**MULTIPLE INTEGRALS WITH RESPECT TO
L-MIXING PROCESSES**

By

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Abstract

We prove a moment inequality for multiple integrals of the Volterra-type with respect to zero-mean L -mixing processes. The result is the extension of an earlier result for single integrals. This investigation was motivated by a problem in the theory of recursive identification for linear systems.

1. Introduction

An important notion in system identification is the notion of L -mixing processes introduced in Gerencsér (1989). To define the class of L -mixing processes we first introduce the following definition, in which the k -dimensional Euclidean space will be denoted by $\mathbb{R}^k$.

Definition 1.1. Let (x_t) be an $\mathbb{R}^k$ -valued stochastic process. We say that (x_t) is M -bounded if for all $1 \leq q < \infty$

$$M_q(x) = \sup_{t \geq 0} \mathbb{E}^{1/q} |x_t|^q < \infty.$$

Let us consider a family of monotone increasing σ -algebras $(\mathcal{F}_t), t \geq 0$ and a family of monotone decreasing σ -algebras $(\mathcal{F}_t^+), t \geq 0$, such that $\mathcal{F}_t, \mathcal{F}_t^+$ are independent for all t and $\mathcal{F}_t = \sigma(\cup_{\epsilon > 0} \mathcal{F}_{t+\epsilon})$ for all t .

Definition 1.2. We say that a stochastic process (x_t) is L -mixing with respect to $(\mathcal{F}_t, \mathcal{F}_t^+)$ if it is $(\mathcal{F}_t)$ -progressively measurable, M -bounded and if we set for $q \geq 1, \tau > 0$

$$\gamma_q(\tau, x) = \gamma_q(\tau) = \sup_{t \geq \tau} \mathbb{E}^{1/q} |x_t - \mathbb{E}(x_t | \mathcal{F}_{t-\tau}^+)|^q$$

then we have

$$\Gamma_q = \Gamma_q(x) = \int_0^\infty \gamma_q(\tau) d\tau < \infty.$$

A fundamental inequality of the theory of L -mixing processes is the following:

Theorem A. (Theorem 1.1 in Gerencsér (1989)) Let $(u_t), t \geq 0$ be an L -mixing process with $\mathbb{E}u_t = 0$ for all t . Let (f_t) be a function in $L_2[0, T]$. Then we have for all $1 \leq m < \infty$

$$\mathbb{E}^{1/2m} |\int_0^T f_t u_t dt|^{2m} \leq C_m \left(\int_0^T f_t^2 dt \right)^{1/2} M_{2m}^{1/2}(u) \Gamma_{2m}^{1/2}(u)$$

where $C_m = (4m - 2)^{1/2}$.

The proof of this theorem is partly based on the following lemma:

Lemma B. (Lemma 2.3 in Gerencsér (1989)) Let (x_t) be a zero-mean L -mixing process with respect to $(\mathcal{F}_t, \mathcal{F}_t^+)$ and let for some $0 < s < t$ η be an $\mathcal{F}_s$ -measurable random variable. Then

$$|Ex_t\eta| \leq 2\gamma_p(t-s, x)E^{1/q}|\eta|^q$$

for every $1 < p, q < \infty$ such that $1/p + 1/q = 1$. The proposition remains true for $p = \infty$ and $q = 1$.

An important problem is whether the deterministic function f_t can be replaced by an adapted stochastic process. It is easy to see that this is not possible without any qualification. Set e.g. $u_t = f_t$, then the integral in Theorem A grows as T rather than $T^{1/2}$. However, we may expect that if the process (f_t) has a lot of “information-content” in the past then the theorem may remain true. A possible assumption is that f_t can be written as

$$f_t = \int_0^t g_s v_s ds$$

where g_s is a deterministic locally square integrable function and (v_s) is a zero-mean L -mixing process. Thus the integral in Theorem A can be represented as a double integral. Estimation of double integrals is important in the theory of recursive identification, E.g. double integrals of the form

$$I_{T_0} = \int_{T_0}^T \frac{1}{t} u_t \int_{T_0}^t \frac{1}{s} v_s ds dt$$

has to be estimated, where (u_t) , (v_s) are zero-mean L -mixing processes (c.f. Gerencsér (1990)) to get a characterization of the parameter estimation error process. The theorem below gives a general answer to the problem raised above. Also the theorem has a potential for being used in the analysis of nonlinear stochastic systems.

