

Essays in International Economics

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Abstract

This dissertation consists of three essays studying different aspects of trade and development, and the implications for economic outcomes.

The first essay considers the effect of trade on consumption inequality. Recent papers have shown that income inequality and consumption inequality do not co-move in the US, I use recent advances in trade theory to examine to what extent this could have been caused by trade. I embed a nonhomothetic hierarchic demand structure into a Melitz-Chaney model of trade with heterogenous models.

The second essay, co-authored with Andrew Glover, examines the role of learning and economic growth. We show that the allocation puzzle first identified by [Gourinchas and Jeanne, 2009] has a time dimension: in particular it grows less severe over time. The allocation puzzle is the observation that amongst the developing countries, the fastest growers have the highest net capital outflow, contrary to standard economic theory. We show that the severity of these outflows decreases over time, and propose a learning mechanism that matches these facts. We introduce a kalman filter into a standard IRBC model so that consumers must learn about the trend of growth: in this model countries with volatile growth rates, such as developing countries, will initially increase savings leading to a capital outflow, which persists until consumers learn that a higher growth rate has been established.

The third essay, empirical in nature, studies the consequences of using the same business cycle filter for developed and developing countries. I calculate the business cycle length for a sample of countries, and find large differences in business cycle length across countries. It is still true, on average, the business cycles and GDP is more volatile in developing countries are shorter than those in developed countries, and that GDP volatility has decreased during the Great Moderation, mirroring results of existing literature.

The ratio of consumption to output volatility for both country groups are larger than what has been found previously, with developed countries approaching a ratio of 1 instead of being far less than one as previously documented. Previous papers documented that developing and developed countries have counter-cyclical government expenditure, while I find the counter-cyclicality of government spending remains weak for developing countries, and actually disappears for most developed countries. The exception is the USA, but this is only when one considers the full sample of years. Net exports are strongly counter-cyclical for emerging economics, while for developed economics these are only weakly counter-cyclical.

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Chapter 1

Trade and Inequality

1.1 Introduction

How does trade affect inequality within a country? The classic approach within the trade literature, based upon the Heckscher-Ohlin model, examines the effect of trade on the relative wages of income groups, and then maps those changes to changes in relative consumption. This approach implies that if trade doesn't change relative earnings then relative consumption doesn't change either. However, recent trade literature has established that cross country price levels of traded goods are systematically positively correlated with per capita income: firms charge higher prices in countries with higher GDP per capita¹, and that this effect is particularly strong in tradeable consumption goods.² As traded goods are part of the consumers consumption bundle, these pricing practices can exacerbate or alleviate the consumption inequality - the difference in aggregate consumption of different income groups - even if income inequality is unchanged. In order to understand how trade changes consumption inequality, it is therefore important to include this underlying price mechanism in a model of trade: that is the goal of this paper.

¹For example, see [Fieler, 2011], [Hummels and Lugovsky, 2009], [Simonovska, 2011]

²[Hsieh and Klenow, 2007a]

I introduce two income groups into a general equilibrium trade model with non-homothetic preferences. The non-homothetic utility in this model results in bounded marginal utility for goods so that there exists a choke price above which consumers are unwilling to buy a good even if they have access to it on the market. Consumers therefore endogenously choose not only the quantity but also the variety of goods to consume. Firms are heterogeneous in productivity and monopolistic price setters, as per [Melitz, 2003] and [Chaney, 2008], but cannot perfectly price discriminate between income groups and must therefore choose whether to make their good available at a low price (such that both consumers can afford the good), or a high price (such that it is exclusively affordable to wealthy consumers).

Previous trade models use homothetic preferences, which limits both income groups to consume the same variety of goods³, and consumers of lower income groups consume a constant fraction of the high income group's consumption. This means that the price indices faced by both consumer groups are identical, and that real income co-move exactly. Therefore, unless trade liberalization affects *relative* incomes, relative consumption inequality is unaffected. With per-capita and inequality dependent mark-ups that exist in the data and in my model, I show that this is no longer true; even if incomes are unaffected by a trade liberalization it is still possible that real income inequality is changed due to the different behaviour in prices faced by the two income groups. A decrease in the cost of trade changes the firms optimal price, and may change which income group market they serve (high income, or both type). This changes the variety and quantity of goods consumers buy. I find that the effect of trade on consumption inequality depends crucially on the characteristics on the trade partner.

The resurgence of nonhomothetic demand in trade has resulted in various possible specifications of demand structures. The interested reader should refer to [Markusen, 2010] for a comprehensive discussion of the usefulness non-homothetic

³Unless explicit assumptions are made that restrict market access of certain income groups, a case I ignore.

preferences in trade. [Bekkers et al., 2011] compares the predictions of hierarchic demand, demand for quality and demand finickyness, all of which have been used in the trade literature to explain the positive correlation between prices and income, and find that only models with hierarchic demand systems can explain the negative correlation between prices and inequality. [Simonovska, 2011] compares hierarchic demand structures with models of search costs and find that these models fail to capture the negative relationship between prices and market size that can be matched by hierarchic demand. Conversely, she finds that the nonhomothetic demand presented by [Feenstra, 2003] and [Melitz and Ottaviano, 2008] capture the negative relationship between prices and market size but do not capture per-capita income effects on prices. With these results in mind, I use a hierarchic demand structure as specified in [Simonovska, 2011] to generate nonhomotheticity in my demand function.

To summarize, in this paper I present a heterogenous-firm model of international trade with two income groups with identical nonhomothetic demand residing in each country. I find that even if trade does not change relative incomes, the optimal pricing strategies of firms change as a result of trade liberalization which in turns changes relative consumption inequality. The layout of the paper is as follows: In section 2, I introduce my model. In section 3, I define consumption inequality, in section 4 I present simulation results.

1.2 Model

I consider a static environment with a finite number of countries, $i \in \{1, \dots, I\}$, trading varieties of final goods. Let s denote the exporter (“source”) and d the importer (“destination”).

1.2.1 Consumer

Country $d \in \{1, \dots, I\}$ has two consumer types, $k = \{H, L\}$, each of population size M_d^k such that $\sum_k M_d^k = M_d$, where M_d denotes the total population. Consumers types differ only by their effective labor, θ_d^k . Both consumer types supply 1 unit of labor inelastically, so that the total effective labor supply is $L_d = M_d^H \theta_d^H + M_d^L \theta_d^L$. Let w_d denote income per unit of effective labor, then income for type k is given by $w_d^k = \theta_d^k w_d$. As $w_d^H = \frac{\theta_d^H}{\theta_d^L} w_d^L$, the assumption that $\theta_d^H > \theta_d^L$ is sufficient to ensure that H-type has higher income than L-type. All consumers have the identical utility function over the available goods, G_s , from countries $s = \{1, \dots, I\}$.

$$U_d^k = \sum_s \int_0^{G_s} \log(c_{sd}^k(i) + q) di \quad (1.1)$$

where $q > 0$ and is assumed to be the same across all goods and countries.

Given income, w_d^k and prices, $p_{sd}(i)$, each type of consumer chooses $c_{sd}^k(z)$ to solve

$$\max_{c_{sd}^k(i)} \sum_s \int_0^{G_s} \log(c_{sd}^k(i) + q) di \quad (1.2)$$

$$\text{s.t.} \quad \sum_s \int_0^{G_s} p_{sd}(i) c_{sd}^k(i) di \leq w_d^k \quad (1.3)$$

$$c_{sd}^k(i) \geq 0 \quad (1.4)$$

The solution to the consumer's problem is

$$c_{sd}^k(i) = \max\left\{0, \frac{w_d^k + qP_d^k}{p_{sd}(i)N_d^k} - q\right\} \quad (1.5)$$

where

$$N_d^k = \sum_s \int_0^{G_s^k} 1 di \quad (1.6)$$

is the number of products purchased by consumer k and

$$P_d^k = \sum_s \int_0^{G_s^k} p_{sd}(i) di \quad (1.7)$$

is the pricing aggregator over the goods purchased by consumer k .

Note that the only variety-specific variable in (1.5) is $p_{sd}^k(i)$; all the other terms are aggregates. It is then obvious that consumers have a bounded marginal utility for any good i : if $p_{sd}^k(i) > \frac{w_d^k + qP_d^k}{qN_d^k}$ (i.e. the price for a specific good i is “too high”) the consumer will choose the optimal boundary solution of $c_{sd}^k(i) = 0$. As consumers have different incomes, it is possible for the variety and price aggregator of each consumer to be different. For future use, define the following terms,

$$\text{Truncated Demand of type } k: \quad Q_d^k = \frac{w_d^k + qP_d^k}{N_d^k} \quad (1.8)$$

$$\text{Aggregate Truncated Demand:} \quad S_d = M_d^H Q_d^H + M_d^L Q_d^L \quad (1.9)$$

Proposition 1.2.1. *If $Q_d^H \geq Q_d^L$ then $\forall z, c_d^H(z) \geq c_d^L(z)$*

Proof: Rewrite equation (1.5) as $c_{sd}^H(i) = \max\{0, \frac{Q_d^H}{p_{sd}(i)} - q\}$ and $c_{sd}^L(i) = \max\{0, \frac{Q_d^L}{p_{sd}(i)} - q\}$. The difference can be written as

$$c_{sd}^H(i) - c_{sd}^L(i) = \frac{1}{p_{sd}(i)} (\max\{0, Q_d^H - p_{sd}(i)q\} - \max\{0, Q_d^L - p_{sd}(i)q\})$$

. As both consumers face the same price, $Q_d^H \geq Q_d^L$ is sufficient to give the result $c_d^H(z) \geq c_d^L(z)$. \square

Proposition (1.2.1) implies that it will never be true that firms only sell the good to L-type; they will either sell to just the H-type or to both types.

1.2.2 Firm Problem

I follow [Melitz, 2003] and assume a continuum of firms in each country using labor-only technology, differentiated by only their productivity. I use the simplification of [Chaney, 2008] and assume that firms can pay a fixed cost, f_c , to draw a productivity z from a Pareto distribution with support $[b_i, \infty]$ where $b_i \geq 1$ denotes the minimum

productivity level of the draw in country i .⁴ Productivity and country uniquely identify each firm. Firms pay a tariff cost, $\tau_{sd} \geq 1$, where $\forall s \quad \tau_{ss} = 1$, and a fixed cost of export, f_{sd} , where $f_{ss} = 0$ and $f_{sd} = f_x$ for $s \neq d$. The fixed cost is a one-time cost incurred to establish trade with each trading partner - it can be thought of as the cost to updating packaging for sales in foreign countries, establishing stores or undertaking initial market research. τ_{sd} is a cost that is paid for each good imported, and can be thought of as representing transportation costs or tariffs. As each firm sells only one good, and goods from different countries are not substitutable, a good i from country s is equivalent to a firm z from country s . I will assume a no-arbitrage constraint across borders - the wages of consumers in neighboring countries do not enter the firms problem - and that firms cannot price discriminate between agents of different income groups: regardless of income all consumers face the same prices.⁵ As firms cannot price discriminate between income groups, each firm can set a low price such that their good is affordable to both consumers and earn a resulting profit of $\pi_{sd}^B(z)$, or it can set a high price such that its good is only affordable to the rich consumer-type earn profit $\pi_{sd}^H(z)$. As I assume a linear production function with only labor and a no-arbitrage condition, I can examine the firms production decision for each destination country, d , separately. The firm's problem then, is to choose the pricing plan that results in

$$\pi_{sd}(z) = \max\{\pi_{sd}^B(z), \pi_{sd}^H(z), 0\} \quad (1.10)$$

⁴The Pareto cdf is $F(z) = 1 - \left(\frac{b_i}{z}\right)^\gamma$, the pdf is $f(z) = \frac{\gamma b^\gamma}{z^{\gamma+1}}$, where the shape parameter $\gamma > 0$. Different b 's denote countries varying levels of productivity, with a higher b denoting a country with a higher average productivity. I will set $b_i = 1$ for all countries, and capture different country-level productivities by allowing different average labor productivities.

⁵It is possible to add a constraint such that $p_{sd}^H(z) \leq (1+r)p_{sd}^L(z)$, where $r > 0$. For example, a firm could choose to locate stores with the lower price goods close to poor neighborhoods but far from wealthy neighborhoods. Poor consumers could then buy the good at the low price, travel to the rich neighborhoods and sell it at a markup of r . A necessary condition for this constraint to bind is $(1+r)^2 < \left(\frac{w_j^H + qP_j^H}{N_j^H}\right) \left(\frac{N_j^L}{w_j^L + qP_j^L}\right)$. This additional constraint complicates the model considerably by adding another equilibrium configuration for cutoffs without considerably changing the results.

where a choice of 0 indicates that the firm has drawn a productivity that is too inefficient to produce under either pricing plan.

Given the consumers indirect demand functions $x^k(p_{sd}(z); P_d^k, N_d^k, w_d^k) = M_d^k c^k(z)$ ⁶, a firm z that sells to both types in country d solves

$$\pi_{sd}^B(z) = \max_{p_{sd}(z), \ell_{sd}(z)} p_{sd}(z)x_{sd}^H(z) + p_{sd}(z)x_{sd}^L(z) - w_s \ell_s(z) \text{ s.t.} \quad (1.11)$$

$$x_{sd}^H(z) + x_{sd}^L(z) = \frac{z}{\tau_{sd}}(\ell_s(z) - f_{sd}) \quad (1.12)$$

where $\ell_{sd}(z) = \theta_s^H \ell_s^H(z) + \theta_s^L \ell_s^L(z)$ represents the effective labor hired by the firm, where market clearing imposes that $\sum_d \int \ell_{sd}(z) dz = M_s^H \theta_s^H + M_s^L \theta_s^L$. The assumption of a linear production technology ensures that firms are indifferent between the two types of labor.

A firm that sells only to the H-type of consumers solves

$$\pi_{sd}^H(z) = \max_{p_{sd}(z), \ell_{sd}(z)} p_{sd}(z)x_{sd}^H(z) - w_s \ell_{sd}(z) \text{ s.t.} \quad (1.13)$$

$$x_{sd}^H(z) = \frac{z}{\tau_{sd}}(\ell_{sd}(z) - f_{sd}) \quad (1.14)$$

Denote by $p_{sd}^H(z)$ the price a firm would charge if the sell only to the H-type, and $p_{sd}^B(z)$ the price a firm would charge if selling to both types. Solving the firms problem we find that the optimal price for a firm selling only to the H-type consumer is

$$p_{sd}^H(z) = \underbrace{\left[\frac{\tau_{sd} w_s}{z} \right]^{\frac{1}{2}}}_{\text{Marginal Cost}} \underbrace{\left[\frac{Q_d^H}{q} \right]^{\frac{1}{2}}}_{\text{Mark-up}} \quad (1.15)$$

which results in a profit

$$\pi_{sd}^H = q^{\frac{1}{2}} M_d^H \left[\left(\frac{Q_d^H}{q} \right)^{\frac{1}{2}} - \left(\frac{\tau_{sd} w_s}{z} \right)^{\frac{1}{2}} \right]^2 - w_s f_{sd} \quad (1.16)$$

⁶I abbreviate this as $x_d^k(z)$.

If the firm sells to both types of consumers optimal price is

$$p_{sd}^B(z) = \underbrace{\left[\frac{\tau_{sd} w_s}{z} \right]^{\frac{1}{2}}}_{\text{Marginal Cost}} \underbrace{\left[\frac{S_d}{qM_d} \right]^{\frac{1}{2}}}_{\text{Mark-Up}} \quad (1.17)$$

which results in a profit

$$\pi_{sd}^B(z) = q^{\frac{1}{2}} M_d \left[\left(\frac{S_d}{qM_d} \right)^{\frac{1}{2}} - \left(\frac{w_s \tau_{sd}}{z} \right)^{\frac{1}{2}} \right]^2 - w_s f_{sd} \quad (1.18)$$

Notice that all firms serving destination d have the same mark-up choices, with variations in the prices of firms with different productivities driven by differing marginal costs of serving market d , or by firms choosing different market segments.

Proposition 1.2.2. *If $Q_d^H \geq Q_d^L$ then $\forall z$ $p_{sd}^H(z) \geq p_{sd}^L(z)$.*

Proof: Rearrange pricing equations (1.15) and (1.17) to find that it is sufficient to show that the markup for $p_{sd}^H(z)$ is greater than the markup for $p_{sd}^L(z)$. The required condition for this to hold is $M_d Q_d^H > M_d^H Q_d^H + M_d^L Q_d^L$, which is true if $Q_d^H > Q_d^L$. \square

1.2.3 Cutoff Productivity

Unlike standard Melitz models in which profit is a linear functions of firm productivity, profits in this model are quadratic and can have multiple cutoff configurations. There are up to three productivities of potential interest: the productivity at which the profit from selling exclusively to the H-type is zero, z^{H0} , the productivity at which the profit from selling to the both types is zero, z^{B0} , and the productivity at which firms are indifferent between selling to the H-type and L-type, z^{HB} .

z_{sd}^{H0} is found by solving $\pi_{sd}^H(z_{sd}^{H0}) = 0$ to obtain

$$z_{sd}^{H0} = \tau_{sd} w_s \frac{qM_d^H}{\left[(M_d^H Q_d^H)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}} \right]^2} \quad (1.19)$$

Similarly, z_{sd}^{B0} is found by solving $\pi_{sd}^B(z_{sd}^{B0}) = 0$

$$z_{sd}^{B0} = \tau_{sd} w_s \frac{q M_d}{\left[S_d^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}} \right]^2} \quad (1.20)$$

Finally, the productivity at which firms are indifferent between selling to the H-type and the L-type, z_{sd}^{HB} is given by solving $\pi_{sd}^B(z) = \pi_{sd}^H(z)$,

$$z_{sd}^{HB} = \tau_{sd} w_s \frac{q \left[M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right]^2}{\left[S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right]^2} \quad (1.21)$$

With three cutoffs there are 10 possible cutoff configurations that begin with a zero profit productivity. The purpose of this section is to show that only one of these configurations is an equilibrium.

