of motion in which the vorticity is confined to certain regions, say rings, in an otherwise potential flow. These are models of flow at high Reynolds numbers. Low Reynolds number studies of the settling and break-up of miscible drops and the formation of rings have been given^{7,8}. To explain the discrepancies between their experimental observations and their Stokes flow calculations, Kojima et al⁷ introduced some notions of transient interfacial tension. This notion is considered in more detail in Ref. (9)

Most of the studies of vortex rings in immiscible liquids are for water and air at high

Reynolds numbers. One example is to let a bubble of air rise through water^{10,11}, and another is to let a drop of water fall through the air¹². The latter method leads to large deformations from the original drop, and the formation of bags which look very different from rings even though these are related. In the former study, it is necessary to inject the air impulsively to create a strong circulation in order to get a ring to form. A recent analysis of this problem has been given in Ref. 13.

Stuke¹⁴ did experiments like those reported here. He mentions that prior to his work, the formation and break-up of immiscible rings happened too fast to be photographed with the technology that was available at that time. Stuke cites the work of Northrup¹⁵ who used paraffin in a water bath, where interfacial tension is large. Northrup needed to inject the paraffin at high speed so that viscous forces would create a circulation of sufficient magnitude, as in the case of air injected into water. The larger the interfacial tension, the faster the intrusive speed necessary to create a ring. Stuke showed that rings would form at slower speeds when water was replaced with amyl alcohol, with a consequent lowering of interfacial tension. The slower speeds allowed these processes to be recorded in photographs which can be compared with the photographs of this paper. Initially, there is a membrane across the hole of the ring (cf. Figs. 9, 10). and then the membrane ruptures. Once formed, the ring starts to expand rapidly and the bulk of the paraffin flows into two or three bulges around the ring (cf. Figs. 11 (f), 12). These heavier bulges fall faster, so that the ring bends and breaks into two or three drops (see his Fig. 5, Fig. 2b of Ref. 12 and our figures.). If a drop were large enough, it would form another vortex ring and the sequence repeats itself. An analogous description of the ring instability for miscible liquids at slow speed is given in Fig. 4 of Ref. 7 and in Ref. 8.

On the whole, it is not surprising that a phenomenon which occurs at zero interfacial

tension also occurs at small values of interfacial tension. And *small* here means *with respect to viscous effects*, so that the actual numerical value of the coefficient of interfacial tension does not have to be small. The processes are similar whether miscible or immiscible liquids are involved, up to the ring stage. However, interfacial tension does affect the break-up pattern: for example¹², it can prevent the smaller-sized drops from repeating the sequence of ring formation and break-up: this limits the vortex-cascade (see Figs. 1 and 2 of Ref. 6 for a description of this cascade for the case of an ink drop in water) to one or two stages. Membrane *rupture* is another form of break-up. The rupture strength or breaking strength of membranes is not well-understood but it may be related to surface tension. We know that the rupture strength generally decreases with surface tension. When the surface tension parameter is small but the surface tension is large, a vortex ring spanned by a tough permanent membrane can form. In other cases in which interfacial tension has been reduced by surfactants, the membrane is blown out and an unstable vortex ring of the type shown in our photographs forms. The difference between strong and weak interfacial tension is illustrated in Figs. 7 and 11. In Fig. 7, a smaller drop of 1000 cS silicone oil is sucked into the wake of an oblate ring-like cap of the same silicone oil falling in a contaminated soy bean oil. The membrane in this system is too tough to break. On the other hand, if a surfactant (97 % dye, 3% Rhodamine B base powder, Aldrich Chem. Co., Milwaukee, WI) is added (as in Fig. 11) the membrane breaks readily. Fig. 1 (d) of Ref. 8 illustrates this well. The existence of a spanning membrane in miscible liquids is hard to understand without invoking the idea of transient interfacial tension induced by momentarily strong gradients of composition.

II. THE NORMAL STRESS BALANCE

It is probable that the parameters which control the deformations of drops to rings in free fall are associated with the stress balance at the interface:

$$-[[p]]\mathbf{n} + 2[[\mu\mathbf{D}[\mathbf{u}]]]\mathbf{n} + S^*\mathbf{n}2H = 0, \quad (2.1)$$

where H is the mean curvature at a point on the interface Σ and

$$2H = \frac{1}{R_1} + \frac{1}{R_2}, \quad \mathbf{D}[\mathbf{u}] = \frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^T),$$

where R_1 and R_2 are the local radii of curvature at a point of Σ ; $\mathbf{n}$ is the outward normal on the interface.