Theorem Let $(u_{i,t})$, $t \geq 0$, $i = 1, \dots, N$ be zero-mean L -mixing processes with respect to a pair of families of σ -algebras $(\mathcal{F}_t, \mathcal{F}_t^+)$ and let $(f_{i,t})$, $t \geq 0$, $i = 1, \dots, N$ be locally square-

integrable functions. Then for the multiple integral

$$x_{N,T} = \int_0^T \int_0^{t_1} \cdots \int_0^{t_{N-1}} (f_{1,t_1} u_{1,t_1}) \cdots (f_{N,t_N} u_{N,t_N}) dt_1 \cdots dt_N. \quad (1.1)$$

we have for any $m \geq 1$

$$E^{1/2m} |x_{N,T}|^{2m} \leq \varphi_{N,2m,T} \quad (1.2)$$

where $\varphi_{N,2m,T}$ are defined recursively as follows: for $N = 0$ we set $\varphi_{0,2m,T} = 1$, for $N = 1$ we have

$$\varphi_{1,2m,T} = 4m^{1/2} M_{2m}^{1/2}(u_1) \Gamma_{2m}^{1/2}(u_1) \left(\int_0^T f_{1,t_1}^2 dt_1 \right)^{1/2}$$

and for $N > 1$ we first define

$$h_s = \int_s^T \gamma_{2mN}(t - r, u_N) |f_{N,t}| dt$$

and then set

$$\begin{aligned} \varphi_{N,2m,T} &= 4m M_{2mN}^{1/2}(u_N) \left(\int_0^T |f_{N,s}| \cdot h_s \cdot \varphi_{N-1,2mN/(N-1),s}^2 ds \right)^{1/2} + \\ &+ 8m M_{2mN}(u_N) \int_0^T |f_{N-1,s}| \cdot h_s \cdot \varphi_{N-2,2mN/(N-2),s} ds. \end{aligned}$$

Remark If the functions $|f_{i,t}|$ are monotone nonincreasing then we have

$$h_s \leq \int_s^t \gamma_{2mN}(t - s, u_N) |f_{N,s}| ds \leq \Gamma_{2mN}(u_N) |f_{N,s}|$$

and a simpler upper bound can be obtained using the recursion

$$\begin{aligned} \varphi_{N,2m,T} &= 4m M_{2mN}^{1/2}(u_N) \Gamma_{2mN}^{1/2}(u_N) \cdot \left(\int_0^T f_{N,s}^2 \varphi_{N-1,2mN/(N-1),s}^2 ds \right)^{1/2} + \\ &+ 8m \cdot M_{2mN}(u_{N-1}) \Gamma_{2mN}(u_N) \int_0^T |f_{N-1,s} \cdot f_{N,s}| \cdot \varphi_{N-2,2mN/(N-2),s} ds. \end{aligned}$$

To illustrate the usefulness of the theorem we shall apply it to estimate the moments of the double integral I_{T_0} above as a function of T_0 . For $N = 1$ we have

$$\varphi_{1,2m,T} = O \left(\int_{T_0}^T \frac{1}{s^2} ds \right)^{1/2} = O(T_0^{-1/2}).$$

For $N = 2$ we note that $h_s = O(s^{-1})$. Hence

$$\varphi_{2,2m,T} = O\left(\int_{T_0}^T \frac{1}{s^2} T_0^{-1} ds\right)^{1/2} + O\left(\int_{T_0}^T \frac{1}{s^2} ds\right) = O(T_0^{-1}).$$

Thus we finally get

$$I_{T_0} = \int_{T_0}^T \frac{1}{t} u_t \int_{T_0}^t \frac{1}{s} v_s ds dt = O_M(T_0^{-1}).$$

Remark Note that if $u_t dt$ and $v_s ds$ are replaced by Gaussian white-noise say du_t and dv_s we have a similar result, i.e.

$$\int_{T_0}^T \frac{1}{t} du_t \int_{T_0}^t \frac{1}{s} dv_s = O_M(T_0^{-1}),$$

which can easily be seen using standard moment inequalities for stochastic integrals.

2. The proof.

We have for $m \geq 1$

$$\frac{d}{dT} |x_{N,T}|^{2m} = 2m |x_{N,T}|^{2m-1} (\operatorname{sgn} x_{N,T}) f_{N,T} u_{N,T} \cdot x_{N-1,T}. \quad (2.1)$$

Setting

$$y_{N,T} = 2m |x_{N,T}|^{2m-1} (\operatorname{sgn} x_{N,T}) \cdot x_{N-1,T}$$

we can write (2.1) also in the form

$$|x_{N,T}|^{2m} = \int_0^T f_{N,t} u_{N,T} \cdot y_{N,t} dt. \quad (2.2)$$

To get a similar integral representation for $y_{N,T}$ we note that

$$\frac{d}{dT} |x_{N,T}|^{2m-1} \operatorname{sgn} x_{N,T} = (2m-1) |x_{N,T}|^{2m-2} \cdot f_{N,T} u_{N,T} \cdot x_{N-1,T}$$

hence

$$\begin{aligned} \frac{d}{dT} y_{N,T} &= 2m(2m-1) |x_{N,T}|^{2m-2} \cdot f_{N,T} u_{N,T} \cdot x_{N-1,T} \cdot x_{N-1,T} \\ &\quad + 2m |x_{N,T}|^{2m-1} (\operatorname{sgn} x_{N,T}) \cdot f_{N-1,T} u_{N-1,T} \cdot x_{N-2,T}. \end{aligned}$$