Proposition 1.2.3. *There is only one zero crossing cutoff in the equilibrium cutoff configuration*

Proof: Let z_{sd}^0 denote the productivity such that $\max\{\pi_{sd}^H(z_{sd}^0), \pi_{sd}^B(z_{sd}^0)\} = 0$. As profit equations (1.16) and (1.18) increase in z , $\forall z > z_{sd}^0$, $\max\{\pi_{sd}^H(z), \pi_{sd}^B(z)\} > \max\{\pi_{sd}^H(z_{sd}^0), \pi_{sd}^B(z_{sd}^0)\} = 0$. \square

Assumption 1.2.4. *Fixed costs don't prevent trade: $M_d^H Q_d^H > w_s f_{sd}$ and $S_d > w_s f_{sd} \forall d$*

Proposition 1.2.5. *If assumption (1.2.4) is satisfied, then in the limit all firms choose to sell to both consumer groups.*

Proof: $\lim_{z \rightarrow \infty} \pi_{sd}^H(z) \rightarrow (M_d^H Q_d^H - f_{sd})$ and $\lim_{z \rightarrow \infty} \pi_{sd}^B(z) \rightarrow (S_d - f_{sd})$. With assumption 1.2.4 in the limit $\pi_{sd}^B > 0$ and firms enter and sell in the market. Since $Q_d^L > 0$ in the limit $\pi_{sd}^B > \pi_{sd}^H$, and firms find it more profitable to sell to both types when they enter. \square

Theorem 1.2.6. *If propositions 1.2.3 and 1.2.5 are satisfied, then there are only 2 potential equilibrium cutoff configurations: $\{z_{sd}^{B0}\}$ and $\{z_{sd}^{H0}, z_{sd}^{HB}\}$.*

Proposition 1.2.3 restricts attention to cutoff configurations with only one zero profit cutoff, reducing the possible cutoff configurations from 10 possible sets to 4: $\{z_{sd}^{H0}\}$; $\{z_{sd}^{B0}\}$; $\{z_{sd}^{H0}, z_{sd}^{HB}\}$; $\{z_{sd}^{B0}, z_{sd}^{HB}\}$. Given that profit functions are increasing and continuously differentiable on the support of productivities, any crossing indicates a switch in market structure. Proposition 1.2.5 further reduces these sets by limiting attention to only sets in which all firms will sell to both groups in the limit, of which there are 2: $\{z_{sd}^{B0}\}$; $\{z_{sd}^{H0}, z_{sd}^{HB}\}$.⁷ \square

Assumption 1.2.7. *Fixed costs of trade are small enough that it is profitable for some foreign firms to sell only to H-type: $\forall s, d f_x < \frac{M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}}} \frac{(M_d Q_d^H)^{\frac{1}{2}} - S_d^{\frac{1}{2}}}{w_s^{\frac{1}{2}}}$*

Proposition 1.2.8. *If $Q_d^H > Q_d^L$ then the only potential domestic equilibrium cutoff configuration is $\{z_{dd}^{H0}, z_{dd}^{HB}\}$. In addition, if the fixed cost of export satisfy assumption (1.2.7), then the only equilibrium cutoff configuration is $\{z_{sd}^{H0}, z_{sd}^{HB}\} \forall s, d$.*

Proof: Suppose that $\{z_{dd}^{B0}\}$ is an equilibrium: then it must be that $z_{dd}^{H0} > z_{dd}^{B0}$. Rearrange equations (1.19) and equations (1.20) to find that this implies

$$S_d^{\frac{1}{2}} > (M_d Q_d^H)^{\frac{1}{2}} \quad (1.22)$$

which cannot be satisfied if $Q_d^H > Q_d^L$. The proof for traded goods cutoffs is identical, but requires the use of the limit of fixed cost of exports. The only remaining potential equilibrium cutoff is $\{z_{dd}^{H0}, z_{dd}^{HB}\}$. \square

The upper bound on the fixed cost of exports is needed because f_x causes the less productive firms that sell only to the H-types, to drop out of the export market. The upper bound on f_x ensures that some firms earn enough positive profit from export even if they can only sell to the H-type.

⁷For example, the configuration $\{z_{sd}^{B0}, z_{sd}^{HB}\}$ implies that firms initially sell to both groups, then switch to selling to just the H-group. This is inconsistent with 1.2.5.

Proposition 1.2.9. *If $Q_d^H < Q_d^L$, the only potential domestic equilibrium cutoff configuration is $\{z_{dd}^{B0}\}$.*

Proof: Follows as prop (1.2.8). \square

Notice that, unlike proposition 1.2.8, proposition 1.2.9 does not require a limit on the fixed cost of exports.

Theorem 1.2.10. *If $w_d^H > w_d^L$ and f_x satisfies assumption (1.2.7), then $\{z_{sd}^{B0}\}$ is not an equilibrium cutoff configuration.*

Proof: Suppose that $\{z_{sd}^{B0}\}$ is the equilibrium for some source-destination pair. By propositions 1.2.8 and 1.2.9, this can only be true if $Q_d^H < Q_d^L$, therefore $\{z_{sd}^{B0}\}$ is the equilibrium for all firms selling in d. This implies that all firms sell to both consumers, so that $N_d^H = N_d^L = N_d$ and $P_d^H = P_d^L = P_d$. We can therefore simplify $Q_d^H = \frac{w_d^H + qP_d}{N_d}$ and $Q_d^L = \frac{w_d^L + qP_d}{N_d}$. If $w_d^H > w_d^L$ however, then $Q_d^H > Q_d^L$, which leads to a contradiction with proposition (1.2.8). \square

In this paper I assume that $w_d^H > w_d^L$ and assumptions (1.2.4) and (1.2.7) are satisfied, therefore the only possible equilibrium configuration is $\{z_{sd}^{H0}, z_{sd}^{HB}\}$, and therefore in equilibrium $Q_d^H \geq Q_d^L$.

Rearrange equations (1.19), and (1.21) and use $f_{dd} = 0$ and $\tau_{dd} = 1$ to equate the cutoffs for each source country with the equivalent cutoff for goods sourced from domestic firms,

$$z_{sd}^{H0^{-\frac{1}{2}}} = \left(\frac{w_d}{w_s \tau_{sd}} \right)^{\frac{1}{2}} z_{dd}^{H0^{-\frac{1}{2}}} - \left(\frac{f_{sd}}{q M_d^H \tau_{sd}} \right)^{\frac{1}{2}} \quad (1.23)$$

$$z_{sd}^{HB^{-\frac{1}{2}}} = \left(\frac{w_d}{\tau_{sd} w_s} \right)^{\frac{1}{2}} z_{dd}^{HB^{-\frac{1}{2}}} \quad (1.24)$$

In a homothetic preference Melitz model domestic and foreign cutoffs are expressed in a manner similar to z_{sd}^{HB} : foreign cutoffs are simply a scaled up (or down) version of the domestic cutoff. This scaling rule is not true for z_{sd}^{H0} . Even though

z_{sd}^{H0} and z_{dd}^{H0} comove⁸, the degree of that comovement ranges from almost zero to $\frac{w_s \tau_{sd}}{w_d}$, depending on the profitability in the destination H-type market, as captured by the value of z_{dd}^{H0} , relative to the fixed export cost, f_x . Profitability in the H-type market matters because the firms that are most affected by f_x are low productivity ones: the ones that sell exclusively to the H-type consumers. Because the cutoffs movements in z_{sd}^{H0} and z_{sd}^{HB} are not perfectly correlated with each other it is possible for any change in cutoffs to increase (or decrease) consumption inequality.

1.2.4 Equilibrium of the World Economy

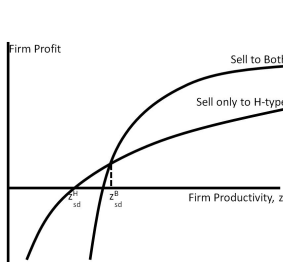


Figure 1.1: Cutoff

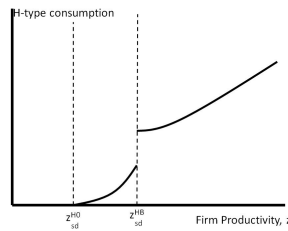


Figure 1.2: H-type consumption

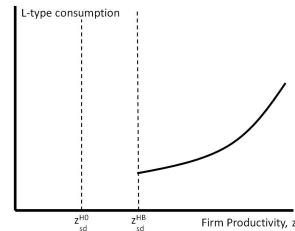


Figure 1.3: L-type consumption

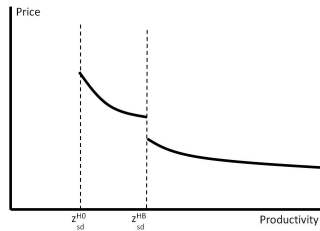


Figure 1.4: Price

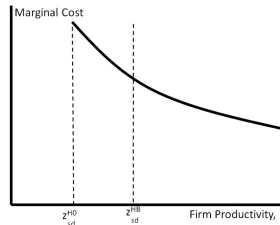


Figure 1.5: Marginal Cost

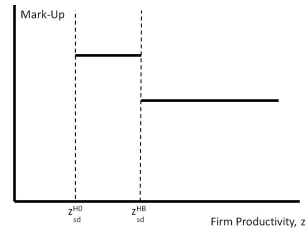


Figure 1.6: Mark-Up

Figure (1.1) plots the profit earned from selling just to H-type, $\pi_{sd}^H(z)$, the profit from selling to both types, $\pi_{sd}^B(z)$, and the zero profit line as a function of productivity. For any given productivity z , if $\pi_{sd}^H(z) > \pi_{sd}^B(z)$ the firm sells to H-type consumers, if $\pi_{sd}^B(z) > \pi_{sd}^H(z)$ the firm sells to both types, and if both profit options are less than zero, the firm choose not to produce. As shown in theorem 1.2.6 and proposition 1.2.8 the only configuration of an economy's cutoffs is as shown:

⁸Obviously, comovements with changes related to w_d , w_s or τ_{sd} are more complicated

$\forall z \geq z_{sd}^{HB}$ firms sell to both consumer types, while for $z_{sd}^{H0} \leq z < z_{sd}^{HB}$ firms sell only to H-type.

Figures (1.2) and (1.3) plots consumption for each group: H and L. All firms with productivity below the productivity threshold do not produce. Firms with $z_{sd}^{H0} \leq z < z_{sd}^{HB}$ are profitable enough to produce, but sell only to H-type consumers, therefore L-type consumers do not consume. For $z \geq z_{sd}^{HB}$ both H and L consume goods. As proven in proposition 1.2.1 for any z , $c_{sd}^H(z) > c_{sd}^L(z)$.

Figure (1.4) plots price against productivity. As proved in proposition 1.2.2, prices charged in the H-region, $z_{sd}^{H0} \leq z < z_{sd}^{HB}$, are greater than those charged when the firms are selling to both. The marginal cost component of price, figure (1.5), is not affected by the cutoff, and decreases steadily with increased productivity. Figure (1.6) plots mark-ups. Mark-ups are dependent on cutoffs and destination but not productivity: Firms selling to just the H-type charge a high markup, while firms selling to both groups the charge a lower mark-up.

Let the number of firms in s who have paid the fixed cost to draw a productivity be denoted by J_s . Of the firms that have drawn the productivity, the number serving H in destination d from source s , N_{sd}^H , is equal to the number of firms who have drawn productivities equal to or greater than z_{sd}^{H0} . With a Pareto distribution of productivities this is given by

$$N_{sd}^H = J_s b_s^\gamma z_{sd}^{H0^{-\gamma}} \quad (1.25)$$

while the number of firms producing for the L-type in d from country s is given by the firms who drew productivities greater than z_{sd}^{HB}

$$N_{sd}^L = J_s b_s^\gamma z_{sd}^{HB^{-\gamma}} \quad (1.26)$$

The total number of firms from which each type in d purchase goods is simply given

by

$$N_d^k = \sum_s N_{sd}^k \quad (1.27)$$

The pricing aggregator for the L-type, P_d^L is given by

$$\begin{aligned} P_d^L &= \sum_s J_s \int_{z_{sd}^{HB}}^{\infty} p_{sd}^B(z) \gamma b_s^\gamma z^{-(\gamma+1)} dz \\ &= \frac{\gamma w_d^L \left[M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} \right]}{q \left[\frac{1}{2} M_d^{\frac{1}{2}} - \gamma M_d^{H\frac{1}{2}} \right]} - \frac{w_d \gamma N_d^L}{(z_{dd}^{H0} z_{dd}^{HB})^{\frac{1}{2}}} \frac{M_d^{H\frac{1}{2}}}{\left[\frac{1}{2} M_d^{\frac{1}{2}} - \gamma M_d^{H\frac{1}{2}} \right]} \end{aligned} \quad (1.28)$$

while the pricing aggregator for the H-type is given by

$$P_d^H = P_d^L + \sum_s J_s \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} p_{sd}^H(z) \gamma b_s^\gamma z^{-(\gamma+1)} dz \quad (1.29)$$

$$\begin{aligned} &= \left[\frac{M_d^{H\frac{1}{2}} + M_d^{\frac{1}{2}}}{M_d^{H\frac{1}{2}}} \right] P_d^L + \frac{2\gamma w_d^H}{q} - \frac{2\gamma w_d^L}{q} \left[\frac{M_d^{H\frac{1}{2}} + M_d^{\frac{1}{2}}}{M_d^{H\frac{1}{2}}} \right] \\ &\quad - \frac{2\gamma}{q} \left(\frac{w_d q}{M_d^H z_{dd}^{H0}} \right)^{\frac{1}{2}} \sum_s b_s^\gamma J_s z_{sd}^{H0-\gamma} (w_s f_{sd})^{\frac{1}{2}} \end{aligned} \quad (1.30)$$

Wage income for consumer of type k is given by

$$w_d^k = \theta_d^k w_s \quad (1.31)$$

Firms will draw productivities until expected profit for firms in s equal the fixed cost of entry

$$w_s f_c = \sum_d \int_{z_{sd}^H}^{z_{sd}^B} \pi_{sd}^H(z) \gamma b_s^\gamma z^{-\gamma+1} dz + \int_{z_{sd}^B}^{\infty} \pi_{sd}^B(z) \gamma b_s^\gamma z^{-\gamma+1} dz \quad (1.32)$$

Equation (1.32) implies that expected profit is zero. Since the law of large numbers is satisfied in this economy expected profit is equal to average profit, and therefore

aggregate profit, which is just average profit multiplied by number of firms, is zero. This means that there is no rebated profit and aggregate consumer income is simply the earned wages. Consumers spend all their income, and income spent on goods has to equal the total sales by $\forall s$.

$$M_d^H w_d^H + M_d^L w_d^L = \sum_s R_{sd} \quad (1.33)$$

Total sales revenue of all firms in s selling to d is given by

$$\begin{aligned} R_{sd} = & \gamma b_s^\gamma J_s \left[\int_{z_{sd}^{H0}}^{z_{sd}^{HB}} p_{sd}^H(z) M_d^H c_d^H(z) z^{-(\gamma+1)} dz \right] \\ & + \gamma b_s^\gamma J_s \left[\int_{z_{sd}^{HB}}^{\infty} p_{sd}^B(z) \left(\frac{S_d}{p_{sd}^B(z)} - M_d q \right) z^{-(\gamma+1)} dz \right] \end{aligned} \quad (1.34)$$

Equations (1.33)-(1.34) can be combined to find a more compact expression for the number of firms

$$J_s = \frac{M_s^H \theta_s^H + M_s^L \theta_s^L}{(\gamma + 1) f_c + b_s^\gamma \sum_d f_{sd} z_{sd}^{H0-\gamma}} \quad (1.35)$$

With a homothetic Melitz model, the final summation term in the denominator would not exist: number of firms would be purely a function of source country characteristics. In this model, the cutoffs in *any* destination affects the number of firms: any movement in H-type cutoffs will change the number of firms in all source countries. The reason for this can be seen in equation (1.32): Firms expected profit over all destinations are considered against the cost of taking the draw. The H-type market is a source of profit, though the ability of low productivity firm to access it is dependent upon the fixed export cost relative to the size of profit than can be earned from serving that destination. A change in H-cutoff will change expected profits and induce entry or exit as the expected profits of firms change.

Definition 1.2.11. For $s, d=1, \dots, I$, given τ_{sd} , M_d , M_d^k , $F(z)$, f_c , f_{sd} , q , w_1 , and b_s

an equilibrium consists of productivity thresholds $\{\hat{z}_{sd}^{H0}, \hat{z}_{sd}^{HB}\}$; measure of entrants, \hat{J}_s ; a measure of firms from country s serving k -type market d , \hat{N}_{sd}^k ; the total measure of firms serving type k in market d , \hat{N}_d^k ; a aggregate price statistic for each k -type, \hat{P}_d^k ; wages for each type, \hat{w}_d^k ; per consumer consumption, $\hat{c}_{sd}^k(z)$; total consumer- K allocation, \hat{C}_{sd}^K , firm pricing rules $\hat{p}_{sd}^k(z)$, production plans, \hat{x}_{sd}^k and firm profits $\hat{\pi}_{sd}(z)$ such that: (i) Given P_d , w_d^k , p_{sd} , $\hat{c}_{sd}^K(z)$ satisfies (1.5) and solves the individual consumers problem, (ii) \hat{C}_{sd}^K is given by $M_d^k c_{sd}^k(z)$ and satisfied aggregate demand for goods by type k , (iii) $\hat{p}_{sd}^H(z)$ is given by (1.15) and solves firm's problem is selling only to H , (iv) $\hat{p}_{sd}^B(z)$ is given by (1.17) and solves firm's problem if selling to both types, (v) $\hat{x}_{sd}^k(s)$ satisfies goods market condition: $\hat{x}_{sd}^k(s) = C_{sd}^K(z)$, (vi) $\hat{\pi}_{sd}^H(s)$ is given by (1.13), (vii) $\hat{\pi}_{sd}^B(s)$ is given by (1.11), (viii) \hat{w}_d^k is given by (A.29), (ix) \hat{z}_{sd}^{H0} ; \hat{z}_{sd}^{HB} ; \hat{N}_d^k ; \hat{P}_d^H ; \hat{P}_d^L ; \hat{w}_d ; \hat{J}_s jointly satisfy (1.19), (1.21), (1.27), (1.28), (1.29), (1.33), (1.35)

1.3 Theoretical Predictions

In this section I study the effect of trade on consumption inequality. When defined by data, consumption inequality reflects the difference in the aggregate quantity of goods purchased by each consumer group. Within the model, the aggregate quantity of consumption goods each consumer purchases is simply the integral over the appropriate interval of $c_{sd}^H(z)$ and $c_{sd}^L(z)$, given by

$$C_d^L = \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^{\infty} \frac{Q_d^L(z)}{p_{sd}^B(z)} z^{-(\gamma+1)} dz - \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^{\infty} q z^{-(\gamma+1)} dz \quad (1.36)$$

$$C_d^H = \sum_s J_s b_s^\gamma \gamma \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} \frac{Q_d^H(z)}{p_{sd}^H(z)} z^{-(\gamma+1)} dz + J_s b_s^\gamma \gamma \int_{z_{sd}^{HB}}^{\infty} \frac{Q_d^H(z)}{p_{sd}^B(z)} z^{-(\gamma+1)} dz - J_s b_s^\gamma \gamma \int_{z_{sd}^{H0}}^{\infty} q z^{-(\gamma+1)} dz \quad (1.37)$$

Trade can affect consumption inequality through two channels: Firstly, foreign countries may sell different proportions of goods to rich and poor consumers, I will term this a composition effect. Secondly, trade may induce a change in the relative price

index of the two agent type by changing mark-ups and varieties. In the next section I compare the provision of goods of foreign and domestic firms, I will then compare the effect of trade of mark-ups and varieties. It is useful to note that there are two components to changes in welfare inequality: A change in the intensive margin - the change in quantity of a good consumed due to the change in prices - and an extensive margin - the change in variety or the number of firms selling to the income group.