We denote

$$[[\cdot]] = (\cdot)_d - (\cdot)_o,$$

to be the jump in the quantity $\cdot$ across the interface, where the subscript d stands for drop and o stands for the outer fluid. In the equilibrium case (that is, no flow), the drop or bubble pulls into a sphere with radius $R_1 = R_2 = a$ and equilibrium pressures satisfying

$$[[p^e]] = p_d^e - p_o^e = \frac{2S^*}{a}.$$

Eq. (2.1) may be decomposed into normal and tangential parts. The tangential part says that shear stress is continuous. By using the condition that the velocity is continuous at the interface

$$[[\mathbf{u}]] = 0 \quad (2.2)$$

and the continuity equation, we will show that

$$[[D_{nn}]] = 0, \quad (2.3)$$

where we denote

$$D_{nn} = \mathbf{n} \cdot \mathbf{D}[\mathbf{u}] \cdot \mathbf{n}.$$

This result will be used in Eq. (2.8).

For the proof, we write the continuity equation

$$\nabla \cdot \mathbf{u} = (\mathbf{n} \frac{\partial}{\partial n} + \nabla_{\parallel}) \cdot \mathbf{u} = 0 \quad (2.4)$$

where $\partial/\partial n = \mathbf{n} \cdot \nabla$, $\nabla_{\parallel}$ is the surface gradient $\mathbf{t}_1(\mathbf{t}_1 \cdot \nabla) + \mathbf{t}_2(\mathbf{t}_2 \cdot \nabla)$, and $\mathbf{t}_1$ and $\mathbf{t}_2$ are the two tangents to the interface. Then on the interface,

$$\begin{aligned} 0 &= \llbracket \nabla \cdot \mathbf{u} \rrbracket = \llbracket \mathbf{n} \cdot \frac{\partial \mathbf{u}}{\partial n} + \nabla_{\parallel} \cdot \mathbf{u} \rrbracket \\ &= \llbracket \mathbf{n} \cdot \frac{\partial \mathbf{u}}{\partial n} \rrbracket + \nabla_{\parallel} \cdot \llbracket \mathbf{u} \rrbracket \\ &= \llbracket \mathbf{n} \cdot \frac{\partial \mathbf{u}}{\partial n} \rrbracket. \end{aligned} \quad (2.5)$$

We note next that

$$D_{nn} = n_i \frac{\partial u_i}{\partial x_j} n_j = \mathbf{n} \cdot \frac{\partial \mathbf{u}}{\partial n}. \quad (2.6)$$

We take the jump of (2.6) and use (2.5) to prove (2.3).

The next step in the reduction of the normal component of the stress balance on Σ is the decomposition of the pressure into an equilibrium part p^e , a hydrostatic part $p^s = \rho g z$, with $p^s = \rho_d g z$ in the drop, and a dynamic part Π due to the motion. We use a coordinate system where the origin is the center of the spherical drop, and we denote the parametrization for the surface of the drop by $z = z_{\Sigma}(x, y)$. Thus, $\llbracket p^s \rrbracket = \llbracket \rho \rrbracket g z_{\Sigma}$, and

$$\begin{aligned} \llbracket p \rrbracket &= \llbracket p^e \rrbracket + \llbracket p^s \rrbracket + \llbracket \Pi \rrbracket \\ &= \frac{2S^*}{a} + \llbracket \rho \rrbracket g z_{\Sigma} + \llbracket \Pi \rrbracket. \end{aligned} \quad (2.7)$$

Combining now (2.3) and (2.7) with (2.1), we find that

$$-[[\Pi]] + 2D_{nn}[[\mu]] + [[\rho]]gz_\Sigma + S^*\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{2}{a}\right) = 0. \quad (2.8)$$

The dynamic pressure is of course an unknown which must be determined from the solution.

III. DIMENSIONLESS PARAMETERS

To identify dimensionless parameters, we scale lengths with a , the radius of the equivalent spherical drop or bubble with the same volume, and velocity with U to be specified later (see next section). The normal stress balance at the interface (see previous section) shows that there are four forces at work: gravity, surface tension, inertia and viscosity.

The viscosity ratio

$$M = \frac{\mu_d}{\mu}, \quad (3.1)$$

where μ_d is the viscosity of the drop and μ is the viscosity of the ambient fluid, is very important.

The ratio of inertia to viscous forces is measured by the Reynolds numbers

$$R = \frac{Ua}{\nu}, \quad R_d = \frac{Ua}{\nu_d}, \quad (3.2)$$

where $\nu = \mu/\rho$, $\nu_d = \mu_d/\rho_d$. To form immiscible vortex rings, inertia is important because the drop will be close to a sphere if R and R_d are sufficiently small¹⁶.