Setting

$$z_{N,T} = 2m(2m-1)|x_{N,T}|^{2m-2} \cdot (x_{N-1,T})^2$$

and

$$w_{N,T} = 2m|x_{N,T}|^{2m-1}(\operatorname{sgn} x_{N,T}) \cdot x_{N-2,T}$$

we can write

$$y_{N,T} = \int_0^T (f_{N,t}u_{N,t} \cdot z_{N,t} + f_{N-1,t}u_{N-1,t} \cdot w_{N,t})dt.$$

Substituting this expression into (2.2) we get

$$|x_{N,T}|^{2m} = \int_0^T f_{N,t}u_{N,t} \int_0^t (f_{N,s}u_{N,s} \cdot z_{N,s} + f_{N-1,s}u_{N-1,s} \cdot w_{N,s})dsdt \quad (2.3)$$

Take expectations of both sides and apply the “improved Hölder’s inequality” given in Lemma B. We have in general

$$|Eu_{N,t} \cdot u_{N,s} \cdot z_{N,s}| \leq 2\gamma_{p_1}(t-s, u_N) \cdot M_{q_1}(u_N) \cdot E^{1/r_1}|z_{N,s}|^{r_1}$$

with any $p_1, q_1, r_1 > 0$ such that $1/p_1 + 1/q_1 + 1/r_1 = 1$. Furthermore

$$E^{1/r_1}|z_{N,s}|^{r_1} \leq 2m(2m-1)E^{1/r_1'}|x_{N,s}|^{(2m-2)r_1'}E^{1/r_1''}|x_{N-1,s}|^{2r_1''}$$

where $r_1', r_1'' > 0$ are such that $1/r_1' + 1/r_1'' = 1/r_1$. Now let us set

$$r_1' = 2m/(2m-2) \quad \text{i.e.} \quad 1/r_1' = 1 - 1/m. \quad (2.4)$$

Then using inductive hypothesis we get

$$|Eu_{N,t} \cdot u_{N,s} \cdot z_{N,s}| \leq 4m(2m-1) \cdot \gamma_{p_1}(t-s, u_N) \cdot M_{q_1}(u_N) \cdot F_{N,s}^{1-1/m} \cdot \varphi_{N-1, 2r_1'', s}^2 \quad (2.4)$$

for any $p_1, q_1, r_1'' > 0$ such that $1/p_1 + 1/q_1 + 1/r_1'' = 1 - 1/r_1' = 1/m$. Note that $2r_1'' > 2$ therefore the use of the inductive hypothesis is justified. Similarly

$$|Eu_{N,t} \cdot u_{N-1,s} \cdot w_{N,s}| \leq 2\gamma_{p_2}(t-s, u_N) \cdot M_{q_2}(u_{N-1}) \cdot E^{1/r_2}|w_{N,s}|^{r_2}$$

with any $p_2, q_2, r_2 > 0$ such that $1/p_2 + 1/q_2 + 1/r_2 = 1$. Furthermore

$$E^{1/r_2} |w_{N,s}|^{r_2} \leq 2m \cdot E^{1/r_2'} |x_{N,s}|^{(2m-1)r_2'} E^{1/r_2''} |x_{n-2,s}|^{r_2''}$$

with any $r_2', r_2'' > 0$ such that $1/r_2' + 1/r_2'' = 1/r_2$. Choosing

$$r_2' = 2m/(2m-1) \quad \text{i.e.} \quad 1/r_2' = 1 - 1/2m \quad (2.5)$$

and using the inductive hypothesis we get

$$|Eu_{N,t} \cdot u_{N-1,s} \cdot w_{N,s}| \leq 4m \cdot \gamma_{p_2}(t-s, u_N) \cdot M_{q_2}(u_{N-1}) \cdot F_{N,s}^{1-1/2m} \cdot C_{N-2,r_2''} \varphi_{N-2,r_2'',s}$$

with any $p_2, q_2, r_2'' > 0$ such that $1/p_2 + 1/q_2 + 1/r_2'' = 1 - 1/r_2' = 1/2m$. Note again that $r_2'' > 2$ hence the use of the inductive hypothesis is justified. Substituting into (2.3) we get

$$\begin{aligned} E|x_{N,T}|^{2m} &\leq \int_0^T \int_0^t f_{N,t} f_{N,s} \cdot 4m(2m-1) \cdot \gamma_{p_1}(t-s, u_N) \cdot \\ &\quad \cdot M_{q_1}(u_N) \cdot F_{N,s}^{1-1/m} \cdot \varphi_{N-1,2r_1'',s}^2 ds dt + \\ &\quad + \int_0^T \int_0^t f_{N,t} f_{N-1,s} \cdot 4m \cdot \gamma_{p_2}(t-s, u_N) \cdot M_{q_2}(u_{N-1}) \cdot \\ &\quad \cdot F_{N,s}^{1-1/2m} \cdot \varphi_{N-2,r_2'',s} ds dt. \end{aligned} \quad (2.6)$$