1.3.1 Composition of Consumption: Foreign and Domestic Goods

Variety and Price

I begin by comparing the relative variety of goods sourced from domestic and foreign sources.

$$\begin{aligned} \frac{N_{sd}^L}{N_{sd}^H} &= \frac{J_s b_s^\gamma z_{sd}^{HB^{-\gamma}}}{J_s b_s^\gamma z_{sd}^{H0^{-\gamma}}} = \frac{z_{sd}^{HB^{-\gamma}}}{z_{sd}^{H0^{-\gamma}}} \\ &= \left[1 - \left(\frac{w_s f_{sd} z_{dd}^{H0}}{w_d q M_d^H} \right)^{\frac{1}{2}} \right]^{-2\gamma} \left(\frac{z_{dd}^{HB}}{z_{dd}^{H0}} \right)^{-\gamma} = \left[1 - \left(\frac{w_s f_{sd}}{M_d^H Q_d^H} \right)^{\frac{1}{2}} \right]^{-2\gamma} \left(\frac{z_{dd}^{HB}}{z_{dd}^{H0}} \right)^{-\gamma} \end{aligned} \quad (1.38)$$

Equation (1.38) show that if the fixed cost of exporting is zero, $f_{sd} = 0$, foreign firms maintain the same ratio of varieties between H- and L-type consumers as domestic firms. As the fixed cost increases, foreign firms reduce the variety sold to the H-types, so that $\frac{N_{sd}^L}{N_{sd}^H} > \frac{N_{dd}^L}{N_{dd}^H} \forall s \neq d$. This occurs because a higher fixed causes the least productive firms: the ones that sell to only the H-type, to drop out of the exporting market, leaving the more productive firms (those that sell to both) unaffected. This effect is immediately obvious when comparing domestic and foreign varieties sold to

each type, equations (1.39) and (1.40),

$$\frac{N_{sd}^L}{N_{dd}^L} = \frac{J_s}{J_d} \left(\frac{w_d}{\tau_{sd} w_s} \right)^\gamma \quad (1.39)$$

$$\frac{N_{sd}^H}{N_{dd}^H} = \frac{J_s}{J_d} \left(\frac{w_d}{\tau_{sd} w_s} \right)^\gamma \left[1 - \left(\frac{f_{sd} z_{dd}^{H0} w_s}{q M_d^H w_d} \right)^{\frac{1}{2}} \right]^{2\gamma} \quad (1.40)$$

An increase in f_{sd} decreases the ratio of foreign H-only varieties, while leaving the L-variety ratio unchanged. The relative domestic-foreign quantity will, however, depend on the number of firms, tariffs and relative wages.

$$\frac{p_{sd}^B(z)}{p_{dd}^B(z)} = \left[\frac{\tau_{sd} w_s}{w_d} \right]^{\frac{1}{2}} \quad \frac{p_{sd}^H(z)}{p_{dd}^H(z)} = \left[\frac{\tau_{sd} w_s}{w_d} \right]^{\frac{1}{2}} \quad (1.41)$$

Comparing the price of foreign firms and domestic firms is simpler. While there will be a set of productivities in which foreign firms and domestic firms will not sell to the same market, the prices of goods for which they do are simply adjusted up by relative wages and tariffs - the standard result in monopolistic pricing models.

Consumption Inequality

Ultimately the interest is in the comparison of consumption of H- and L-types. The consumption generated from s enjoyed by consumers of each type in d can be shown to equal

$$C_{sd}^L = \frac{q J_s b_s^\gamma}{\gamma - \frac{1}{2}} \left(\frac{w_d}{w_s \tau_{sd}} \right)^\gamma \left[\gamma \frac{M_d^{\frac{1}{2}} S_d^{\frac{1}{2}} + (M_d^H Q_d^H)^{\frac{1}{2}}}{M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} S_d^{\frac{1}{2}}} - \left(\gamma - \frac{1}{2} \right) \right] z_{dd}^{HB^{-\gamma}} \quad (1.42)$$

$$C_{sd}^H = \frac{q J_s b_s^\gamma}{\gamma - \frac{1}{2}} \left(\frac{w_d}{w_s \tau_{sd}} \right)^\gamma \left[\gamma \frac{\left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right) Q_d^{H\frac{1}{2}}}{S_d^{\frac{1}{2}}} \frac{(M_d Q_d^H)^{\frac{1}{2}} - S_d^{\frac{1}{2}}}{S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}}} \right] z_{dd}^{HB^{-\gamma}} \quad (1.43)$$

$$+ \frac{q J_s b_s^\gamma}{\gamma - \frac{1}{2}} \left(\frac{w_d}{w_s \tau_{sd}} \right)^\gamma \left[\frac{\frac{1}{2} (M_d^H Q_d^H)^{\frac{1}{2}} + (\gamma - \frac{1}{2}) (w_s f_{sd})^{\frac{1}{2}}}{(M_d^H Q_d^H)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}} \left[1 - \left(\frac{w_s f_{sd}}{M_d^H Q_d^H} \right)^{\frac{1}{2}} \right]^{2\gamma} \right] z_{dd}^{H0^{-\gamma}}$$

We can compare the consumption ratio of L- and H-type consumers in equation (1.44) to understand how trade can change consumption inequality.

$$\frac{C_{sd}^H}{C_{sd}^L} = \frac{\gamma M_d^L \left[(M_d Q_d^H)^{\frac{1}{2}} - S_d^{\frac{1}{2}} \right] Q_d^{H\frac{1}{2}}}{\left[S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right] \left[\gamma M_d^{H\frac{1}{2}} \left[(M_d Q_d^H)^{\frac{1}{2}} - S_d^{\frac{1}{2}} \right] + \frac{1}{2} \left[M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} \right] S_d^{\frac{1}{2}} \right]} + \frac{\left[\frac{1}{2} (M_d^H Q_d^H)^{\frac{1}{2}} + (\gamma - \frac{1}{2}) (w_s f_{sd})^{\frac{1}{2}} \right] \left(M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} \right) S_d^{\frac{1}{2}} \left[1 - \left(\frac{w_s f_{sd}}{M_d^H Q_d^H} \right)^{\frac{1}{2}} \right]^{2\gamma} \left(\frac{z_{dd}^{HB}}{z_{dd}^{H0}} \right)^\gamma}{\left[(M_d^H Q_d^H)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}} \right] \left[\gamma M_d^{\frac{1}{2}} \left[S_d^{\frac{1}{2}} + (M_d^H Q_d^H)^{\frac{1}{2}} \right] - (\gamma - \frac{1}{2}) \left(M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} \right) S_d^{\frac{1}{2}} \right]} \quad (1.44)$$

Once again, the only source dependent variable is f_{sd} , so that if there is no fixed cost the *relative* quantity provided to each group is identical to that provided by the domestic country.

$$\frac{C_{sd}^L}{C_{dd}^L} = \frac{J_s b_s^\gamma z_{sd}^{HB^{-\gamma}}}{J_d b_d^\gamma z_{dd}^{HB^{-\gamma}}} = \frac{N_{sd}^L}{N_{dd}^L} = \frac{J_s b_s^\gamma}{J_d b_d^\gamma} \left(\frac{w_d}{\tau_{sd} w_s} \right)^\gamma \quad (1.45)$$

In comparing the relative quantities we find that L-type quantities follow the same considerations as L-type varieties: what matters is only the relative number of firms, wages, and tariffs. If $f_{sd} = 0$ then $\frac{C_{sd}^H}{C_{dd}^H} = \frac{J_s b_s^\gamma}{J_d b_d^\gamma} \left(\frac{w_d}{\tau_{sd} w_s} \right)^\gamma = \frac{C_{sd}^L}{C_{dd}^L}$. As before, a change in consumption inequality due a composition effect will occur if there is a fixed cost. Without a fixed cost, the relative quantities of H- and L-goods sourced from foreign countries is identical to domestic countries. Any composition effect arising from a fixed cost is caused by a decrease in a H-type goods.

1.3.2 Log Linearization

Given the nonlinearity of the model, log-linearization is required to understand the effects of trade on firm entry and exist and mark-ups. Let $\hat{x} = \log(x') - \log(x)$, where x denotes the original value, and x' denotes new value. \hat{x} therefore approximately denotes the percent change in variable. Log-linearizing the equations for variety we

obtain

$$\begin{aligned}
\hat{N}_{sd}^H &= \hat{J}_s - \gamma \hat{\tau}_{sd} + \gamma \hat{w}_d - \gamma \hat{z}_{dd}^{H0} - \frac{\gamma (M_d^H Q_d^H)^{\frac{1}{2}} \hat{w}_s}{(M_d^H Q_d^H)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}} - \frac{\gamma (f_{sd} w_s)^{\frac{1}{2}} [\hat{f}_{sd} - \hat{M}_d^H - \hat{Q}_d^H]}{(M_d^H Q_d^H)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}} \\
\hat{N}_{sd}^L &= \hat{J}_s - \gamma \hat{\tau}_{sd} + \gamma \hat{w}_d - \gamma \hat{z}_{dd}^{HB} - \gamma \hat{w}_s
\end{aligned} \tag{1.46}$$

Notice that if cutoffs and/or mark-ups faced by H and L types will have change differently, consumption inequality will change. For variety inequality to change the cutoffs faced by the two groups needs to change by differently.

$$\begin{aligned}
\hat{N}_{sd}^H - \hat{N}_{sd}^L &= \gamma \frac{\left(S_d^{\frac{1}{2}} + (M_d^H Q_d^H)^{\frac{1}{2}} \right)}{S_d^{\frac{1}{2}}} \left(\hat{Q}_d^L - \hat{Q}_d^H \right) \\
&\quad - \frac{\gamma (f_{sd} w_s)^{\frac{1}{2}}}{(M_d^H Q_d^H)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}} \left[\hat{w}_s + \hat{f}_{sd} - \hat{M}_d^H - \hat{Q}_d^H \right]
\end{aligned} \tag{1.47}$$

If equation (1.48) is positive, then the H-types gain more varieties than the L-types and variety inequality increases. For prices we obtain

$$\hat{p}_{sd}^H(z) = \frac{1}{2} \underbrace{(\hat{\tau}_{sd} + \hat{w}_s - \hat{z})}_{\text{Marginal Cost}} + \frac{1}{2} \underbrace{(\hat{Q}_d^H - \hat{q})}_{\text{Mark-Up}} \tag{1.48}$$

$$\begin{aligned}
\hat{p}_{sd}^B(z) &= \frac{1}{2} \underbrace{(\hat{\tau}_{sd} + \hat{w}_s - \hat{z})}_{\text{Marginal Cost}} \\
&\quad + \frac{1}{2} \underbrace{\left(\frac{M_d^H (Q_d^H - Q_d^L)}{S_d} \hat{M}_d^H - \frac{S_d - M_d Q_d^L}{S_d} \hat{M}_d + \frac{M_d^L Q_d^L}{S_d} \hat{Q}_d^L + \frac{M_d^H Q_d^H}{S_d} \hat{Q}_d^H - \hat{q} \right)}_{\text{Mark-Up}}
\end{aligned} \tag{1.49}$$

The difference in prices

$$\hat{p}_{sd}^B(z) - \hat{p}_{sd}^H(z) = \frac{1}{2} \left[\frac{M_d^H (Q_d^H - Q_d^L)}{S_d} (\hat{M}_d^H - \hat{M}_d) + \frac{S_d - M_d^H Q_d^H}{S_d} (\hat{Q}_d^L - \hat{Q}_d^H) \right]$$

If equation (1.50) is positive, then the mark-up on goods sold to both types change increases (decreases) by more than (less than) the mark-up on goods sold to the just the high types.

1.4 Numerical Example: Consumption Inequality

Parameter		Value
N	# of countries	2
γ	Pareto Shape	4.5
M_d	Population	1
M_d^H	20% of population	0.2
b_s	Minimum Productivity	1
θ_d^H	Effective labor of H-type	2
θ_d^L	Effective labor of L-type	1.5
f_c	Fixed entry cost	1e-3
f_x	Fixed cost of export	1e-4
q	Non homothetic parameter	2

Table 1.1: Parameters used in Numerical Exercise

In this section I examine the effect that a change in trade cost has on the relative variety, quantity consumed and utilities of consumer groups, H and L, in a 2 country trade example. I then compare the model's predictions regarding each group's gain with different kinds trade partners. In all exercises the destination country has the parameters indicated in table 1.4, which imply an upper 20% income share of 25%.

1.4.1 Changing Trade Barriers

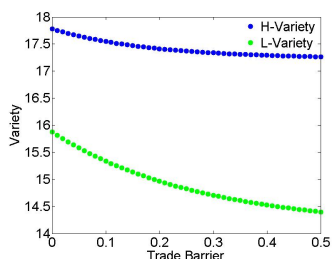


Figure 1.7: Variety

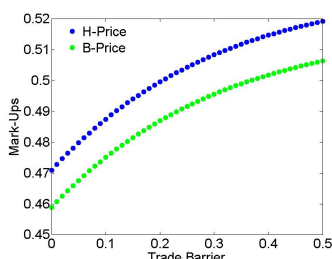


Figure 1.8: Mark-Ups

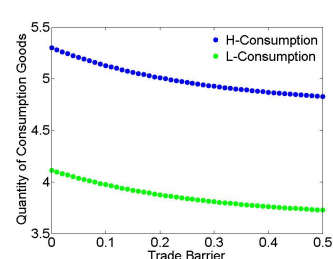
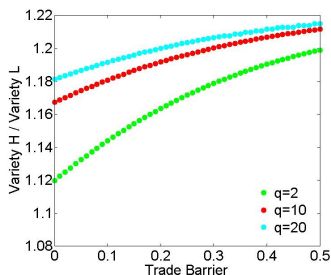
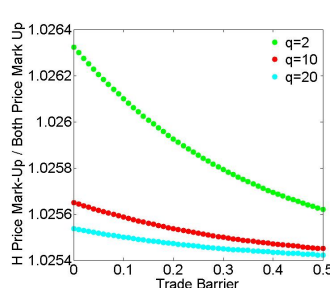
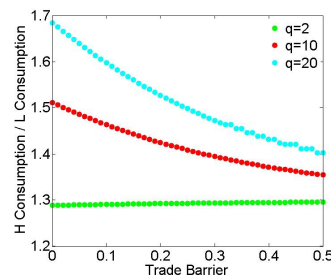


Figure 1.9: Consumption

In the following exercise I change trade barrier ($\tau_{sd} - 1$) from zero (free trade) to 0.5 (a 50% cost for each good imported) for two identical countries to examine the effect of trade cost on consumption inequality.

Figure 1.7 shows the varieties purchased by each type of consumer with respect to trade cost. For both consumers the variety, N_d^k , decreases with higher trade barriers. Figure 1.8 shows the mark-up component of prices indicated in equations (1.15) and (1.17). As the variety decreases, the mark-ups charged by the remaining firms increase for all firms, resulting in an increase in prices of the remaining goods - in addition to the increase that results from the increase of marginal cost. Finally, figure 1.9 the quantity of consumption goods, C_d^k purchased by consumers at different trade costs. For both consumers C_d^k increases with lower trade barriers. Figure 1.9 shows the quantity of consumption goods, C_d^k purchased by consumers at different trade costs. For both consumers C_d^k the decrease in variety and increase in prices combine to decrease aggregate consumption.

Figure 1.10: $\frac{N_d^H}{N_d^L}$ Figure 1.11: $Q_d^H \frac{M_d}{S_d}$ Figure 1.12: $\frac{C_d^H}{C_d^L}$

The focus of this paper however, is on relative number of goods purchased. In a model with homothetic preferences the ratio of varieties would be constant and equal to 1, $\frac{N_d^H}{N_d^L} = 1$, as all consumers will buy all the goods available to them on the market. With non-homothetic preferences it is no longer true that this ratio is equal to one, yet it is still possible that the ratio is constant. Figure 1.10 shows that this is not the case. As the trade cost increases, the ratio of goods the H consumer relative to that that both consume increases. A decrease in trade cost therefore decreases variety inequality. Figure (1.11) shows that the markup component on the price of goods sold to both consumers increases by slightly more than the price of goods sold to just the rich - but the increase is only slight. Figure 1.12 shows that a decrease in trade cost decreases quantity of consumption goods purchased by the H consumer relative to that purchased by the L-consumer increases. Figures (1.10)-(1.12) also show the change for different values of the non-homothetic parameter, q . As the non-homothetic parameters increases, the inequality effects of changing the trading decreases.

1.4.2 Changing Source Country Income Inequality

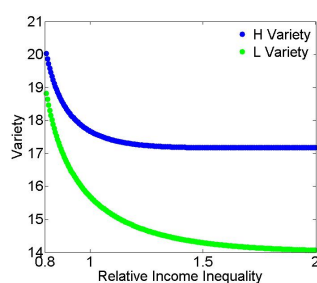


Figure 1.13: Variety

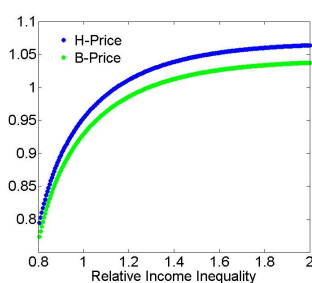


Figure 1.14: Mark-Ups

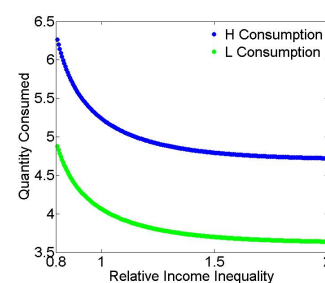
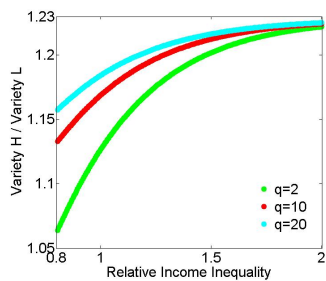
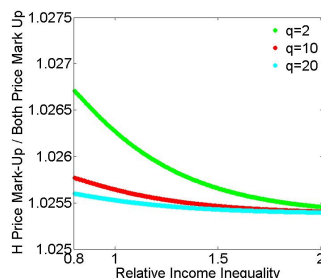
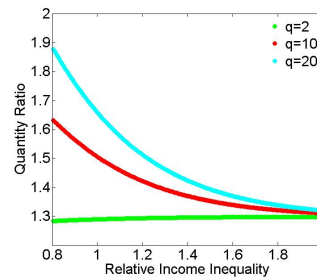


Figure 1.15: Consumption

Figure 1.16: $\frac{N_d^H}{N_d^L}$ Figure 1.17: $Q_d^H \frac{M_d}{S_d}$ Figure 1.18: $\frac{C_d^H}{C_d^L}$

In this exercise I study the effect of a change in income inequality of a source country. To accomplish this I keep all destination parameters fixed, while changing the productivity each labor type in the source country to keep aggregate labor supply constant and equal to the destination country, $M_d^H \theta_d^H + M_d^L \theta_d^L = M_s^H \theta_s^H + M_s^L \theta_s^L$. Income inequality, measured as the income share of the upper 20th percentile, increases from 20% (equality) to 50%. In all figures, the vertical line shows the income inequality in the destination country (fixed and equal to 25%).