The viscous part of the normal stress in the drop is scaled by $U\mu_d/a$ and in the exterior fluid by $U\mu/a$. The interfacial tension term in the stress balance is scaled by S^*/a . The ratio of the stress associated with interfacial tension to the viscous part of the normal stress in the outer fluid is

$$\frac{S^*}{\mu U} = \frac{J}{R}, \quad (3.3)$$

where

$$J = \frac{S^* a}{\nu^2 \rho}, \quad (3.4)$$

is Chandrasekhar's capillary number (used in his study of the capillary instability of a jet; see §111 of Ref. 17) for the outer fluid. Similarly, for the inner fluid we have

$$\frac{S^*}{\mu_d U} = \frac{J_d}{R_d}, \quad (3.5)$$

where

$$\frac{M J_d}{R_d} = \frac{J}{R}. \quad (3.6)$$

For a ring to form, the tendency for interfacial tension to keep the drop spherical should be overcome by the effect of viscosity to distort it. Thus, we expect to see rings when $M \gg 1$ and $J/R \ll 1$.

The inertial part of the dynamic pressure for the outer fluid is scaled with ρU^2 and the drop with $\rho_d U^2$. The ratio of interfacial tension to inertia in the outside fluid is

$$\frac{(S^*/a)}{\rho U^2} = \frac{J}{R^2}, \quad (3.7)$$

and in the drop is

$$\frac{(S^*/a)}{\rho_d U^2} = \frac{J_d}{R_d^2}. \quad (3.8)$$

Obviously,

$$\frac{\rho_d}{\rho} \frac{J_d}{R_d^2} = \frac{J}{R^2}. \quad (3.9)$$

Since ρ and ρ_d do not differ greatly in our experiments, the ratio of interfacial tension to inertia is nearly the same in the outside fluid and the drop.

IV. PHYSICAL AND OTHER PROPERTIES

The physical properties are density, viscosity and interfacial tension. Other properties used in our discussion are the velocity U and the drop size a . First, we discuss the fluid properties.

Table I lists the fluids used in the experiments. The densities were measured using a Curtin Scientific hydrometer at approximately 21 degrees Celsius. The viscosities were measured using standard Cannon Fenske tube viscometers at approximately 21 degrees Celsius. Canola oil is also termed rapeseed oil. The glycerin listed is .99 pure USP glycerin. The percentages listed for golden syrup and glycerin are dilutions with water. Alconox is an industrial glass cleaner and is used as a surfactant with water. The interfacial tension S^* was measured with the spinning rod tensiometer.

Baumann's¹⁸ experiments on vortex rings were carried out in a vortex ring box, a plexiglass tube three inches square and eight feet long. The top of Baumann's apparatus is open to allow introduction of the drop and the bottom is closed by a valve. The valve holds the host fluid in and allows the removal of the dropped fluids that collect at the bottom of the box. The plexiglass is clear to allow good visualization and photographic recording. The vortex ring box is back-lighted by two eight feet long fluorescent lights that are separated from the box by a translucent plexiglass sheet. The most recent experiments were carried out in a vortex ring tube, which differed from Baumann's apparatus in that it is a glass tube of circular cross-section with a three inch diameter. The tube is four feet long, and like Baumann's apparatus, it is open at the top, and closed at the bottom with a valve.

The method for introducing the drop into the vortex ring box is as follows. A 10 cc drop of the more dense fluid was carefully placed on top of the host fluid with a calibrated

beaker. This gives

$$\frac{4}{3}\pi a^3 = 10 \text{ cc} \quad \text{or} \quad a = (2.39)^{1/3} \text{ cm} = 1.34 \text{ cm}.$$

Care was taken to insure that the drop was not splashed or accelerated into the host fluid.

A parametric study of drop size was also carried out with volumes other than 10 cc. The results of these studies are summarized in Fig. 14.

The velocity scale we use to calculate the Reynolds numbers is U given by (see Sec. 4.9 of Ref. 19)

$$U = \frac{1}{3} \frac{a^2 g}{\mu} (\rho_d - \rho) \left(\frac{\mu + \mu_d}{\mu + \frac{3}{2}\mu_d} \right). \quad (4.1)$$

The assumptions leading to this formula are that the fluid is a sphere falling at constant speed in Stokes flow, and that if the shear stress and velocity are matched at the interface, then the normal stress is automatically matched. The analysis does not say anything about what the drop would do if it were not falling at the terminal speed; for example, in the experiments, the drop starts at rest, some drops do not attain any steady speed, and moreover appear not to reach the speed predicted by this formula. It is difficult to decide *a priori* on a velocity scale because we do not have a formula for predicting the velocity as the drop changes shape. For each experiment, one could measure the maximum speed attained by the drop and use that as U , and this type of data is available for some of the experiments.