Interchanging the order of integration and introducing the notations

$$h_{1,s} = \int_s^T f_{N,t} \cdot \gamma_{p_1}(t-s, u_N) dt \quad \text{and} \quad h_{2,s} = \int_s^T f_{N,t} \cdot \gamma_{p_2}(t-s, u_N) dt$$

we get

$$\begin{aligned} E|x_{N,T}|^{2n} &\leq \int_0^T f_{N,s} h_{1,s} \cdot 4m(2m-1) \cdot M_{q_1}(u_N) \cdot F_{N,s}^{1-1/m} \cdot \varphi_{N-1,2r_1'',s}^2 ds + \\ &\quad + \int_0^T f_{N-1,s} h_{2,s} \cdot 4m \cdot M_{q_2}(u_{N-1}) \cdot F_{N,s}^{1-1/2m} \cdot \varphi_{N-2,r_2'',s} ds \end{aligned} \quad (2.7)$$

and here

$$1/p_1 + 1/q_1 + 1/r_1'' = 1/m \quad (2.8)$$

and

$$1/p_2 + 1/q_2 + 1/r_2'' = 1/2m. \quad (2.9)$$

Setting $\psi_s = E^{1/2m}|x_{N,s}|^{2n}$ we get the inequality

$$\psi_T^{2m} \leq \int_0^T (a_s \psi_s^{2m-2} + b_s \psi_s^{2m-1}) ds \quad (2.10)$$

with

$$a_s = C_a^2 f_{N,s} h_{1,s} \phi_{n-1,s}^2 \quad \text{and} \quad b_s = C_b f_{N-1,s} h_{2,s} \phi_{N-2,s}$$

where

$$C_a^2 = 4m(2m-1)M_{q_1}(u_N) \quad \text{and} \quad C_b = 4mM_{q_2}(u_{N-1}).$$

We need the following lemma below:

Lemma 2.1. Let (ψ_t) , $t \geq 0$ be a nonnegative monotone increasing measurable function satisfying

$$\psi_T^{2m} \leq \int_0^T (a_s \psi_s^{2m-2} + b_s \psi_s^{2m-1}) ds \quad (2.11)$$

where (a_s) , (b_s) are nonnegative locally integrable functions and $n \geq 1$. Then we have

$$\psi_T \leq \sqrt{2} \left(\int_0^T a_s ds \right)^{1/2} + \frac{\sqrt{2+1}}{2} \left(\int_0^T b_s ds \right).$$

Proof: Increase the right hand side of (2.11) replacing ψ_s by ψ_T and then divide both sides by ψ_T^{2m-2} . (If $\psi_T = 0$ then the proposition is trivial). Then we get

$$\psi_T^2 \leq \left(\int_0^T a_s ds \right) + \left(\int_0^T b_s ds \right) \psi_T = A + B \psi_T.$$

from which we get

$$\psi_T \leq \frac{1}{2} (B + (B^2 + 4A)^{1/2}). \quad (2.12)$$

Now $B^2 + 4A \leq 2 \max(B^2, 4A)$, hence $(B^2 + 4A)^{1/2} \leq 2^{1/2} \max(B, 2A^{1/2}) \leq 2^{1/2} (B + 2A^{1/2})$, and substituting into (2.12) we get the proposition of the lemma.

Applying Lemma (2.1) to (2.10) and using $(\sqrt{2} + 1)/2 \leq 2$ we get

$$\begin{aligned} E^{1/2m}|x_{N,t}|^{2m} &\leq 4mM_{q_1}^{1/2}(u_N) \left(\int_0^T f_{N,s} h_{1,s} \varphi_{N-1,2r_1'',s}^2 ds \right)^{1/2} \\ &\quad + 8mM_{q_2}(u_{N-1}) \int_0^T f_{N-1,s} h_{2,s} \varphi_{N-2,r_2'',s} ds. \end{aligned} \quad (2.13)$$

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Let us now choose $p_1 = q_1 = p_2 = q_2 = 2nN$. Then we get from (2.8) that $r_1'' = mN/(N - 1)$ and from (2.10) that $r_2'' = 2nN/(N - 2)$, and the proposition of the theorem follows.

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