As inequality in the source country increases the variety enjoyed by both groups in the destination country decreases. This decrease is less for the H-type and variety inequality increases. Quantity consumed decreases with increases inequality, while consumption inequality decreases. In summary, as the inequality in the source country increases, the consumption inequality decreases.

1.5 Conclusion

In this paper I have studied the effect of trade on consumption inequality, using the channel of market and price discrimination. I show that even in the absence of income changes, trade liberalization changes the optimal pricing decision of firms which may cause changes in consumption inequality. I find that for identical countries trade liberalization decreases consumption inequality. Furthermore, I find that

trade with a country with different income inequality decreases consumption inequality.

Chapter 2

Learning About Growth

2.1 Introduction

We¹ introduce a model in which agents learn the composition of growth of TFP. Agents don't know how much of the observed growth is permanent or transitory. The permanent income hypothesis implies agents who believe growth is mostly transitory follow different consumption and investment paths than those who believe shocks are permanent. In particular, agents in a country in which, historically, the growth rate is mostly permanent would believe most of the currently observed growth is permanent. This implies that they would decrease their savings (foreign bond holdings in an SOE economy) in response to an increase in growth (a positive relationship between growth and the capital account). Agents in countries in which growth rates are highly volatile however, would view current growth to be mostly transitory and would increase their savings. This results in a negative relationship between growth and the capital account. However, even in countries with a historically highly volatile growth rates, should TFP continued to grow (as would be the case in an emerging economy), agents will update their beliefs about the composition of growth to reflect a permanent growth of TFP, endogenously leading to behavior

¹This chapter is co-authored with Andy Glover

that matches stable growth regime.

[Aguiar and Gopinath, 2007] show most of the volatility at the business cycle level of Mexico’s TFP comes from transitory shocks to the trend growth (i.e. volatile growth), whereas for Canada most of the volatility at the business cycle level comes from transitory shocks around a stable growth trend. Instead of looking only at Canada and Mexico we generalize our sample to Non-OECD and OECD countries, and look at long-run TFP growth rates instead of business cycle level. We use GMM estimation on TFP data to obtain the relative contributions of the permanent and transitory components in our sample of countries, and find that non-OECD countries experience volatile long run trend growth, while OECD countries have stable growth rates. We do not intend to address the causes for the differences in volatility of TFP growth rates in this paper.

According to standard neoclassical growth theory, a country that experiences relative productivity growth should receive an inflow of capital to equate marginal productivity and an inflow of savings for consumption smoothing - a positive correlation between productivity growth and capital inflows. [Gourinchas and Jeanne, 2009] document that amongst developing countries this correlation is not strongly positive - and may even be negative.² This has been termed “the allocation puzzle” - the *allocation* of the flows amongst DVC’s is inconsistent with theory.³

This project introduces the method of kalman filtering, used within other branches of economics (i.e. [Güvenen, 2007]), to models of trade and development. It is important to notice that this model induces different behavior in developing countries not because of an exogenous parameter (i.e. corruption), but because of easily measurable difference in the historical TFP processes (which informs the initial beliefs). This means that this model is equally applicable to both undeveloped and developed

²This pattern remains even after adjusting for foreign aid inflows.[Rajan et al., 2007] investigate further and show that the puzzle is driven by savings outflows exceeding any additional investment inflows after a growth episode.

³It is different from the Lucas puzzle, which states that the *level* of flows from developed (DC) to developing (DVC) countries is too low

countries. [Boz et al., 2008] imbed kalman filtering in the Aguir-Gopinath Mexico model and find that the model can reproduce some of the business-cycle statistics without the need for excessive volatility in the transitory shocks.⁴

A few other papers have proposed solutions to the allocation puzzle. [Sandri, 2009] models the behavior of entrepreneurs facing incomplete financial markets and risky investment; poor financial markets lead to entrepreneurs to increase saving to self-finance investment for firm growth, causing capital outflows. [Buera and Shin, 2009] also consider entrepreneurial funding, but through a different mechanism: Heterogeneous agents have the choice of being either an entrepreneur or a worker; capital outflows follow liberalization due to the rapid disinvestment of less productive entrepreneurs and slow capital absorption by new entrepreneurs due to borrowing constraints. [Aghion et al., 2009] describes a model in which local firms familiar with domestic monitoring technology accumulate assets to attract foreign investors who are familiar with frontier technology and are willing to fund projects co-financed and monitored by local firms.

In contrast with this strand of the literature, we do not require that the institutions within the developing countries be significantly worse than those of developed countries, though adding these sort of frictions would amplify our results. Even with the same financial institutions, we would still see capital outflows due to the uncertainty of TFP growth. While we do not intend to address the cause of the difference in TFP processes in this paper, it suggests a potentially fertile line of future research. Additionally, in our model TFP growth is exogenous, and causes the savings outflows, while in the above papers savings outflows causes TFP growth. Adding any of the above mechanisms would only strengthen our results.

In section 2 we introduce our model. In section 3 we provide analytical intuition for a simplified version of our model that shows how kalman filtering works in this class of models, and discuss limitations. In section 4 we estimate the TFP process

⁴Specifically, the model predicts a higher variability of consumption relative to output and a strongly negative correlation between the trade balance and output

for developed and developing countries, and in section 5 we discuss how to derive the allocation puzzle from the data and provide an analysis of the time series evolution of the correlation between capital flows and growth - an aspect of the puzzle that has not been studied before. In section 6 we compare numerical results from our model to the data, and section 7 concludes.

2.2 Model

Our model is a small open economy (SOE) with neoclassical production. Time is discrete and infinite. The only shock is to labor augmenting TFP, where agents have uncertainty about the composition of a given shock. We restrict uncertainty to the first T periods, after which every country grows at the same, known rate without any further shocks.

2.2.1 TFP Process

We specify a process of TFP in which the average growth rate of TFP over the first T periods is heterogeneous across countries. At $t = 0$, each country draws an average, unobserved growth rate, denoted η^i . A country's actual growth rate deviates from η^i due to a unobserved shock which we denote by $\epsilon_t^i \sim N(0, \sigma_\epsilon^2)$, where σ_ϵ^2 is known.

$$A_{t+1}^i = e^{\eta^i + \epsilon_{t+1}^i} A_t^i$$

After T periods, where T is known, every country's TFP grows at the same uniform rate γ . Thus for $t \geq T$:

$$A_{t+1}^i = \gamma^{T-t} A_T^i$$

For future reference, note that the process for the growth rate is simply:

$$g_t^i = \eta^i + \epsilon_t^i$$

Denote $\hat{x} = \log(x)$ and write the TFP process for $t=0, \dots, T-1$ in logs as:

$$\hat{A}_{t+1}^i = (\eta^i + \epsilon_{t+1}^i) + \hat{A}_t^i$$

2.2.2 Initial Conditions and Information

We set initial conditions so that all countries are ex-ante identical, hence $k_0^i = k_0^j$ and $A_0^i = A_0^j$, and we normalize $k_0^i = 1$ and $A_0^i = 1$. We assume that countries draw their average growth rate, η^i , and their initial belief regarding the average growth rate, $\tilde{\mu}_0^i$ from:

$$\begin{bmatrix} \eta^i \\ \tilde{\mu}_0^i \end{bmatrix} \sim N \left(\begin{bmatrix} \mu \\ \mu \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right) \quad (2.1)$$

where we assume that countries know the values of μ, σ^2, ρ but not η^i . Making belief dogmatic, keeping $\tilde{\mu}_0^i$ constant, corresponds to the assumption that there is no relationship between initial realized growth rates and beliefs. This implies that a country cannot learn about the true correlation through cross-sectional information.

2.2.3 Learning Process

A country faces a signal extraction problem in determining how much growth is due to their mean growth rate and how much is due to transitory shocks. Since the initial belief, mean growth rates and their errors are normal, the Kalman Filter can be applied.

For $0 < t \leq T$ agents are learning about the TFP growth rate - trying to separate the two unobserved components (η_t^i and ϵ^i) using the sequence of observed growth rates $\{g_n^i\}_{n=0}^{t-1}$ to make a prediction about next periods TFP. Let $(\tilde{\mu}_t^i, \tilde{\sigma}_t^{i^2})$ be the forecast mean and variance of growth before the realization of g_t^i . We get the recursive

filtering equations:

$$\underbrace{\tilde{\mu}_{t+1}^i}_{\text{New Prediction}} = \underbrace{\tilde{\mu}_t^i}_{\text{Current Prediction}} + \underbrace{\kappa_t^i(g_t^i - \tilde{\mu}_t^i)}_{\text{Weighted realized error in current prediction}} \quad (2.2)$$

$$\tilde{\sigma}_{t+1}^{i^2} = (1 - \kappa_t^i)\tilde{\sigma}_t^{i^2} \quad (2.3)$$

$$\text{where the Kalman gain is } \kappa_t^i \equiv \frac{1}{1 + \frac{\sigma_\epsilon^2}{\tilde{\sigma}_t^{i^2}}} \quad (2.4)$$

Note the role of the Kalman gain κ . If agents are very sure about the growth rate estimate ($\tilde{\sigma}_t^i \approx 0$) or if the variance of the transitory shocks (σ_ϵ) is very large then $\kappa_t^i \approx 0$ and agents will make only very small adjustments in their predictions if today realizations vary greatly from their predictions.

Using the definition of g_t^i , we can relate this equation back to TFP. The mean and variance of the forecast of $t + 1$ log TFP conditional on information at time t , $(A_t^i, g_t^i, \tilde{\mu}_t^i, \tilde{\sigma}_t^i)$, are:

$$\tilde{\mathbb{E}}_t \hat{A}_{t+1}^i = \tilde{\mu}_{t+1}^i + \hat{A}_t^i \quad (2.5)$$

$$\tilde{\mathbb{V}}_t \hat{A}_{t+1}^i = \tilde{\sigma}_{t+1}^{i^2} + \sigma_\epsilon^2 \quad (2.6)$$

Thus the forecast of the level of TFP, A_{t+1}^i , conditional on information available at time t is log-normal. Using equations 2.5 and 2.6 we can summarize the forecasting distribution with the first two moments:

$$\begin{aligned} \tilde{\mathbb{E}}_t A_{t+1}^i &= A_t^i \exp\left(\tilde{\mu}_t^i + \kappa_t^i(g_t^i - \tilde{\mu}_t^i) + \frac{1}{2}(\tilde{\sigma}_{t+1}^{i^2} + \sigma_\epsilon^2)\right) \\ \tilde{\mathbb{V}}_t A_{t+1}^i &= \left(e^{\tilde{\sigma}_{t+1}^{i^2} + \sigma_\epsilon^2} - 1\right) \exp\left(2(\tilde{\mu}_t^i + \kappa_t^i(g_t^i - \tilde{\mu}_t^i)) + (\tilde{\sigma}_{t+1}^{i^2} + \sigma_\epsilon^2)\right) (A_t^i)^2 \end{aligned}$$

For future reference, it will be useful to note that a s period ahead forecast of log TFP conditional on information at time t has mean and variance:

$$\tilde{\mathbb{E}}_t \hat{A}_{t+s}^i = \hat{A}_t^i + s \tilde{\mu}_{t+1}^i \quad (2.7)$$

$$\tilde{\mathbb{V}}_t \hat{A}_{t+s}^i = s^2 \tilde{\sigma}_t^2 + s \sigma_\epsilon^2 \quad (2.8)$$

Hence the forecast mean of the level of TFP is simply:

$$\tilde{\mathbb{E}}_t A_{t+s}^i = A_t^i \exp \left(s \tilde{\mu}_{t+1} + \frac{s^2 \tilde{\sigma}_t^2 + s \sigma_\epsilon^2}{2} \right)$$

2.2.4 Consumer Problem

For what follows we suppress the country specific superscripts. We assume a small open economy with immobile capital. For $t \geq T$ no further learning takes place and the problem takes the form of a standard growth model with a growth rate of γ :

$$V_T(A, b, k) = \max_{c, b', k'} u(c) + \beta V_T(\gamma A, b', k') \quad (2.9)$$

$$c + b' + k' = A^{1-\alpha} k^\alpha + (1 - \delta)k + (1 + r)b \quad (2.10)$$

while for $t < T$ the agents are learning η

$$V_t(A, b, k, g, \tilde{\mu}, \tilde{\sigma}) = \max_{c, b', k'} u(c) + \beta \int V_{t+1}(A', b', k', \hat{A}' - \hat{A}, \tilde{\mu}', \tilde{\sigma}') dF(A'; \tilde{\mu}', \tilde{\sigma}') \quad (2.12)$$

s.t.

$$c + b' + k' = A^{1-\alpha} k^\alpha + (1 - \delta)k + (1 + r)b \quad (2.13)$$

where $\tilde{\mu}'$ and $\tilde{\sigma}'$ are given by equations (2.2)-(2.4)

In our model, b_t is the capital flows into and out of the country. $b_t > 0$ indicates savings, and therefore a capital outflow, while $b_t < 0$ indicates borrowing - a capital inflow. The allocation puzzle couched in the terms of our model is therefore $\text{corr}(b_t, g_t) \geq 0$ - growth does not induce agents to borrow.

2.3 Analysis

2.3.1 Solution After Uncertainty is Realized

Proposition 2.3.1. *Assume the utility function has the two properties that $u(\gamma^k x) = h(\gamma)^k u(x)$ and $u'(\gamma^k x) = g(\gamma)^k u'(x)$. If $\beta h(\gamma) < 1$ and $1 = \beta(1+r)g(\gamma)$, then there is a real valued function $V_T(A_T, k_T, b_T)$ that solves the recursive problem for $t \geq T$ and there are some functions:*

$$\bar{c}(A_T, b_T, k_T), \bar{k}(A_T, b_T, k_T), \bar{b}(A_T, b_T, k_T)$$

such that, for $t \geq T$:

- $c_t = \gamma^{t-T} \bar{c}(A_T, b_T, k_T)$
- $k_{t+1} = \gamma^{t+1-T} \bar{k}(A_T, b_T, k_T)$
- $b_{t+1} = \gamma^{t+1-T} \bar{b}(A_T, b_T, k_T)$

To see why proposition 2.3.1 is true, note that the first order conditions for $t \geq T$ are given by:

$$u'(c_t) = \beta u'(c_{t+1}) ((\gamma^{t+1} A_T)^{1-\alpha} k_{t+1}^\alpha + 1 - \delta) \quad (2.14)$$

$$u'(c_t) = \beta u'(c_{t+1})(1+r) \quad (2.15)$$

Under our assumptions, we can replace each variable with the appropriate version from the theorem to get:

$$1 = \beta g(\gamma) (A_T^{1-\alpha} \bar{k}^\alpha + 1 - \delta) \quad (2.16)$$

$$1 = \beta g(\gamma)(1+r) \quad (2.17)$$

The importance of the assumption on $g(x)$ can be seen immediately from the first order condition on bonds, equation(2.17). Once we have verified that these conditions

can be met we can compute the value as:

$$V_T(A_T, k_T, b_T) = \frac{1}{1 - \beta h(\gamma)} u(\bar{c}(A_T, k_T, b_T))$$

which is real valued so long as $\beta h(\gamma) < 1$.

2.3.2 Analytical Example

In general, learning makes the problem analytically intractable before uncertainty is resolved. However, if we make the assumption that utility is quadratic and time variant, and abstract from production, an analytical solution is possible.

Proposition 2.3.2. *Assume that $u_t(x) = F_1\gamma^t x - F_2x^2$, $\alpha = 0$, $1 = \beta\gamma(1+r)$, and $1 > \beta\gamma^2$. Then the policy functions that solve the maximization problem of a country are given by: For $t \geq T$:*

$$c_t(A_T, b_T) = \gamma^{t-T} (A_T + (1+r-\gamma)b_T) \quad (2.18)$$

$$b_{t+1}(A_T, b_T) = \gamma^{t+1-T} b_T \quad (2.19)$$

$$(2.20)$$

For $t < T$:

$$c_{T-t} = \frac{1+r-\gamma}{1+r} A_{T-t} + (1+r-\gamma)b_{T-t} + \frac{1+r-\gamma}{1+r} \tilde{\mathbb{E}}_{T-t} \sum_{j=1}^{t-1} \frac{A_{T-t+j}}{(1+r)^j} + \tilde{\mathbb{E}}_{T-t} \frac{A_T}{(1+r)^t}$$

$$b_{T-t+1} = \frac{\gamma}{1+r} A_{T-t} + \gamma b_{T-t} - \frac{1+r-\gamma}{1+r} \tilde{\mathbb{E}}_{T-t} \sum_{j=1}^{t-1} \frac{A_{T-t+j}}{(1+r)^j} - \tilde{\mathbb{E}}_{T-t} \frac{A_T}{(1+r)^t}$$

The formulas for \bar{c} and \bar{b} are found by using the budget constraint for $t > T$ and for $t = T$. Once the endowment process is deterministic, the country simply consumes its endowment plus the growth adjusted savings at T . Given this consumption policy, the sequential problem (in appendix for reference), is used to

find Euler Equations for $t < T$ and we conduct backwards iteration to get the expression for c_{T-t} . The consumption function can be understood as a growth and belief-augmented version of what would occur with quadratic preferences under uncertainty, in which consumption at t is just the present value of expected future income.

It is instructive to consider the bond policy function. Define $B_{t+1} = \frac{b_{t+1} - \gamma b_t}{A_t}$. Then the covariance between B_{t+1} and g_t will clearly take the opposite sign as the correlation between the beliefs about growth and true growth rates. Therefore once this covariance is positive (and it must be eventually, no matter how negatively beliefs and reality are assumed to initially covary), the savings and growth must become negatively correlated as in the neoclassical model. We conjecture, but cannot prove, that the correlation is monotonic, in the sense that with each passing year the model would generate a correlation between savings and growth that is closer to that of the full information version. In absence of a general proof, we provide an example in which the covariance between beliefs and reality changes sign after one period, as does the covariance between savings and growth.