The velocity of the drop as it falls in the vortex ring box has been measured for some situations and found to be much smaller than the value from (4.1). Measurements of the velocity of a falling drop were made by recording the time it took for the drop to cross a six inch region of the box. Five such regions were selected to best capture the rate of fall at critical sections. The records were taken ten times for each region and the average velocity was calculated. Fig. 2 shows the average velocity versus distance down the tube for three

types of glycerin (e.g. 90% glycerin means 90% glycerin in 10 % water) and silicone oil falling in soy bean oil. (Rings were observed in the 100% glycerin case, but not in the other two cases, which happen to have higher velocities in the figure.) Take, for example, the data in this figure. Compared with this, the value of U from (4.1) is approximately 110 inches per second, which is about twenty times the actual average velocity. This is consistent with the notion that a spherical drop would fall faster than a flattened spheroid or a ring. The swings in the measured speed reflect the changes in the shape of the drop as it evolves into a ring and decays. We should therefore keep in mind when looking at the tables that the true Reynolds numbers are probably an order of magnitude less than those tabulated.

There are also situations where U from (4.1) turns out to be large which is inconsistent with one of the assumptions in the derivation of (4.1); but since Stokes drag is less than the actual drag at higher Reynolds numbers, we expect that the U is an upper bound on the actual maximum velocity. Thus, our tabulated values of J_d/R_d and J/R in the sequel underestimate the importance of surface tension, but consistently, so that they should probably be an order of magnitude larger than they are. This would imply that the switch in the behavior from ring formation to no ring is actually occurring at a value of J/R of order 1. This, in fact, is what one would expect.

It is interesting that the condition for ring formation (on M and J/R) appears to hold for the entire wide range of Reynolds numbers encountered in the experiments. Why? In the normal stress condition at the interface, the only term we have not really commented on above is the pressure term, which is multiplied by the Reynolds number R . It appears that this term does not affect the ability to give birth to a ring: indeed, the factor R appearing there can be made to disappear just by changing the way the pressure is non-

dimensionalized.

The formation of vortex rings always involves the breaking of a membrane, by poke-through or blow-out, and the breaking strength (toughness) of a membrane is very difficult to control, especially in silicone-vegetable oil systems. Paul Mohr repeated the experiments of Nick Baumann with good success except for the breaking of silicone-vegetable oil membranes. Mohr did some experiments with a contaminated safflower oil with various additives that Baumann used to complete his studies. We could never break a membrane in a silicone-contaminated safflower oil system (Figs. 7 and 10). The breaking strength of a membrane may be related to interfacial tension since we could get tough membranes to break by adding certain types of surfactants to the silicone oil (trace amounts of 97% dye, 3% Rhodamine B base powder in Fig. 11; trace amounts of Igepal in Fig. 12). We also had difficulty breaking membranes in a silicone-soy bean oil system, even when uncontaminated fluids were used. However, the oils used in the most recent experiments were not exactly the same as those used by Baumann, and it is possible that the newer oils had an interfacial tension large enough to prohibit vortex ring formation. As was the case for the contaminated oil system, rings were formed when the above mentioned surfactants were added to the silicone oil.

V. DISTORTION OF THE SPHERICAL DROP

It is well known that a spherical drop or bubble, moving slowly, in Stokes flow can keep a spherical shape even when interfacial tension is suppressed^{19,20}. There is numerical and experimental evidence to suggest that these solutions are stable to small disturbances^{21,22} but not to large disturbances.

When viscous effects win over the effect of interfacial tension, a falling drop cannot maintain a spherical shape. Numerical solutions have been obtained²³ for steady streaming flow past an axisymmetric drop over a wide range of Reynolds numbers, interfacial tension, viscosity ratios and density ratios. Their results indicate that at lower Reynolds numbers, the shape of the drop tends toward an indented oblate shape with decreasing interfacial tension, and at higher Reynolds numbers the drop becomes more disk shaped with decreasing interfacial tension.

In our experiments, the drop is released at zero speed and undergoes accelerations and decelerations, so that the results mentioned above concerning steady motions cannot strictly be used to infer anything about what our drop is doing. Moreover, as mentioned in the previous section, measurements of the drop speed indicate that it often does not reach the steady speed predicted by the formula (4.1). However, there are similarities with these analyses and what we have seen.