Proposition 2.3.3. *Continue making the assumptions from propositions 2.3.1 and 2.3.2. Also assume the alternate TFP process of $A_{t+1}^i = (\eta^i + \epsilon_{t+1}^i)A_t^i$ and put $T = 2$. Assume that $\rho = 0$ Then:*

$$COV(b_1, A_1) = -\frac{1+r-\gamma}{(1+r)^2} \rho \sigma^2 = 0$$

but

$$COV(B_2, \frac{A_2}{A_1}) = -\frac{\sigma^2}{1+r} (\kappa_0 + (1-\kappa_0)\rho) < 0$$

This is just algebra and computing covariances, but it is illustrative of the prior point. Note that the correlations themselves are much more difficult to compute, which is why we have found conditions where the covariance changes signs so that we can say something definite about the correlations, in particular that is possible

to generate a zero-correlation between growth and capital flows in the short run, but in the long run the correlation will become positive. In section 5.3, we study the time series behaviour of the allocation puzzle and show that this is consistent with the data.

2.4 TFP Estimation

2.4.1 Data Methodology

We set initial capital stock to $k_0 = I_0/(g_i + \delta)$ where g_i is growth rate of real investment for the first 10 years of data, and I_0 is the initial level of real investment from version 6.1 of the Penn World Tables (PWT [Heston et al., 2002]).⁵ Future capital stock is constructed using $k_t = (1 - \delta)k_{t-1} + I_t$, where $\delta = 0.06$.

This method of calculating initial capital stock comes from [Caselli, 2004]. On a balanced growth path, $\bar{x} = (\delta + \gamma)\bar{k}$, where δ is capital depreciation and γ is the growth rate. If a country is on a balanced growth path and we have information on investment data at date T, we can calculate k_T using $x_T = (\delta + g_i)k_T$, where g_i is the mean growth rate which we take to be the growth rate of real investment for the next ten years. Clearly, this requires that k_0 and the 10 years of investment data we use to calculate g_i do not overlap the dates of interest, as the learning mechanism implies that the country is no longer on the balanced growth path. As can be seen in the appendix, k_0 starts in 1950 for most countries, (i.e. we assume balanced growth path from 1950-1960), well outside the date range we are considering for capital flows. [Caselli, 2004] examines the affect of different ways of estimating initial capital stock and concludes that “improv[ements to] the initial capital stock estimates [by using different methods of construction] is not likely to lead to major revisions to the baseline result”.

⁵PWT 7.0 is slated for release in the last quarter of 2009. Among other things, it will include an update to ICP 2005, the pricing survey data that comprises the bulk of the data that PWT uses to construct the international dollar estimate. We will update all our data work to PWT 7.0 once it is released.

We construct labor-augmenting TFP using RGDP in Laspeyres chained International Dollars and population from PWT, fraction of population aged 15-64 (“working age”) from WDI, and the constructed series for capital stock:

$$A_t = \left(\frac{y_t}{k_t^\alpha} \right)^{1/(1-\alpha)} \frac{1}{L_t}$$

where $\alpha = 0.3$ and L_T , the size of the workforce, is population multiplied by fraction of population that is working age. The growth of TFP between $t = 0$ and T is given by

$$\Delta A_0^T = \left(\frac{A_T}{A_0} \right)^{1/T} - 1 \quad (2.21)$$

2.4.2 TFP Process Estimation

We estimate σ_ϵ^2 , μ_η , σ_η^2 for growth process using our constructed TFP process, A_t . Let $\hat{A}_t = \log(A_t)$. Then $\Delta \hat{A}_t = \hat{A}_t - \hat{A}_{t-1} = \eta^i + \epsilon_t^i$ where η^i is the mean growth rate for country i and ϵ_t^i is the transitory shock. We choose our parameters to match the following moments:

Moment	Equation	DVC Estimation	DC Estimation
μ	$\frac{1}{N} \sum_{i=1}^N (\eta^i)$	0.020	0.012
σ	$\frac{1}{N} \sum_{i=1}^N (\eta^i - \mu)^2$	0.003	0.0001
σ_ϵ	$\frac{1}{T} \sum_{t=0}^T \frac{1}{N} \sum_{i=1}^N (\Delta \hat{A}_t - \eta^i)^2$	0.008	0.001

Table 2.1: Parameters of TFP Process

Using these equations and our constructed TFP series we estimate the following moments.

σ for developed countries is one-sixth than that of developing countries, showing developed countries growth rates are more similar to each other than what we observe for developing countries. σ_ϵ is more than four times larger for developing countries, so that developing countries have growth rates that are more than four times as volatile as that of developed countries. We interpret this as developed countries having stable growth trend compared to developing countries.

2.5 The Allocation Puzzle

2.5.1 Constructing Change in Net External Position

We use the Net International Investment Position (NIIP) and cumulated errors and omissions (CEO) from [Lane and Milesi-Ferretti, 2007] to construct the net external position (a stock variable) in year t ⁶

$$NEP_t = CEO_t - NIIP_t \quad (2.22)$$

Next, we need to convert from current US dollars to constant international dollars. We use the PPP adjustment method in [Hsieh and Klenow, 2007b]. We divide by the PPP Adjustor, Q_t ,

$$Q_t = P_t \frac{CGDP_t}{RGDP_t} \quad (2.23)$$

where $CGDP_t$ ($RGDP_t$) is GDP in current (constant) international dollars and P_t is the price of investment goods in PWT. As described in GJ, the PPP adjustment method may decrease our estimates of the volume capital flows, but will not cause a change in signs. Our methodology for constructing the change in net external position is slightly different than GJ. While GJ adds the sum of the (PPP adjusted) yearly capital flows from the IFS and CEO_t from LMF to NEP_{1980} to obtain an

⁶The sign convention is tricky: NIIP uses current account sign convention, so a negative NIIP indicates foreign capital inflow. Following standard practice, cumulative errors and omissions (CEO) are considered unreported capital flows and carries capital account sign conventions - foreign capital inflow in CEO is a positive entry. NEP_t is capital account units.

estimate for the net external position in 2000, we repeat the procedure in equation (2.22). While these two approaches are theoretically equivalent, there is a difference in the way the data is constructed: Lanes-Milesi-Feretti take valuation effects (due to asset price and currency movements) into account, while the data from the IFS does not. Given that our primary motivator is that developing countries face more uncertainty - which includes behavior of asset prices and currency movements - taking valuation effects into account is more consistent with our model. The change in net external position from $t=0$ to T is

$$\Delta NEP_0^T = \frac{NEP_T}{Q_T} - \frac{NEP_0}{Q_0} \quad (2.24)$$

If $\Delta NEP_0^T < 0$ then, on net, capital has left the country, if $\Delta NEP_0^T > 0$, capital has entered the country. Finally, we adjust for country size for dividing by initial RGDP.

2.5.2 The Allocation Puzzle

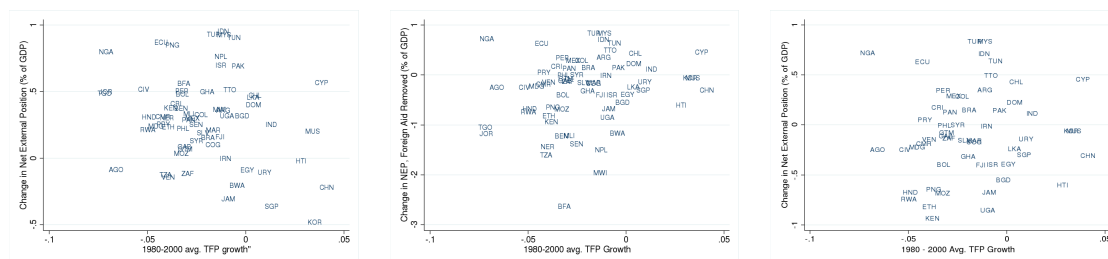


Figure 2.1: 1980-2000 avg. TFP growth: (L) All countries, (M) Foreign Aid Excluded, (R) No extreme aid

To establish the allocation puzzle, we take $\text{corr}(\frac{\Delta NCF_0^T}{Y_0}, \Delta \text{TFP})$ across the 66 countries in our sample for the years 1980-2000. The year 1980 is chosen as a result of [Chinn and Ito, 2008], who find that countries in our sample had completed their financial liberalization by 1980. This means that the capital outflows we measure aren't caused by the relaxation of international capital mobility constraints. We

find that the correlation is -0.2311, with a significance level of 0.0620. This negative correlation between capital flows and TFP growth is the allocation puzzle.

Foreign financial aid is included in capital flows, and this may skew results. We use net disbursements (ND_t)⁷ from the UN Development Assistance Committee - Overseas Development Assistance to construct net foreign aid inflows, ΔB_0^T .

$$\Delta B_0^T = \sum_{t=0}^{T-1} \frac{ND_t}{Q_t} \quad (2.25)$$

The change in capital flows adjusted for foreign aid becomes:

$$\Delta D_0^T = \Delta NCF_0^T - \Delta B_0^T \quad (2.26)$$

where we assumed that all foreign aid remains within the country. Since $\Delta B_0^T > 0$ for all countries in our sample (all countries are, on net, recipients of foreign aid) equation (2.26) clearly shows that $\Delta D_0^T < \Delta NCF_0^T$. In other words, by excluding foreign aid implies we are overestimating capital outflows. Furthermore, within our sample of countries the correlation of 1980 - 2000 avg. TFP growth and net foreign aid inflows is negative and significant⁸, so the fastest growing countries receive the least foreign aid inflows. The combination of these effect implies that our foreign aid adjusted numbers are strongly biased *against* the correlation puzzle. However, when we remove foreign aid, we find a positive but insignificant correlation of 0.2025 (0.1030). This positive correlation arises from the assumption that all aid remains within the country, the resulting overestimation of capital outflows effects mostly the slow growing economies.

Examination of figure 2 reveals that using this method of adjusting foreign aid some countries experienced net capital outflows greater than their 1980 GDP. If we drop

⁷Net disbursements are defined in the data as total grants + concessional development loans - principal repayments on loans

⁸We find a correlation coefficient of -0.3137, with a significance level of 0.0092, or significance at 1%.

all countries for which net capital outflows between 1980 and 2000 exceeds their 1980's GDP (i.e. $\frac{\Delta D_0^T}{Y_{1980}} < -1$) we find a correlation of 0.0799 (0.5622). We will refer to these as extreme aid countries. Extreme aid countries are Botswana, Benin, Burkina Faso, Ethiopia, Jordan, Kenya, Malawi, Mali, Nepal, Niger, Papua New Guinea, Senegal, Tanzania, Togo and Uganda.

	Non Aid-Adjusted	Aid Adjusted	Drop Extreme Aid
1980-2000	-0.2311	0.2025	0.0799
	(0.0620)	(0.1030)	(0.5622)

Table 2.2: Allocation Puzzle

Our model has strong implications for the evolution of the correlation of TFP growth and capital flows through time, a dimension of the allocation puzzle which has not been studied before. Intuitively, our model predicts that as time passes agents learn the true growth rate, therefore we should observe the correlation between TFP growth and capital outflows become less negative.

We choose 10 years as our smallest window to avoid business cycle effects and compare correlations from 1980-1990 and 1980-2000. We observe an increase in the correlation from -0.2607 to -0.2311. While each individual correlation is still negative, the correlation becomes more positive as we increase the length of time. Dropping Jordan (JOR) and Congo (COG), potentially biasing outliers, changes the 1980-1990 correlation to -0.2451 (0.0509) and the 1980-2000 correlation to -0.2193 (0.0817), so that our result still holds.

	1980-1990	1980-2000
All countries	-0.2607	-0.2311
	(0.0345)	(0.0620)
Aid Adjusted	-0.0405	0.2025
	(0.7506)	(0.1030)

Table 2.3: Allocation puzzle in a 10 year and 20 year window

2.6 Numerical Results

In this section we simulate analytical version of the model analyzed in the previously. Using the consumption and savings equations derived under quadratic utility in the endowment economy, we compute the time series of bond holdings. We follow Gourinchas and Jeanne by putting $\beta = 0.96$, which along with our estimate for the growth rate of developed countries implies that $r = 2.98\%$. We compute the correlation of the endogenous variables of interest over a large cross section of countries. We are able to match the correlation between debt and growth over a twenty year period, and find that the correlation implied over the first ten years is very close (but slightly higher) than in the data. While we are forced to impose an extremely unrealistic initial correlation between the mean beliefs and reality of $\rho = -0.602$, we see some evidence that learning is a relevant friction.

	1980-1990	1980-2000
Data	-0.261	-0.231
Model	-0.274	-0.231

Table 2.4: Simulation results

2.7 Time Series Behavior

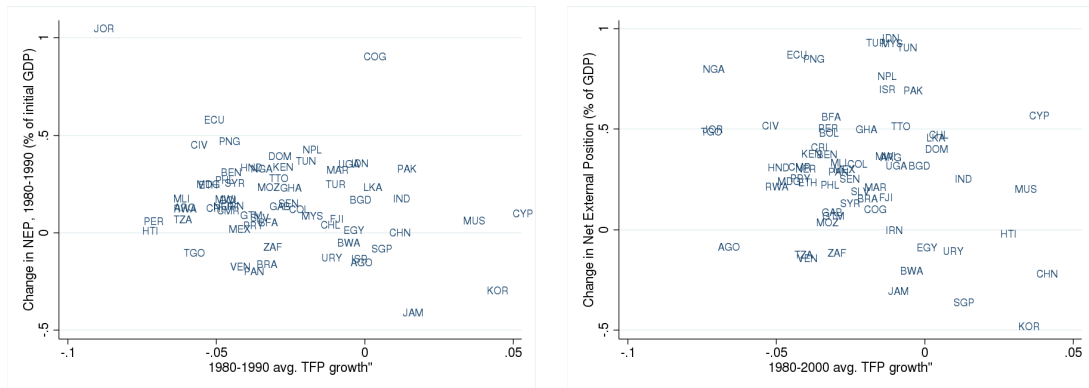


Figure 2.2: Allocation puzzle: 10 year to 20 year window

Given our model, a theory about learning TFP growth, 1980 may seem arbitrary year to use as $t=0$. To address this concern we redo the data work using three different events as our initial dates: First, we use the date of the beginning of a growth acceleration episode (12 countries). Next, we use as our initial date the year of a stable regime change (16 countries). Finally, we consider the date of joining WTO as our initial year (9 countries). For each of these three experiments we find the same behaviour: There is a negative correlation between inflows and TFP growth for the first five years after the event, which becomes less negative (and in some cases positive) as we increase the length of time to 10 years after the event.⁹ It is also noteworthy all of these occurrences, with the exception of the regime change with foreign aid included, seem to trigger a capital outflow, as is evidenced by the larger negative correlation we find when we compare net capital flows for the first five years after the event (T to $T+5$) to the net capital flows for the five years before the event ($T-5$ to T). Unfortunately, we also find that almost all of these correlations are not significant, with or without foreign aid. This insignificance is almost certainly due to the small sample sizes, and we hope that when we update to PWT 7.0 (which extends the data a further 8 years) the increase in sample size will result in significance for these correlations. Figures associated with each of the reported correlations are available in the appendix.

First, we use as our initial year is the beginning of a growth acceleration episode as identified by [Hausmann et al., 2005], in which a growth acceleration episode is identified as a date t such that the growth rate of real gdp per capita from t to $t+n$ ($g_{t,t+n}$, $n=[0,1,\dots,7]$) satisfies the following three criteria

1. $g_{t,t+n} \geq 3.5\%$
2. $\Delta g_{t-n,t+n} \geq 0.2\%$

⁹We would have preferred to make the smallest window 10 years and then scale up to 20 years, but doing so would have reduced our sample size to 0, 5 and 1 country respectively.

$$3. y_{t+n} \geq \max y_i \quad \forall i \leq t + n$$

Condition 1 is simply that the growth rate exceeds that of the OECD's. Condition 2 states that growth needs to accelerate. Condition 3 removes economic recovery episodes by requiring that rgdp per cap needs to be higher than any prior year on record.

	T-5	T+5	T+10
Non Aid-Adjusted	0.0421 (0.8967)	-0.3547 (0.2580)	-0.2840 (0.3709)
Aid-Adjusted	0.1502 (0.6413)	0.0680 (0.8336)	0.2363 (0.4597)

Table 2.5: Growth Acceleration Episodes

The correlations evolve as one would expect given the existence of the allocation puzzle - the growth acceleration episode triggers a capital outflow. However, none of these correlation are clearly significant.

Next, We use as our initial date for each country the year of a stable regime change - a regime change that is neither preceded or anteceded by another regime change for a period of 10 years between the years 1975 and 1990. We find 17 countries within our sample. To determine the date of the regime change, we use [Marshall and Jaggers, 2007] Polity IV database, which tracks all political regime changes (positive or negative) in independent countries with total population greater than 500,000 in 2007.

As table 4 shows, there is a net outflow of capital after a regime change, and this outflow decreases the longer the regime is in power. It seems reasonable that countries that are politically unstable, or seem on the brink of a political regime change, may experience a surge in foreign aid, for either humanitarian or political motives. When we remove foreign aid flows we still have the same behavior, though

once again the correlations are not significant.

	T-5	T+5	T+10
Non Aid-Adjusted	-0.1542 (0.5686)	-0.0491 (0.8516)	0.0037 (0.9888)
Foreign Aid Removed	-0.0362 (0.8941)	0.0548 (0.8402)	0.1758 (0.5148)
Foreign Aid Removed, Drop Chile	0.0644 (0.8196)	-0.4473 (0.0823)	-0.0766 (0.7780)

Table 2.6: Regime Change

Finally, we use the date of joining WTO as $t=0$. Nine countries in our sample joined GATT/WTO between 1975 and 1990. Once again we find the same result both with and without foreign aid flows - the negative correlation between capital outflows and TFP growth becomes less negative the longer the country has been a member.

	T-5	T+5	T+10
Non Aid-Adjusted	-0.2512 (0.5145)	-0.4513 (0.2228)	0.2179 (0.5733)
Foreign Aid Removed	-0.3697 (0.3274)	-0.6362 (0.0655)	0.-0.0234 (0.9524)

Table 2.7: GATT/WTO Membership

2.8 Conclusion

We introduce learning into a standard neoclassical growth model and use it to explain the allocation puzzle. While we have limited success - we need to impose a

significant negative relationship between reality and initial beliefs - we are capable of matching both the initial correlation and the evolution of the allocation puzzle.

As a next step, we plan to solve the model with production and precautionary savings in hopes that this will allow for a more reasonable initial correlation, and to see if we can solve the Feldstein-Horoika puzzle using the same model.