Fig. 3 shows an indented oblate drop like those computed in Ref. 22 at low Reynolds numbers (see, for example, their Fig. 3). Experimental observations suggest that the streamline pattern on the concave side of the cap is probably like that of Fig. 4; there are no points of separation or vortices in this guess about the underlying fluid dynamics. The suction in the cap, call it a wake, is large and small drops and even large drops are easily captured by the indented drop, as shown in Figs. 5, 6, 7 (a) and 9 (a). If the conditions

are right, the drop in the wake will poke through the membrane spanning the indentation, as in Figs. 9 and 11, but if the membrane is tough, as in Figs. 7 and 10, the poke-through will fail.

VI. FORMATION OF RINGS

If the conditions are right, if the drop is much more viscous than the host fluid ($M \gg 1$) and the ratio J/R of interfacial to viscous forces is not too large, then the spherical drop will evolve toward a vortex ring. The entries in Tables II to IV for drops of silicone oils in soy bean oil exemplify these effects well. The viscosity of silicone oil can be varied through careful mixing without changing their density or surface tension appreciably. It was observed that when the viscosity of the drop was lower than a certain value (about 500 centistokes here), rings did not form. In particular, when the drop was less viscous than the bath, even with very low interfacial tension, rings did not form, as in Fig. 8.

Inertia alone will not cause a ring to form. Indented oblate drops like those shown in Fig. 3 are the most robust of the falling drops. If conditions are such that the viscous action of the host fluid can create a permanent circulation (like that sketched in Figs. 4 (b) and 6) of sufficient strength, the drop will begin to look like a ring, spanned by a membrane, as in Figs. 5, 6, 9 (a) and especially 10 (a). A free ring will form only if the membrane breaks. The membrane may or may not break. If it breaks, it does so either by poke-through of a smaller drop caught in the wake as in Figs. 9 and 10, or by blow-out. Blow-out can best be understood by the failure of blow-out shown in Fig. 10 (a). Blow-out can occur only if the membrane is very weak as in miscible liquids or in low interfacial tension systems like those shown in Figs. 12 and 13.

Vortex rings are unstable; whether or not the membrane has broken, the ring will expand rapidly. The rapid extension is a universal characteristic of the instability. If a

membrane remains and no drop rests in the wake to poke through, the membrane will stretch and either rupture or fold as in Fig. 10 (b).

Due to capillarity, draining, or other causes, bulges develop on the ring; these fall faster than the rest of the ring, and fluid drains rapidly to the heavy bulges, exacerbating the instability. This instability can be considered as a manifestation of the Rayleigh-Taylor instability of the heavy fluid into the light, when the heavy fluid has the shape of a vortex ring. In our experiments, the draining almost always occurs at just two points of the ring, more or less at opposite points on the ring as in Figs. 9 (d) and 11 (f), and in the figures of Refs. 7 and 8. This type of instability scenario can occur even for ring-like structures like the one shown in Fig. 10 (a) in which the membrane does not break and it leads to the folded ring shown in Fig. 10 (b). The heavier places fall faster and the ring bends and breaks into drops.

O'Brien²⁵, in reviewing her own work and that of Ref. 14, noted that the number of bulges which develop on the ring depends on the Reynolds number and is two for Reynolds numbers of order one or less. Basically, we observed only two bulges even at Reynolds numbers of order 100, with some very rare exceptions. Perhaps the number of bulges on the ring is related to capillary breakup and is strongly influenced by the value of interfacial tension. We saw many bulges when soap was added to water in soy bean oil (see the last entry of Table II). This interpretation is also suggested by the closely similar instability in miscible liquids¹², in which case the lack of surface tension promotes the formation of many more nodules around the ring, the ring breaks into many drops, and those drops subsequently repeat the cycle and there is a vortex cascade⁶. Surface tension can keep subsequent drops spherical if they are small enough, and thus inhibits the cascade.

The effect of the wall on the drop and ring needs further study²⁴. For example, when

a ring approaches a wall of the apparatus, it expands considerably before touching it. Also, experiments done with a vortex tube of smaller diameter show that the walls inhibit ring formation. Observations about the way a ring behaves (in the miscible case) at a variety of boundaries is reported in Ref. 15.

The dynamics leading to formation of vortex rings is not well understood. Data presented in the next section show that rings form from drops started from rest when the viscosity of the drop is relatively great and the interfacial forces do not dominate viscous forces.

A falling drop is relentlessly sheared by the host fluid, but only small portions of the host fluid come under the influence of the falling drop, and these only momentarily. If we move with the drop, we can think that we have a uniform flow around the drop as in Fig. 4 (b), and this picture is also suggestive of why circulations develop in the drop and not in the host fluid. The flow around the ring would, in the case where the ring fluid is very viscous, resemble that of Fig. 1 (a), where the flow is analogous to the rigid-body rotation of Fig. 1 (b) and would then be almost potential flow, with potential flow at infinity (uniform flow), and the vorticity is localized to the interface region between the fluids.

VII. TWO-FLUID SYSTEMS THAT DO AND DO NOT FORM

VORTEX RINGS

We used formula (4.1) to compute the velocity of a falling drop (with $a = (2.39)^{1/3}$) and rising bubble (with volume 5 cc, $a = (1.19)^{1/3}$) and evaluated many of the dimensionless parameters. The parameters are listed in Tables II, III and IV. Parameters which are not set down explicitly in these tables can be computed readily from the listed values. Table II tabulates the systems that were observed to form rings. The other two tables list the two-fluid pairs that were observed not to form rings.

We find that to form a ring, it is necessary that the drop fluid be much more viscous than the host fluid. Another criterion which appears to be necessary is that the effect of interfacial tension should be smaller than viscous effects, which may be expressed as

$$\frac{J}{R} < O(1), \quad (7.1)$$

where the notation is defined in (3.4) and (3.6). The last column of Table II is comfortably in agreement with (7.1). However, the velocity U used to compute R was computed from (4.1) and the true J/R may be an order of magnitude larger than in the table. A very dramatic illustration of the importance of J/R and M is exhibited by data for Palmolive soap dropped in soy bean oil (a ring forms; see last entry of table IX.2) and water dropped in soy bean oil (a ring does not form). Palmolive soap is essentially water modified with a surfactant which reduces the interfacial tension enough to move J/R down to a sufficiently small value. The viscosity of Palmolive soap is greater than that of water and this alters M such that the combination of J/R and M produces a vortex ring. The evolution of the ring in this soapy solution is exceptionally rapid and the torus breaks up rapidly into small bubbles, as in the case of miscible liquids.

Table III displays systems that do not form vortex rings. The data show that a modification of the fluids will switch a system that forms a vortex ring to one that does not. An example is the glycerin and soy bean oil system. When 9 percent water is added to glycerin, the resulting diluted solution will not form a vortex ring because the viscosity ratio M has decreased and J/R has increased to the borderline level. A similar adjustment was made for golden syrup by adding water.

Table IV lists systems where bubbles of the less dense liquid were released and left to rise through the more dense liquid. In each case, an oil was released into water. No vortex rings were observed. The related case of air bubbles released into water has been shown to yield rings (see Fig. 7 of Ref. 10). The case of 12500 centistoke silicone oil was inconclusive because the silicone oil showed an affinity for the plexiglass box.

In Fig. 14, we display the results of a study of drop size on the distance required for ring formation. This distance decreases monotonically with volume and seems to asymptote to some small value less than 15 inches for large volumes.

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TABLES

TABLE I

Fluid Properties

Fluid	Density (g/cm ³)	Kinematic Viscosity (cstokes)	Viscosity (g/cm sec)
Canola Oil	0.915	67	0.61
Glycerin	1.265	656	8.29
.95 Gly*	1.245	244	3.03
.91 Gly	1.240	113	1.40
.92 Golden Syrup	1.400	2606	36.49
Lyle's Golden Syrup	1.440	20804	299.58
Motor Oil 30W	0.886	316	2.80
Olive Oil	0.914	69	0.63
Palmolive Soap	1.05	238	2.50
Safflower Oil	0.920	51	0.47
Sesame Oil	0.920	64	0.59
Shell Research Oil	0.895	2037	18.23
Sil** 5cS	0.930	5	0.05
Sil 100cS	0.960	100	0.96
Sil 200cS	0.970	200	1.94
Sil 300cS	0.970	300	2.91
Sil 400cS	0.970	400	3.88
Sil 500cS	0.971	500	4.86
Sil 600cS	0.971	600	5.83

Sil 1000cS	0.971	1000	9.71
Sil 10000cS	0.975	10000	97.50
Sil 30000cS	0.975	30000	292.50
Soy Bean Oil	0.922	53	0.49
Water	1.000	1	0.01
Water+Alconox	1.080	33	0.36
Walnut Oil	0.925	51	0.47