Chapter 3

Re-evaluating Stylized Business Cycle Facts

3.1 Introduction

In papers studying business cycle behaviour, one of the first tasks is to isolate the business cycle component of the data, usually with either a high pass (HP) or band pass (BP) filter. In a HP filter authors provide the minimum duration of the business cycle, while a BP filter requires both a minimum and maximum duration. Almost without fail, the duration most authors cite was the one established by [Burns and Mitchell, 1946] (BM): a minimum business cycle length of 16-22 months and a maximum of 100-106 months (5-7 to 33-35 quarters, or 2 to 8 years). What most authors fail to realize is that BM found these numbers using USA data from 1885 to 1931. A review of the NBER business cycle dates reveals that over 1937-2009 the business cycle length has changed to a minimum of 18 months and a maximum of 128 months (7 - 42 quarters or 2 - 11 years). If focus is on the Great Moderation years of 1985-2011, this business length is further changed to 81-128 months (27-42 quarters) within the USA. Misspecifying the business cycle length leads to incorrect identification of the business cycle elements within the data, affecting the imputed

volatility and correlation statistics for all business cycle variables.

The first goal is to obtain updated business cycle lengths for a sample of developed and developing countries. I simplify the [Harding and Pagan, 2002] version of the [Bry and Boschan, 1971] procedure to identify turning points as detailed in section 2. Limiting attention to 1985-2011, I find that the average min-max business length for developed countries is 35-66 quarters, while for developing countries it is a shorter 28-59 quarters, longer than the 5-35 quarter window specified by BM.

Using the updated business cycle length I re-examine stylized business cycle facts for both developed and developing countries as detailed in section 3.

3.2 Identifying Business Cycles

3.2.1 Method

There are a number of papers that try to determine the turning points in the business cycles, usually using only USA data. The methods they propose can be broadly be divided into parametric and non-parametric approach. The nonparametric approach declares a definition of a business cycle, then find points in the data that matches that definition. The procedure provided by [Bry and Boschan, 1971] procedure (BBP) is the most enduring of these approaches as it best matches NBER dates of USA business cycles while using only GDP data. Parametric approaches usually employ Markov switching techniques, i.e. [Hamilton, 1989]. Given its transparency, I will use the nonparametric approach.¹

At the heart of BBP is the definition of the turning points of a business cycle: usually based upon a single data series such as GDP, Y_t .² Define a peak (trough) turning

¹[Harding and Pagan, 2002] compares parametric and non-parametric approaches and find in favor of the non-parametric approach

²I will use the log value of GDP. This is a monotonic transformation and does not change the procedure or definitions that follow.

point as one that is higher (lower) than any two points preceding or following it:

$$\text{Peak at } t: y_{t-2} < y_t, y_{t-1} < y_t \text{ and } y_{t+2} < y_t, y_{t+1} < y_t \quad (3.1)$$

$$\text{Trough at } t: y_{t-2} > y_t, y_{t-1} > y_t \text{ and } y_{t+2} > y_t, y_{t+1} > y_t \quad (3.2)$$

BBP also applies several censoring procedures.³

1. Turning points are not to be situated within the first or last 2 quarters.
2. The first (last) peak and trough must be higher (lower) than the values closer to the beginning (end) of the data series.
3. A peak (trough) must be followed by a trough (peak).
4. A phase must have a duration of at least 2 quarters.
5. A cycle must have a duration of at least 4 quarters.

With the turning point definition and censoring rules established, what remains is to identify these points in the data. While BBP introduce a method to smooth their monthly data, the quarterly data I use allows for great simplification of their original process. This process I use proceeds in 5 rounds, with each round removing a layer of smoothing, and using information from the previous step to identify the initial region in which to find the turning point.

The first step is to identify the extreme values to reduce their influence on cycle identification. Extreme values are defined as values whose ratio to 3 quarter moving average are greater than 3.5 standard deviations of all the ratios. Extreme values are substituted by the 3 quarter moving average around that point. Once the extreme values have been removed, I take a 3 quarter moving average of the series adjusted for extreme points, and identify potential turning points: all points meeting the definition of a turning point. For the third step, I use data points generated by a

³These were originally stated in term monthly durations, here I use the quarterly equivalents.

2-quarter moving average of the series using the no-extreme-values series generated in step 1. Again I search for points meeting the definition of turning points. This now proceeds in 2 steps: (1) I identify all points that meet the definition of turning point, and (2) only keep turning points within 2 quarters of a turning point located in step two. For the fourth round I use data points from a 2-quarter moving average of the original series. Once again, I select tentative turning points within 2 quarters of the turning points found using step (3). Finally, I use the original data series. I identify turning points within 2 quarters of the turning points found in the fourth round. I then apply the censoring rules in the order above. At any step in the censoring, turning points are removed in the following order: first, turning points that violate any of the remaining censoring rules are removed. In the case of a tie, the lowest (highest) peak (trough) is removed. If there is still a tie, the most recent turning point is kept.

3.2.2 Correct Business Cycle Length

To verify that the procedure works I test the turning points identified by my simplified Bry-Boshan Procedure (SBBP) against those selected by the NBER using quarterly RGDP for the USA from 1960Q1-2011Q4 obtained from the OECD. Results can be found in table (C.1).⁴ First, notice that although SBBP uses only one source of economic data, it matches the official business cycle dates provided by the NBER very well. The only business cycle omission is the recession in 2001. A brief examination of the data reveals why this is missed: other than a small dip in RGDP in 2001Q3 RGDP was unaffected. As this procedure considers only RGDP when identifying recessions, not the full spectrum of data considered by the NBER, the 2001 recession is not selected.⁵ Other than this omission, all business cycles identified by NBER are identified by SBBP. Furthermore, the turning point

⁴As my data covers 1960Q1-2011Q4 the first turning point I find will be 1960Q4 or later by censoring rule(4): this is why the peak in 1960Q2 is not amongst my turning points.

⁵Future work will also consider alternate data series: for example, GDP per capita or growth rate of GDP.

dates selected by the SBBP line up reasonably well with those identified by the NBER: 9 out of the 13 turning points have the same date, of the remaining 4 three are wrong by 1 quarter, and one is wrong by 2 quarters.

Table (C.2) expands the method to other countries in the OECD. The start date for each series is given in the second column. Table (C.3) provides the characteristics of the identified business cycles. The characteristics are the number of cycles (count) the maximum duration (max), minimum duration (min) and average duration (avg) for peak to peak cycles and trough to trough cycles. These characteristics viewed over 3 different periods: the "Entire Sample" columns consider the full data series for the indicated countries, the 1985-2011 reduces focus on the Great Moderation in the USA, and 1993Q-2011 reduces the focus to time periods in which all countries in the sample have data.⁶ The table is summarized in table (3.1).

	Entire Sample						1985-2011 (Great Moderation)					
	Peak to Peak			Trough to Trough			Peak to Peak			Trough to Trough		
	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min
USA	30	69	5	31	72	5	52	69	34	54	72	35
<i>Developed</i>	39	66	15	41	66	18	51	66	35	50	61	30
<i>Emerging</i>	36	59	24	36	54	26	44	59	28	39	54.0	26

Table 3.1: Business Cycle Characteristics (Summary)

Several things are obvious from Tables (3.1) and (C.3): Firstly, and most importantly, the 6 to 35 quarters business cycle length of Burns and Mitchell does not describe current business cycles. When considering data from 1960 - 2011, the business cycle for the USA ranges from 5-72 quarters. Once attention is limited to post-Great Moderation, the USA business cycle length changes to 34-69 quarters. Secondly, the business cycle length varies substantially across countries, highlighting the importance of using country-specific filter lengths. During the Great Moderation era, the business cycle length for the UK is 9-70 quarters, while for South Africa it

⁶Mexico's data is the limiting factor, covering only 1993Q1-2011Q4. All other countries have data for the entirety of 1985-2011.

is 20-75 quarters. Unsurprisingly, business cycle lengths in emerging countries are shorter than those in developed: the average business cycle length is for developed countries is 35.4-65.6 quarters, while the equivalent measure is 28.3-58.7 quarters for emerging countries during the Great Moderation.

3.3 Stylized Business Cycle Facts

Even though the business cycle length implied by commonly used filters is incorrect, it could be that using the correct length does not affect business cycle statistics very much. The next section evaluates whether this is the case, using data that has been filtered using a bandpass filter⁷ with the indicated minimum and maximum business cycle. I use 3 different filter lengths: the 6-35 quarters proposed by (BM), the more recent length of 7-42 quarters generated by NBER data, and finally a country specific length calculated using my SBBP method.

3.3.1 GDP Volatility

	Entire Sample			1985-2011		
	Burns Mitchell 6-35 quarters	NBER 7-42 quarters	Entire Sample Country Specific	Burns Mitchell 6-35 quarters	NBER 7-42 quarters	Great Moderation Country Specific
USA	2.42	3.20	4.92	1.39	2.05	1.21
Canada	1.80	2.27	3.28	1.82	2.44	3.11
U.K.	2.03	2.74	5.94	1.72	2.97	5.63
France	1.09	1.73	2.20	1.12	1.60	0.73
Australia	1.57	2.02	1.81	0.82	1.30	-
<i>Developed</i>	1.78	2.39	3.63	1.37	2.07	2.67
Korea	5.62	6.65	5.59	5.18	5.49	3.14
Mexico	8.36	8.95	1.99	-	-	-
South Afr.	2.81	3.32	4.52	2.09	3.15	3.41
<i>Emerging</i>	5.60	6.31	4.03	3.64	4.32	3.28

Table 3.2: Business Cycle: Volatility of GDP

⁷Pre-built bandpass written by Eduard Pelz based on Christiano and Fitzgerald (1999)

Table 3.2 reports the $1e^4$ variance of business cycle GDP⁸, which I interpret as volatility of GDP. Output in developing countries is more volatile than developed countries, which one would also expect given their shorter business cycle length. As expected, GDP volatility is lower during the period of the Great Moderation. Using the correct business cycle length, however, decreases the relative average volatility of developing to developed countries from three times as volatile to less than twice as volatile. On a country level, using the correct business cycle length usually increases the volatility of GDP of developed countries, while decreasing volatility of developing countries, with the largest change in volatility occurring for the UK.

3.3.2 Relative Standard Deviation

	BM		NBER		Country specific	
	Emerging	Developed	Emerging	Developed	Emerging	Developed
<u>Full Sample</u>						
Consumption	1.18	0.65	1.16	0.85	1.07	0.96
Government Spending	0.69	0.74	0.69	0.73	0.85	0.78
Imports	3.65	5.57	3.55	3.38	3.12	2.58
Exports	2.38	2.70	2.46	2.55	2.16	2.14
<u>1985-2011</u>						
Consumption	1.17	0.63	1.28	0.79	1.40	1.05
Government Spending	0.53	0.51	0.58	0.60	0.75	0.68
Imports	3.13	2.98	3.42	3.36	3.89	2.23
Exports	2.28	2.31	2.55	2.55	2.57	1.79

Table 3.3: Relative Std. Deviation, Summary

Tables C.4 and C.5 shows the standard deviation of consumption, government, imports and exports to that of standard deviation of GDP for each country in the sample, summarized by table 3.3.

Previous papers have documented that consumption in developed countries is less volatile than GDP, while consumption in emerging countries is more volatile than

⁸BM denotes Burns Mitchell window length of 6-35 quarters, NBER denotes the window length of 7-42 quarters, CS refers to the country specific business window. The country specific window is recalculated during 1985-2011

GDP. Unsurprisingly, this finding is duplicated both when using a full sample and the Great Moderation years using either the BM or NBER window. Using a country specific business cycle window reveals a different story: the average consumption to GDP volatility in developed countries has been greater than 1 in the Great Moderation, though it is still true that consumption in developing countries is more volatile than consumption in developed countries.

The volatility of government spending in both developed and emerging countries is less than the volatility of GDP. Using the BM or NBER windows would lead one to conclude that government spending in emerging markets was less volatile than spending in developed markets, though the country specific window shows that this is not the case. All windows show that that government spending has become less volatile during the great moderation.

For all countries imports and exports are more volatile than output, and imports are more volatile than exports. During the Great moderation imports were more volatile in emerging countries than in developed countries, while exports are revealed to be more volatile in emerging countries when using the country specific windows. For the full sample, one finds that imports and exports are more volatile in emerging countries than developing countries when using country specific windows, contrary to the conclusions reached when using BM. When using country specific filters reveals that both imports and exports are more volatile for developing countries than developed countries on average: though there are some country specific deviations that may change this result with a larger sample of countries.

3.3.3 Correlation with own business cycle

	BM		NBER		Country specific	
	Emerging	Developed	Emerging	Developed	Emerging	Developed
<u>Full Sample</u>						
Consumption	0.85	0.76	0.85	0.78	0.66	0.75
Government Spending	-0.08	-0.32	-0.05	-0.31	-0.10	0.12
Imports	0.86	0.75	0.86	0.77	0.74	0.73
Exports	0.46	0.50	0.52	0.56	0.61	0.48
<u>1985-2011</u>						
Consumption	0.74	0.80	0.90	0.84	0.85	0.95
Government Spending	-0.01	-0.41	-0.07	0.44	-0.06	0.16
Imports	0.86	0.76	0.89	0.84	0.78	0.84
Exports	0.62	0.52	0.67	0.59	0.30	0.53

Table 3.4: Correlation with own business cycle, Summary

Tables *C.6* and *C.7* show the correlation of consumption, government, imports and exports with GDP, summarized in table (3.4).

For all countries consumption is strongly pro-cyclical, using a country-specific window reveals that consumption is more pro-cyclical in developed than emerging countries, and this pro-cyclicality has increased during the great moderation.

Government spending is the most interest result. Using BM cycle length would lead one to believe that government spending is counter-cyclical: especially in developed countries. Using country-specific filters reveal a different story: government spending is actually pro-cyclical. Furthermore, amongst emerging countries government spending is strongly counter-cyclical in Mexico but weakly pro-cyclical in all other countries.

For all countries imports and exports are positively correlated with GDP. Using BM cycles would lead one to believe that this effect is stronger for emerging countries than developed countries during the Great Moderation, while using country specific filters reveals that the opposite is true: imports and exports correlate more with the output for developed countries.

3.3.4 Correlation with USA business cycles

	Canada	U.K.	France	Developed	Korea	Mexico	South Africa	Emerging
<u>Full Sample</u>								
BM	0.76	0.66	0.58	0.67	0.27	0.87	0.03	0.39
NBER	0.81	0.75	0.36	0.64	0.38	0.75	0.11	0.41
CS	0.64	0.70	0.37	0.57	0.29	0.61	0.22	0.37
<u>1985-2011</u>								
BM	0.86	0.83	0.74	0.81	0.21	0.79	0.63	0.54
NBER	0.86	0.83	0.74	0.81	0.21	0.82	0.63	0.55
CS	0.94	0.86	0.56	0.79	0.25	0.18	0.50	0.31

Table 3.5: Correlation with USA business cycle, Summary

Tables C.8 and C.9 present the correlation of output across the various countries, summarized in table 3.5 summarizes the results. The output of developed countries are more correlated with USA business cycles than emerging countries, though perhaps not as much as previously believed. During the great moderation correlation with USA business cycles increased for all developed countries, but the same is not true for developing countries. Mexico is the most drastic change when using the country specific windows during the great moderation: its correlation with USA business cycles is greatly reduced and more in line with other developing countries.

3.4 Conclusion

	BM		CS	
	Emerging	Developed	Emerging	Developed
GDP Volatility	3.64	1.37	3.28	2.67
Correlation with USA business cycles	0.54	0.81	0.31	0.79
Relative Std: Consumption	1.17	0.63	1.40	1.05
Relative Std: Government Spending	0.53	0.51	0.75	0.68
Relative Std: Imports	3.13	2.98	3.89	2.23
Relative Std: Exports	2.28	2.31	2.57	1.79
Correlation: Consumption	0.74	.074	0.85	0.95
Correlation: Government Spending	-0.01	-0.41	-0.06	0.16
Correlation: Imports	0.86	0.76	0.78	0.84
Correlation: Exports	0.62	0.52	0.30	0.53

Table 3.6: Result Summary

In this paper I calculate the business cycle length for a sample of countries, and find large differences in business cycle length across countries. It is still true, on average, the business cycles and GDP is more volatile in developing countries are shorter than those in developed countries, and that GDP volatility has decreased during the Great Moderation.

Previous papers⁹ established that business cycles of developed countries are more correlated with USA business cycles than developing countries. Consumption is less volatile than output in developed countries, while it is more volatile in developing countries. For both developing and developed countries government expenditure is counter-cyclical. Net exports are strongly counter-cyclical for emerging economics, while for developed economics these are only weakly counter-cyclical.

Using the country-specific windows I find that output volatility of developing on average is only slightly greater than that of developed countries and using the corrected business cycle window reveals interesting deviations from this trend: the UK is more volatile than Korea, Mexico or South Africa, while France and Australia both have a very low volatility. The ratio of consumption to output volatility for both country groups are larger than what has been found previously, with developed countries approaching a ratio of 1 instead of being far less than one as previously documented. The counter-cyclicity of government spending remains weak for developing countries, and actually disappears for most developed countries. The exception is the USA, but this is only when one considers the full sample of years.

⁹I use [Canova, 1998] for business cycle statistics for USA. Developing country business cycle statistics come from [Calderon and Fuentes, 2010], [Agenor et al., 1999], [?], [Male, 2010], [Perri and Neumeyer, 2005], [Bergoeing and Soto, 2002]

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Appendix A

Trade and Inequality

A.A Detailed Derivations

A.A.1 Model

To find P_d^L begin with

$$P_d^L = \sum_s J_s \int_{z_{sd}^{HB}}^{\infty} p_{sd}^B(z) \gamma b_s^\gamma z^{-(\gamma+1)} dz \quad (\text{A.1})$$

This evaluates to

$$P_d^L = \frac{\gamma}{\gamma + \frac{1}{2}} \left[\frac{S_d w_d}{q M_d} \right]^{\frac{1}{2}} z_{dd}^{HB-\frac{1}{2}} \sum_s J_s b_s^\gamma z_{sd}^{HB-\gamma} \quad (\text{A.2})$$

Use the definition of N_d^L and z_{dd}^{HB} to obtain

$$P_d^L = \frac{\gamma}{\gamma + \frac{1}{2}} \frac{N_d^L}{q} \left[\frac{S_d}{M_d} \right]^{\frac{1}{2}} \left[\frac{S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}}}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \right] \quad (\text{A.3})$$

Next, P_d^H is given by

$$P_d^H = P_d^L + \sum_s J_s \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} p_{sd}^H(z) \gamma b_s^\gamma z^{-(\gamma+1)} dz \quad (\text{A.4})$$

which evaluates to

$$\begin{aligned}
P_d^H &= P_d^L + \frac{\gamma}{\gamma + \frac{1}{2}} \left[\frac{Q_d^H}{q} \right]^{\frac{1}{2}} \sum_s J_s \left[\frac{\tau_{sd} w_s}{z_{sd}^{H0}} \right]^{\frac{1}{2}} z_{sd}^{H0^{-\gamma}} \\
&\quad - \frac{\gamma}{\gamma + \frac{1}{2}} \left[\frac{Q_d^H}{q} \right]^{\frac{1}{2}} \sum_s J_s \left[\frac{\tau_{sd} w_s}{z_{sd}^{HB}} \right]^{\frac{1}{2}} z_{sd}^{HB^{-\gamma}} \tag{A.5}
\end{aligned}$$

$$\begin{aligned}
&= P_d^L + \frac{\gamma}{\gamma + \frac{1}{2}} \frac{N_d^H Q_d^H}{q} - \frac{\gamma}{\gamma + \frac{1}{2}} \frac{N_d^L}{q} Q_d^{H\frac{1}{2}} \left[\frac{S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}}}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \right] \tag{A.6} \\
&\quad - \frac{\gamma}{\gamma + \frac{1}{2}} \frac{1}{q} \left[\frac{Q_d^H}{M_d^H} \right]^{\frac{1}{2}} \sum_s b_s^\gamma J_s z_{sd}^{H0^{-\gamma}} [w_s f_{sd}]^{\frac{1}{2}}
\end{aligned}$$

Next, substitute $S_d = M_d^H Q_d^H + M_d^L Q_d^L$, and use $Q_d^L = \frac{w_d^L + q P_d^L}{N_d^L}$ and $M_d^L = \left(M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right) \left(M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} \right)$ into the expression for P_d^L to obtain

$$P_d^L = \frac{\gamma w_d^L}{q} \frac{\left[M_d^{\frac{1}{2}} + M_d^{H\frac{1}{2}} \right]}{\left[\frac{1}{2} M_d^{\frac{1}{2}} - \gamma M_d^{H\frac{1}{2}} \right]} - \frac{\gamma N_d^L}{q} \frac{(M_d^H Q_d^H)^{\frac{1}{2}} \left[S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right]}{\left[\frac{1}{2} M_d^{\frac{1}{2}} - \gamma M_d^{H\frac{1}{2}} \right] \left[M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}} \right]} \tag{A.7}$$

Use the definition of Q_d^H and P_d^L to simplify P_d^H and obtain

$$\begin{aligned}
P_d^H &= \left[\frac{M_d^{H\frac{1}{2}} + M_d^{\frac{1}{2}}}{M_d^{H\frac{1}{2}}} \right] P_d^L + \frac{2\gamma w_d^H}{q} - \frac{2\gamma w_d^L}{q} \left[\frac{M_d^{H\frac{1}{2}} + M_d^{\frac{1}{2}}}{M_d^{H\frac{1}{2}}} \right] \\
&\quad - \frac{2\gamma}{q} \frac{Q_d^{H\frac{1}{2}}}{M_d^{H\frac{1}{2}}} \sum_s b_s^\gamma J_s z_{sd}^{H0^{-\gamma}} (w_s f_{sd})^{\frac{1}{2}} \tag{A.8}
\end{aligned}$$

Finally, substitute the definition of the cutoffs into these expressions to obtain equations (1.28) and (1.29).

Firms will continue to draw productivities until the expected profit equals the cost

of taking the productivity draw

$$w_s f_c = \sum_d \int_{z_{sd}^H}^{z_{sd}^B} \pi_{sd}^H(z) dz + \sum_d \int_{z_{sd}^B}^{\infty} \pi_{sd}^B(z) dz$$

Expected profit, $\int \pi_{sd}^k(z) dz$ has two components, expected revenue, R_{sd}^k and expected cost, TC_{sd}^k .

$$w_s f_c = \sum_d E(\pi_{sd}) \quad (\text{A.9})$$

$$= \sum_d E(R_{sd}) - E(T_{sd}) \quad (\text{A.10})$$

Total cost, TC is $J_s \int_{z^*} b^\gamma w_s \ell_s z^{-(\gamma+1)} dz$, where the cost for firm z is $w_s \ell_s(z) = \frac{w_s \tau_{sd}}{z} M_d^H c_d^H(z) + \frac{\tau_{sd} w_s}{z} M_d^L c_d^L(z) + w_s f_{sd}$.

$$\begin{aligned} TC_{sd} &= \gamma J_s b_s^\gamma w_s \tau_{sd} \int_{z^{H0}}^{z^{HB}} M_d^H c_d^H(z) J_s z^{-(\gamma+2)} dz + \int_{z^{H0}}^{z^{HB}} (M_d^H c_d^H + M_d^L c_d^L(z)) z^{-(\gamma+2)} dz \\ &\quad + \gamma J_s b_s^\gamma \int_{z_{sd}^H}^{\infty} w_s f_{sd} z^{-(\gamma+1)} dz \\ &= \frac{\gamma N_{sd}^H M_d^H Q_d^H}{2(\gamma+1)(\gamma+\frac{1}{2})} + \frac{\gamma^2 J_s b_s^\gamma z_{sd}^{H0-\gamma}}{(\gamma+1)(\gamma+\frac{1}{2})} [M_d^H Q_d^H w_s f_{sd}]^{\frac{1}{2}} + \frac{J_s b_s^\gamma z_{sd}^{H0-\gamma} (w_s f_{sd})}{\gamma+1} \quad (\text{A.11}) \\ &\quad + \frac{\gamma N_{sd}^L \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right)}{2(\gamma+1)(\gamma+\frac{1}{2})} \left[S_d^{\frac{1}{2}} + (M_d^H Q_d^H)^{\frac{1}{2}} + \frac{2\gamma M_d^H}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \left((M_d Q_d^H)^{\frac{1}{2}} - S_d^{\frac{1}{2}} \right) \right] \end{aligned}$$

Trade value , R , is given by $\sum_k J_s b_s^\gamma \int_z M_d^k p_{sd}^k(z) c_{sd}^k(z) z^{-(\gamma+1)} dz$

$$R_{sd} = \int_{z_{sd}^{H0}}^{z_{sd}^{HB}} \gamma J_s b_s^\gamma p_{sd}^H(z) M_d^H c_d^H(z) z^{-(\gamma+1)} dz \quad (\text{A.12})$$

$$\begin{aligned} & + \int_{z_{sd}^{HB}}^{\infty} \gamma b_s^\gamma J_s p_{sd}^B(z) (M_d^H c_d^H(z) + M_d^L c_d^L(z)) z^{-(\gamma+1)} dz \\ & = \frac{N_{sd}^H M_d^H Q_d^H}{2(\gamma + \frac{1}{2})} + \frac{\gamma J_s b_s^\gamma z_{sd}^{H0-\gamma}}{(\gamma + \frac{1}{2})} [M_d^H Q_d^H w_s f_{sd}]^{\frac{1}{2}} \\ & + \frac{N_{sd}^L \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right)}{2(\gamma + \frac{1}{2})} \left[S_d^{\frac{1}{2}} + (M_d^H Q_d^H)^{\frac{1}{2}} + \frac{2\gamma M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \left((M_d Q_d^H)^{\frac{1}{2}} - S_d^{\frac{1}{2}} \right) \right] \end{aligned} \quad (\text{A.13})$$

Equations (A.11) and (A.13) can be rearranged to show that

$$TC_{sd} = \frac{\gamma}{\gamma + 1} R_{sd} + \frac{w_s f_{sd}}{\gamma + 1} b_s^\gamma J_s z_{sd}^{H0-\gamma} \quad (\text{A.14})$$

Under the assumption of balanced trade, sales to the rest of the world equal purchases from the rest of the world, $\sum_{d \neq s} R_{sd} = \sum_{s \neq d} \sum_k p_{sd}^k C_{sd}^k$. Aggregate income is $\sum w_s^k \ell_s^k$ and is equal to total spending, $\sum_s \sum_k p_{sd}^k C_{sd}^k$.

$$\sum_{d \neq s} R_{sd} = \sum_k w_s^k \ell_s^k - p_{ss}^k C_{ss}^k \quad (\text{A.15})$$

$$\sum_d R_{sd} = \sum_k w_s^k \ell_s^k = Y_s \quad (\text{A.16})$$

Because of the large number of firms, expected sales equal average sales and total sales equal the average sales multiplied by number of firms, J_s .

$$\sum_d R_{sd} = J_s \sum_d E(R_{sd}) \quad (\text{A.17})$$

$$\Rightarrow \sum_d E(R_{sd}) = \frac{\sum_d R_{sd}}{J_s} \quad (\text{A.18})$$

$$\sum_d E(R_{sd}) = \frac{\sum_k w_s^k \ell_s^k}{J_s} \quad (\text{A.19})$$

We can use this expression to simplify free entry as

$$w_s f_c = \sum_d \frac{\sum_k w_s \ell_s^k}{J_s} - \frac{\gamma}{\gamma+1} \frac{\sum_k w_s \ell_s^k}{J_s} - \frac{b_s^\gamma w_s f_{sd} z_{sd}^{H0-\gamma}}{\gamma+1} \quad (\text{A.20})$$

$$= \frac{Y_s}{J_s(\gamma+1)} - \sum_d \frac{b_s^\gamma w_s f_{sd} z_{sd}^{H0-\gamma}}{\gamma+1} \quad (\text{A.21})$$

$$\Rightarrow J_s = \frac{Y_s}{(\gamma+1)w_s f_c + w_s b_s^\gamma \sum_d f_{sd} z_{sd}^{H0-\gamma}} \quad (\text{A.22})$$

A.A.2 Calibration

Begin by observing that $w_d M_d^H \theta_d^H = IncShare_d Y_d$ and $w_d M_d^L \theta_d^L = (1 - IncShare_d) Y_d$. This in turn implies that $M_d^L \theta_d^L = \frac{1 - IncShare_d}{IncShare_d} M_d^H \theta_d^H$. Next, notice that θ_d^H can be expressed as

$$\theta_d^H = \frac{IncShare_d Y_d}{M_d^H w_d}$$

The value imported from t into d is simply the total revenue received by firms from t selling to d. Define $A_d = S_d^{\frac{1}{2}} + (M_d^H Q_d^H)^{\frac{1}{2}} + \frac{2\gamma M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \left((M_d Q_d^H)^{\frac{1}{2}} - S_d^{\frac{1}{2}} \right)$, then the import share of country s in d is given by

$$\begin{aligned} \lambda_{td} &= \frac{R_{td}}{\sum_s R_{sd}} \quad (\text{A.23}) \\ &= \frac{b_t^\gamma J_t z_{td}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{td}^{HB}}{z_{td}^{H0}} \right)^{-\gamma} \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right) + 2\gamma [M_d^H Q_d^H w_t f_{td}]^{\frac{1}{2}} \right]}{\sum_s J_s b_s^\gamma z_{sd}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{sd}^{HB}}{z_{sd}^{H0}} \right)^{-\gamma} \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right) + 2\gamma [M_d^H Q_d^H w_s f_{sd}]^{\frac{1}{2}} \right]} \end{aligned}$$

where $\frac{z_{sd}^{HB}}{z_{sd}^{H0}} = \left[\frac{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}}{M_d^{H\frac{1}{2}}} \right]^2 \left[\frac{(M_d^H Q_d^H)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}}{S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}}} \right]^2$. Since $\sum_s R_{sd} = Y_d$ total value of imports can be written as $R_{sd} = \lambda_{sd} Y_d$. The value of all imports into d equals

the value of all exports from d, $\sum_s R_{sd} = \sum_s R_{ds}$

$$\begin{aligned}
Y_t &= \sum_d R_{td} = \sum_d Y_d \lambda_{td} \tag{A.24} \\
&= \sum_d Y_d \frac{b_t^\gamma J_t z_{td}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{td}^{HB}}{z_{td}^{H0}} \right)^{-\gamma} \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right) + 2\gamma [M_d^H Q_d^H w_t f_{td}]^{\frac{1}{2}} \right]}{\sum_s J_s b_s^\gamma z_{sd}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{sd}^{HB}}{z_{sd}^{H0}} \right)^{-\gamma} \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right) + 2\gamma [M_d^H Q_d^H w_s f_{sd}]^{\frac{1}{2}} \right]}
\end{aligned}$$

The number of firms trading with a country is given by N_{sd}^H . The average sales per firm is given by

$$\frac{R_{sd}}{N_{sd}^H} = \frac{M_d^H Q_d^H + 2\gamma [M_d^H Q_d^H w_t f_{td}]^{\frac{1}{2}} + A_d \left(\frac{z_{td}^{HB}}{z_{td}^{H0}} \right)^{-\gamma} \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right)}{2 \left(\gamma + \frac{1}{2} \right)} \tag{A.25}$$

Finally the number of firms from s that serve destination s relative to destination j is given by

$$\frac{N_{sd}^H}{N_{sj}^H} = \left(\frac{\left[\frac{M_d^H}{M_j^H} \right] \left[\frac{\left(M_j^H Q_j^H \right)^{\frac{1}{2}} - (w_s f_{sj})^{\frac{1}{2}}}{\left(M_d^H Q_d^H \right)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}} \right]^2}{\left[\frac{M_d^H}{M_j^H} \right] \left[\frac{\left(M_j^H Q_j^H \right)^{\frac{1}{2}} - (w_s f_{sj})^{\frac{1}{2}}}{\left(M_d^H Q_d^H \right)^{\frac{1}{2}} - (w_s f_{sd})^{\frac{1}{2}}} \right]^2} \right)^{-\gamma} \tag{A.26}$$

Calibration if $f_{sd} = 0$

The value imported from s into d is simply the total revenue received by firms in s selling to d. Define $A_d = S_d^{\frac{1}{2}} + (M_d^H Q_d^H)^{\frac{1}{2}} + \frac{2\gamma M_d^{H\frac{1}{2}}}{M_d^{\frac{1}{2}} - M_d^{H\frac{1}{2}}} \left((M_d^H Q_d^H)^{\frac{1}{2}} - S_d^{\frac{1}{2}} \right)$, then the import share of country t in d is given by

$$\lambda_{td} = \frac{b_t^\gamma J_t z_{td}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{td}^{HB}}{z_{td}^{H0}} \right)^{-\gamma} \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right) \right]}{\sum_s b_s^\gamma J_s z_{sd}^{H0-\gamma} \left[M_d^H Q_d^H + A_d \left(\frac{z_{sd}^{HB}}{z_{sd}^{H0}} \right)^{-\gamma} \left(S_d^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}} \right) \right]} \tag{A.27}$$

Using $\frac{z_{sd}^{HB}}{z_{sd}^{H0}} = \left[\frac{(M_d Q_d^H)^{\frac{1}{2}} - (M_d^H Q_d^H)^{\frac{1}{2}}}{S_d^{\frac{1}{2}} - (M_d^H Q_d^H)} \right]^2$ and $z_{sd}^{H0} = \frac{q}{Q_d^H} \tau_{sd} w_s$ and $J_s = \frac{Y_s}{(\gamma+1)w_s f_c}$

$$\lambda_{td} = \frac{b_t^\gamma J_t z_{td}^{H0^{-\gamma}}}{\sum_s b_s^\gamma J_s z_{sd}^{H0^{-\gamma}}} = \frac{\frac{Y_t}{w_t} \left(\frac{b_t}{\tau_{td} w_t} \right)^\gamma}{\sum_s \frac{Y_s}{w_s} \left(\frac{b_s}{\tau_{sd} w_s} \right)^\gamma} \quad (\text{A.28})$$

Next, since $\sum_s R_{sd} = Y_d$ total value of imports can be written as $R_{sd} = \lambda_{sd} Y_d$. Finally, the value of all imports into d equals the value of all exports from d, $\sum_s R_{sd} = \sum_s R_{ds}$, so

$$\begin{aligned} Y_d &= \sum_t R_{dt} = \sum_t Y_t \lambda_{dt} = \sum_t Y_t \frac{\frac{Y_d}{w_d} \left(\frac{b_d}{\tau_{dt} w_d} \right)^\gamma}{\sum_s \frac{Y_s}{w_s} \left(\frac{b_s}{\tau_{st} w_s} \right)^\gamma} \\ \frac{w_d^{\gamma+1}}{b_d^\gamma} &= \sum_t \frac{Y_t}{\sum_s \frac{Y_s}{w_s} \left(\frac{b_s \tau_{dt}}{w_s \tau_{st}} \right)^\gamma} \end{aligned} \quad (\text{A.29})$$

Equation (A.29) implicitly solves wages in d as a function of wages in all other countries. Setting a wage to be the numeraire solves for all wages.