* .95 Gly = 95 percent glycerin in 5 percent water

** Sil = Silicone oil with indicated viscosity

TABLE II

Systems that form vortex rings

System	$\Delta\rho$ (g/cm ³)	$\Delta\nu$ (cS)	S^* (dyn/cm)	U (cm/sec)	R_d	ρ_d/ρ	M	J_d/R_d	J/R
Gly/Soy	0.343	606	18.45	278.5	57	1.37	16.9	.008	.135
500cS Sil	0.049	447	1.68	40.3	11	1.05	9.9	.008	.079
/Soy									
Gly	0.350	589	35.78	227.5	46	1.38	13.6	.02	.272
/Canola									
500cS Sil	0.056	433	2.33	37.0	10	1.06	8.0	.01	.080
/Canola									
Gly	0.351	588	10.50	242.0	49	1.38	13.2	.005	.066
/Olive Oil									
500cS Sil	0.057	431	3.10	58.0	16	1.06	7.7	.01	.077
/Olive Oil									
Gly	0.345	606	15.65	291.5	59	1.38	17.6	.006	.106
/Safflower									
500cS Sil	0.051	450	7.44	43.6	12	1.06	10.3	.03	.309
/Safflower									
Gly	0.340	605	39.51	285.8	58	1.37	17.6	.02	.352
/Walnut Oil									
500cS Sil	0.046	449	4.15	39.1	10	1.05	10.3	.02	.206
/Walnut Oil									

Gly	0.345	592	15.20	216.3	44	1.38	14.1	.008	.113
/Sesame Oil									
500cS Sil	0.051	436	3.40	34.9	9	1.06	8.2	.02	.164
/Sesame Oil									
500cS Sil	0.049	447	1.68	40.3	11	1.05	9.9	.008	.079
/Soy									
600cS Sil	0.049	547	2.68	40.1	9	1.05	11.9	.01	.119
/Soy									
1000cS	0.049	947	2.41	39.7	5	1.06	19.8	.006	.119
Sil/Soy									
10000cS	0.053	9947	3.29	42.3	0.6	1.06	199.0	.0008	.159
Sil/Soy									
30000cS	0.053	29947	6.49	42.3	0.2	1.06	597.0	.0005	.298
Sil/Soy									
Golden	0.518	20751	42.20	413.3	2.7	1.56	611.4	.0003	.183
Syrup/Soy									
.92Gold.	0.478	2553	28.46	382.6	19.6	1.52	74.47	.002	.149
Syrup/Soy									
Palmolive	0.128	185	18.10	108.0	61.0	1.14	5.10	.0006	.003
/Soy									

Gly denotes 100 percent glycerin; .95Gly denotes 95 percent glycerin in 5 percent water; Shell denotes Shell Research Oil; Sil denotes silicone oil with the indicated viscosity; Soy denotes soy bean oil. The difference $\Delta\rho$ denotes (density of dropped fluid) — (density of host fluid); $\Delta\nu$ denotes (viscosity of dropped fluid) — (viscosity of host fluid); this difference is negative in some of the systems in the tables.

TABLE III

Systems that do not form vortex rings

System	$\Delta\rho$ (g/cm ³)	$\Delta\nu$ (cS)	S^* (dyn/cm)	U (cm/sec)	R_d	ρ_d/ρ	M	J_d/R_d	J/R
.95 Gly /Soy	0.323	194	13.43	269.8	148	1.35	6.2	.02	.124
.91 Gly /Soy	0.318	63	11.49	277.3	328	1.34	2.9	.03	.87
Water /30W Motor Oil	0.114	315	9.22	23.7	3172	1.13	0.004	38.85	.156
Water /Shell	0.105	2036	42.14	3.4	450	1.12	0.0005	1251	.625
Gly /Shell	0.370	1381	27.83	10.3	2	1.41	0.45	.34	.153
500cS Sil /Shell	0.076	1537	5.82	2.2	0.6	1.08	0.27	.53	.143
5cS Sil /Soy	0.008	48	1.14	9.2	245	1.01	0.10	2.7	.27
100cS Sil /Soy	0.038	47	2.75	34.1	46	1.04	2.0	.08	.16
200cS Sil /Soy	0.048	147	2.16	40.9	28	1.05	4.0	.03	.12
300cS Sil	0.048	247	2.71	40.2	18	1.05	5.9	.02	.118

/Soy

400cS Sil	0.048	347	2.67	39.7	13	1.05	7.9	.02	.158
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/Soy

Water	0.078	-52	3.39	92.3	12337	1.08	0.02	.37	.074
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/Soy

Water+	0.158	-20	4.64	156.0	632	1.17	0.73	.08	.056
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Alconox/Soy

.60Gold.	0.342	-27.5	7.42	341.0	1788	1.37	0.659	.07	.046
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Syrup/Soy

TABLE IV

Systems of injected bubbles

System	S^*	ρ_d/ρ	U	R_d	M	J_d/R_d	J/R
	(dyn/cm)	(μ_d/μ)		(cm/sec)			
100cS Sil	24.67	0.96	984	1043	96	0.03	2.88
/Water							
12500cS	27.11	0.975	621	5.3	12188	0.0004	4.8
Sil/Water							
Soy Oil	3.39	0.922	1917	3829	49	0.004	0.196
/Water							
30W Motor	9.22	0.886			280		
Oil/Water							
Olive Oil	16.42	0.914	2119	3257	63	0.012	0.72
/Water

Fig. 1 (a) The two circular cross-sections of a ring vortex are shown. The flow is as indicated. The dashed line denotes the axis at the center of the ring. The ring as a whole turns about this axial line. The continual turning of the ring is analogous to the rigid-body rotation of a straight cylinder shown in (b) if the ring were cut and straightened. (b) An infinitely long solid cylinder is rotating with azimuthal velocity Ωr , where Ω is the angular speed. The flow outside the cylinder is irrotational and given by the streamfunction $\psi = c \ln r$.

Fig. 2 The average velocity versus distance down the vortex ring tube for the designated liquids falling in soy bean oil.

Fig. 3 Indented oblate drops falling in safflower oil. (a) Water $M = 0.02$, $J = 4530$, $S^* = 3.39$ dyn/cm. Indentation never develops in water and oil systems without surfactants. (b) 500 cS silicone oil. These are the most common shapes when falling. The high viscosity drop develops a circulation which brings it closer to a vortex ring.

Fig. 4 Development of vorticity in a drop falling from rest. The streamlines are sketched in a frame moving with the drops: (a) from experimental observations at sufficiently small velocity; (b) larger velocity.

Fig. 5 Spheres nested in the wake of an indented oblate drop. (a) Glycerin falling in soy bean oil $M = 16.9$, $J = 0.45$, $S^* = 18.45$ dyn/cm. (b) 500 cs silicone oil in contaminated safflower oil.

Fig. 6 A streamline pattern for Fig. 7. The flow in the wake could pull out a tail from the nested sphere if the wake were strong as in Fig. 7.5 (b), or the surface tension weak as in the case of miscible liquids.

Fig. 7 Failure of poke-through of captured drop of 1000 cS silicone oil in an indented oblate drop of the same silicone oil falling through contaminated safflower oil. (a) The captured drop is sucked strongly into the wake behind the oblate drop. There is a tail drawn out of the captured drop by the motion of safflower oil in the wake which reminds one of the tail behind drops in miscible liquids (cf. Fig. 2 (c)). (b) The drops are sucked into strong contact. (c) The captured drop decelerates under the restraining action of the silicone oil membrane on the oblate drop which never breaks.

Fig. 8 Crisco rising in a column of water with surfactant (Alconox). The value of interfacial tension has been reduced from 3.39 dyn/cm to 0.158 dyn/cm by the surfactant. The membrane does not break, despite the low tension. (a) A torus is formed inside the water bag. (b) The water in the torus is dragged out in the wake.

Fig. 9 Poke-through of 1000 cS silicone oil in safflower oil $M = 19.8$, $J = 0.03$, $S^* = 2.41$ dyn/cm. (a) Silicone oil spheres nested in the wake of an indented oblate drop of the same oil. (b) Poke-through leads to a vortex ring. (c) Vortex ring (Rayleigh-Taylor) instability is the rapid expansion of the ring diameter and the draining of the oil into the falling bulges. (d) Two new indented oblate drops form from the falling bulges in a replication of the dynamic sequence.

Fig. 10 1000 cS silicone oil falling in contaminated safflower oil. (a) A vortex ring with circulation has developed but a tough membrane spans the ring. (b) The ring is unstable in the usual way (Rayleigh-Taylor instability) forming the characteristic drops (cf. Fig. 11 (d)), but the membrane breaks.

Fig. 11 Ring formation in 1000 cS silicone oil with a surfactant falling in soy bean oil. The surfactant is a trace amount of 97% dye with 2% Rhodamine B base powder. (a) One indented oblate sphere accelerates in the wake of another; (b) they come close. (c) Poke-through: the large ring loses its membrane. (d) The small ring never pokes through: it retains the oblate indented shape. (e) Beginning of the two-lobe instability of the Rayleigh-Taylor type. (f) The instability can be compared with Fig. 12 (b) where the membrane does not break and with the miscible ring in Fig. 2 (d).

Fig. 12 Vortex ring of 1000 cS silicone oil with trace amounts of surfactant (Igepal) falling in soy bean oil after blow-out.

Fig. 13 Vortex ring of dyed glycerine falling in soy bean oil after blow-out.

Fig. 14 Distance travelled by a falling drop before a vortex ring forms as a function of drop volume. The formation of the ring occurs when the membrane spanning the ring breaks.