Appendix B

Learning about Growth

B.A Sequential Problem

We write the sequential problem for completeness. A country chooses functions in $(c_t, k_{t+1}, b_{t+1})_{t=0}^{\infty}$ such that $(c_t, k_{t+1}, b_{t+1}) : \mathbb{R}^t \rightarrow \mathbb{R}_+^3$ for $0 \leq t \leq T$ and $(c_t, k_{t+1}, b_{t+1}) : \mathbb{R}^T \rightarrow \mathbb{R}_+^3$ for $t \geq T$. These functions must solve the maximization problem:

$$\max \sum_{t=0}^T \beta^t \int u(c_t(A^t)) d\tilde{F}_t(A^t; \tilde{\mu}_0, \tilde{\sigma}_0^2) + \beta^T \int \sum_{t=T}^{\infty} u(c_t(A^T)) d\tilde{F}_T(A^T, \tilde{\mu}_0, \tilde{\sigma}_0^2)$$

subject to

$$\text{For } t = 0, \dots, T-1$$

$$c_t + k_{t+1} + b_{t+1} = A_t^{1-\alpha} k_t^\alpha + (1-\delta)k_t + (1+r)b_t$$

$$\text{For } t \geq T$$

$$c_t + k_{t+1} + b_{t+1} = (\gamma^{t-T} A_T)^{(1-\alpha)} k_t^\alpha + (1-\delta)k_t + (1+r)b_t$$

B.B Tables

B.B.1 Developing Countries

Country	Isocode	k_0 Date	TFP Start	TFP End	NCF Start	NCF End	For. Aid Start	For. Aid End
Angola	(AGO)	1960	1960	1996	1985	2000	1970	1996
Argentina	(ARG)	1950	1960	2000	1970	2000	1970	2000
Bangladesh	(BGD)	1959	1960	2000	1973	2000	1971	2000
Benin	(BEN)	1959	1960	2000	1970	2000	1970	2000
Bolivia	(BOL)	1950	1960	2000	1970	2000	1970	2000
Botswana	(BWA)	1960	1960	1999	1974	1999	1970	2000
Brazil	(BRA)	1950	1960	2000	1970	2000	1970	2000
Burkina Faso	(BFA)	1959	1960	2000	1974	2000	1970	2000
Cameroon	(CMR)	1960	1960	2000	1970	2000	1970	2000
Chile	(CHL)	1951	1960	2000	1970	2000	1971	2000
China, P.R.	(CHN)	1952	1960	2000	1981	2000	1979	2000
Colombia	(COL)	1950	1960	2000	1970	2000	1970	2000
Congo, Rep.	(COG)	1960	1960	2000	1970	2000	1970	2000
Costa Rica	(CRI)	1950	1960	2000	1970	2000	1970	2000
Cote d'Ivoire	(CIV)	1960	1960	2000	1970	2000	1970	2000
Cyprus	(CYP)	1950	1960	1996	1973	1996	1970	1996
Dominican Rep.	(DOM)	1951	1960	2000	1970	2000	1970	2000
Ecuador	(ECU)	1951	1960	2000	1970	2000	1970	2000
Egypt, Arab Rep.	(EGY)	1950	1960	2000	1970	2000	1970	2000

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Country	Isocode	k_0	TFP	TFP	NCF	NCF	For. Aid	For. Aid
		Date	Start	End	Start	End	Start	End
El Salvador	(SLV)	1950	1960	2000	1970	2000	1970	2000
Ethiopia	(ETH)	1950	1960	2000	1970	2000	1970	2000
Fiji	(FJI)	1960	1960	2000	1977	2000	1970	2000
Gabon	(GAB)	1960	1960	2000	1970	2000	1970	2000
Ghana	(GHA)	1955	1960	2000	1970	2000	1970	2000
Guatemala	(GTM)	1950	1960	2000	1970	2000	1970	2000
Haiti	(HTI)	1967	1967	1998	1970	1998	1970	2000
Honduras	(HND)	1950	1960	2000	1970	2000	1970	2000
India	(IND)	1950	1960	2000	1970	2000	1970	2000
Indonesia	(IDN)	1960	1960	2000	1981	2000	1970	2000
Iran, Islamic Rep	(IRN)	1955	1960	2000	1970	2000	1970	2000
Israel	(ISR)	1950	1960	2000	1970	2000	1970	1996
Jamaica	(JAM)	1953	1960	2000	1970	2000	1970	2000
Jordan	(JOR)	1954	1960	2000	1970	2000	1970	2000
Kenya	(KEN)	1950	1960	2000	1970	2000	1970	2000
Korea, Rep.	(KOR)	1953	1960	2000	1970	2000	1970	1999
Madagascar	(MDG)	1960	1960	2000	1970	2000	1970	2000
Malawi	(MWI)	1954	1960	2000	1970	2000	1970	2000
Malaysia	(MYS)	1955	1960	2000	1970	2000	1970	2000
Mali	(MLI)	1960	1960	2000	1970	2000	1970	2000
Mauritius	(MUS)	1950	1960	2000	1970	2000	1970	2000

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Country	Isocode	k_0	TFP	TFP	NCF	NCF	For. Aid	For. Aid
		Date	Start	End	Start	End	Start	End
Mexico	(MEX)	1950	1960	2000	1970	2000	1970	2000
Morocco	(MAR)	1950	1960	2000	1970	2000	1970	2000
Mozambique	(MOZ)	1960	1960	2000	1980	2000	1970	2000
Nepal	(NPL)	1960	1960	2000	1970	2000	1970	2000
Niger	(NER)	1960	1960	2000	1970	2000	1970	2000
Nigeria	(NGA)	1950	1960	2000	1970	2000	1970	2000
Pakistan	(PAK)	1950	1960	2000	1970	2000	1970	2000
Panama	(PAN)	1950	1960	2000	1970	2000	1971	2000
Papua New Guinea	(PNG)	1960	1960	1999	1973	1999	1970	2000
Paraguay	(PRY)	1951	1960	2000	1970	2000	1970	2000
Peru	(PER)	1950	1960	2000	1970	2000	1970	2000
Philippines	(PHL)	1950	1960	2000	1970	2000	1970	2000
Rwanda	(RWA)	1960	1960	2000	1970	2000	1970	2000
Senegal	(SEN)	1960	1960	2000	1970	2000	1970	2000
Singapore	(SGP)	1960	1960	1996	1970	1996	1970	1995
South Africa	(ZAF)	1950	1960	2000	1970	2000	1993	2000
Sri Lanka	(LKA)	1950	1960	2000	1970	2000	1970	2000
Syrian Arab Rep	(SYR)	1960	1960	2000	1970	2000	1970	2000
Tanzania	(TZA)	1960	1960	2000	1970	2000	1970	2000
Togo	(TGO)	1960	1960	2000	1970	2000	1970	2000

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Country	Isocode	k_0	TFP	TFP	NCF	NCF	For. Aid	For. Aid
		Date	Start	End	Start	End	Start	End
Trinidad & Tobago	(TTO)	1950	1960	2000	1970	2000	1970	2000
Tunisia	(TUN)	1961	1961	2000	1970	2000	1970	2000
Turkey	(TUR)	1950	1960	2000	1970	2000	1970	2000
Uganda	(UGA)	1950	1960	2000	1970	2000	1970	2000
Uruguay	(URY)	1950	1960	2000	1970	2000	1970	2000
Venezuela, RB	(VEN)	1950	1960	2000	1970	2000	1970	2000

Table B.1: DVC Country list (66)

B.B.2 Developed Countries

Country	Isocode	k_0	TFP	TFP
		start	Start	End
Australia	(AUS)	1950	1960	2000
Austria	(AUT)	1950	1960	2000
Belgium	(BEL)	1950	1960	2000
Canada	(CAN)	1950	1960	2000
Denmark	(DNK)	1950	1960	2000
Finland	(FIN)	1950	1960	2000
France	(FRA)	1950	1960	2000
Germany	(GER)	1970	1970	2000
Greece	(GRC)	1950	1960	2000
Iceland	(ISL)	1950	1960	2000

Continued on Next Page...

Country	Isocode	k_0	TFP	TFP
		start	Start	End
Ireland	(IRL)	1950	1960	2000
Italy	(ITA)	1950	1960	2000
Japan	(JPN)	1950	1960	2000
Luxembourg	(LUX)	1950	1960	2000
Netherlands	(NLD)	1950	1960	2000
New Zealand	(NZL)	1950	1960	2000
Norway	(NOR)	1950	1960	2000
Portugal	(PRT)	1950	1960	2000
Spain	(ESP)	1950	1960	2000
Sweden	(SWE)	1951	1960	2000
Switzerland	(CHE)	1950	1960	2000
United Kingdom	(GBR)	1950	1960	2000
United States	(USA)	1950	1960	2000

Table B.2: Developed Countries list (23)

B.B.3 Time Series Behavior, Initial Dates

Country	Growth Acceleration	Regime Change	GATT/WTO Membership
Argentina	1990		
Bolivia			1990
Botswana			1987
Brazil		1985	
Chile	1986	1989	

China	1987		
Colombia			1981
Costa Rica			1990
Ecuador		1979	
El Salvador		1984	
Fiji		1990	
Honduras		1982	
India	1982		
Indonesia	1987		
Iran		1982	
Jordan		1989	
Korea	1984		
Malaysia	1988		
Mauritius	1983		
Mexico		1977	1986
Morocco			1987
Nigeria		1979	
Panama		1989	
Papua New Guinea	1987	1975	
Peru		1980	
Philippines			1979
Senegal		1978	
Syrian Arab Republic	1989		
Tunisia		1987	1990
Uganda	1989	1980	
Uruguay	1989	1985	
Venezuela			1990

Table B.3: Initial Years

B.C Aid-Adjusted Figures

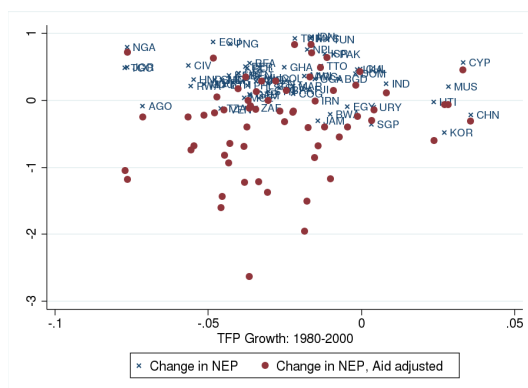


Figure B.1: Comparing No-Aid Adjusted and Aid-Adjusted changes in Net External Position

Note that the Aid-Adjusted case is simply the downward vertical shift of the Aid-included case for all countries except Korea and Singapore. For example,

Angola (AGO) aid adjusted capital flows (the circle) is beneath the non-aid-adjusted net capital flows (the x). Korea and Singapore have a shift in TFP growth as well because the foreign aid data series ends before the capital flows does, meaning that we need to recalculate TFP growth for the shorter time span.

Appendix C

Re-evaluating stylized business cycle facts for developed and developing countries

C.A Business Cycles Identification

Simplified Bry Boschan Procedure

1. Identify extreme values (ratio to the 3 quarter moving average outside 3.5 of standard deviations) and substitute values of the corresponding 3 point moving average.
2. Using a 3-quarter moving average of the data series constructed in step 1:
 - (a) Identification of turning points (higher or lower than 2 quarters on each side).
3. Using a 2-quarter moving average of the data series constructed in step 1:
 - (a) Identification of peaks (troughs) within 2 quarters of turns found in step 2.

4. Using a 2-quarter moving average of the original data series:
 - (a) Identification of peaks (troughs) within 2 quarters of turns found in step 3.
5. Using the original data series:
 - (a) Identification of peaks (troughs) within 2 quarters of turns found in step 4.
 - (b) Elimination of turning points within 2 quarters of the beginning and the end of the data series.
 - (c) Elimination of peaks (troughs) at both ends of the data series which are lower (higher) than values closer to the end.
 - (d) Elimination of cycles whose duration is less than 5 quarters.
 - (e) Elimination of phases whose duration is less than 2 quarters.

C.B Tables

	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough
NBER	1961Q1	1969Q4	1970Q4	1973Q4	1975Q1	1980Q1	1980Q3	1981Q3	1982Q4	1990Q3	1991Q1	2001Q1	2001Q4						
SBBP	1960Q4	1969Q3	1970Q4	1973Q4	1975Q1	1980Q1	1980Q3	1981Q3	1982Q2	1990Q2	1991Q1								

Table C.1: Comparison of SBBP and NBER turning points

	Start	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough
USA	1960Q1	1960Q4	1969Q3	1970Q4	1973Q4	1975Q1	1980Q1	1980Q3	1981Q3	1982Q2	1990Q2	1991Q1						
Canada	1961Q1																	
U.K.	1960Q1	1961Q2	1961Q4	1973Q2	1974Q3	1975Q2	1974Q3	1975Q3	1979Q2	1981Q1	1990Q2	1992Q2						
France	1960Q1			1974Q3	1974Q3	1975Q2	1980Q1	1980Q4										
Australia	1960Q1	1961Q3		1974Q3	1974Q3	1975Q4	1977Q2	1983Q2										
Korea	1970Q1	N/A	N/A	N/A	N/A	N/A	N/A	1980Q4	1979Q2	1980Q4	1997Q3	1998Q2						
Mexico	1993Q1	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	1995Q1	1995Q4	2001Q1	2002Q2				
South Afr.	1960Q1	1974Q3	1975Q1	1976Q3	1976Q3	1977Q3	1981Q4	1983Q1	1984Q2	1986Q1	1989Q3	1992Q4						

Table C.2: Turning points identified by SBBP

	Entire Sample						1985-2011 (Great Moderation)						1993Q3-2011 (Same Length)							
	Peak to Peak			Trough to Trough			Peak to Peak			Trough to Trough			Peak to Peak			Trough to Trough				
	#	Avg	Max	Min	#	Avg	Max	Min	#	Avg	Max	Min	#	Avg	Max	Min	#	Avg	Max	Min
USA	5	29.6	69	5	6	31.3	72	5	2	51.5	69	34	2	53.5	72	35	1	69.0	-	-
Canada	2	53.5	73	34	2	52.0	72	32	2	53.5	73	34	2	52	72	32	1	73.0	-	-
U.K.	6	31.8	70	4	5	37.0	67	5	3	40.7	70	9	2	55.5	67	44	2	39.5	70	9
France	4	47.0	63	21	4	48.0	63	21	2	55.0	63	47	2	55.5	63	48	1	63.0	-	-
Australia	2	32.5	53	12	3	38.7	56	29	1	53	-	-	1	31	-	-	0	N/A	-	-
<i>Developed</i>	3.8	38.9	65.6	15.2	4	41.4	66	18.4	2	50.7	65.6	35.4	1.8	49.5	61	30	1.3	61.1	68.8	53.5
Korea	2	57.5	72	43	2	55.5	69	42	2	57.5	72	43	2	55.5	69	42	2	57.5	72	43
Mexico	2	25.5	29	22	2	27.0	28	26	2	25.5	29	22	2	27	28	26	2	25.5	29	22
South Afr.	5	26.2	75	7	5	26.4	65	9	2	47.5	75	20	3	34	65	11	1	75.0	-	-
<i>Emerging</i>	3	36.4	58.7	24	3	36.3	54	25.7	2	43.5	58.7	28.3	2.3	38.8	54.0	26.3	1.7	52.7	58.7	46.7

Table C.3: Business Cycle Characteristics

		Consumption	Government	Imports	Exports
USA	BM	0.80	0.57	3.07	2.65
Canada	BM	0.63	0.81	3.46	2.88
U.K.	BM	1.14	0.71	2.62	1.95
France	BM	0.78	0.58	3.49	2.98
Australia	BM	0.89	1.01	5.20	3.06
<i>Developed</i>		<i>0.65</i>	<i>0.74</i>	<i>5.57</i>	<i>2.70</i>
Korea	BM	1.20	0.46	2.93	2.52
Mexico	BM	1.00	0.41	2.61	2.40
South Afr.	BM	1.33	1.19	5.42	2.21
<i>Emerging</i>		<i>1.18</i>	<i>0.69</i>	<i>3.65</i>	<i>2.38</i>
USA	NBER	0.81	0.56	3.18	2.72
Canada	NBER	0.76	0.77	3.47	2.77
U.K.	NBER	1.08	0.72	2.41	1.98
France	NBER	0.78	0.58	3.19	2.52
Australia	NBER	0.80	1.03	4.66	2.76
<i>Developed</i>		<i>0.85</i>	<i>0.73</i>	<i>3.38</i>	<i>2.55</i>
Korea	NBER	1.11	0.53	2.81	2.51
Mexico	NBER	1.08	0.46	2.64	2.42
South Afr.	NBER	1.29	1.09	5.20	2.44
<i>Emerging</i>		<i>1.16</i>	<i>0.69</i>	<i>3.55</i>	<i>2.46</i>
USA	ES	0.90	0.58	3.17	2.46
Canada	ES	1.12	0.94	2.30	2.45
U.K.	ES	1.25	0.75	2.29	1.62
France	ES	0.93	0.68	2.46	1.97
Australia	ES	0.62	0.95	2.67	2.19
<i>Developed</i>		<i>0.96</i>	<i>0.78</i>	<i>2.58</i>	<i>2.14</i>
Korea	ES	1.13	1.13	2.87	2.90
Mexico	ES	0.77	0.25	1.54	1.34
South Afr.	ES	1.31	1.16	4.95	2.24
<i>Emerging</i>		<i>1.07</i>	<i>0.85</i>	<i>3.12</i>	<i>2.16</i>

Table C.4: Relative Std. Dev. to GDP (Full Sample)

		Consumption	Government	Imports	Exports
USA	BM	0.68	0.62	2.76	2.65
Canada	BM	0.50	0.49	2.51	2.23
U.K.	BM	0.72	0.39	2.13	1.77
France	BM	0.49	0.45	2.64	2.83
Australia	BM	0.78	0.60	4.85	2.07
<i>Developed</i>		<i>0.63</i>	<i>0.51</i>	<i>2.98</i>	<i>2.31</i>
Korea	BM	1.42	0.44	3.04	2.06
Mexico	BM	1.00	0.41	2.61	2.40
South Afr.	BM	1.09	0.74	3.73	2.39
<i>Emerging</i>		<i>1.17</i>	<i>0.53</i>	<i>3.13</i>	<i>2.28</i>
USA	NBER	0.80	0.70	3.15	3.33
Canada	NBER	0.70	0.77	2.90	2.38
U.K.	NBER	0.99	0.43	2.59	2.06
France	NBER	0.61	0.47	3.01	2.96
Australia	NBER	0.86	0.62	5.16	2.04
<i>Developed</i>		<i>0.79</i>	<i>0.60</i>	<i>3.36</i>	<i>2.55</i>
Korea	NBER	1.42	0.53	3.13	2.47
Mexico	NBER	1.08	0.46	2.64	2.42
South Afr.	NBER	1.33	0.74	4.48	2.76
<i>Emerging</i>		<i>1.28</i>	<i>0.58</i>	<i>3.42</i>	<i>2.55</i>
USA	GM	1.14	0.79	2.66	2.51
Canada	GM	0.72	0.52	1.68	1.89
U.K.	GM	1.34	0.71	2.19	1.61
France	GM	0.99	0.69	2.37	1.14
Australia	GM	-	-	-	-
<i>Developed</i>		<i>1.05</i>	<i>0.68</i>	<i>2.23</i>	<i>1.79</i>
Korea	GM	1.88	0.59	4.03	2.72
Mexico	GM	1.00	0.41	2.61	2.40
South Afr.	GM	1.31	1.24	5.03	2.58
<i>Emerging</i>		<i>1.40</i>	<i>0.75</i>	<i>3.89</i>	<i>2.57</i>

Table C.5: Relative Std. Dev. to GDP (1985-2011)

		Consumption	Government	Imports	Exports
USA	BM	0.89	-0.70	0.83	0.50
Canada	BM	0.86	-0.38	0.77	0.73
U.K.	BM	0.84	-0.20	0.75	0.47
France	BM	0.72	-0.11	0.81	0.80
Australia	BM	0.48	-0.21	0.57	0.01
<i>Developed</i>		<i>0.76</i>	<i>-0.32</i>	<i>0.75</i>	<i>0.50</i>
Korea	BM	0.82	-0.26	0.85	0.31
Mexico	BM	0.89	-0.11	0.87	0.86
South Afr.	BM	0.85	0.14	0.85	0.20
<i>Emerging</i>		<i>0.85</i>	<i>-0.08</i>	<i>0.86</i>	<i>0.46</i>
USA	NBER	0.92	-0.81	0.86	0.48
Canada	NBER	0.89	-0.46	0.80	0.77
U.K.	NBER	0.80	-0.29	0.80	0.62
France	NBER	0.82	0.04	0.84	0.81
Australia	NBER	0.45	-0.02	0.55	0.13
<i>Developed</i>		<i>0.78</i>	<i>-0.31</i>	<i>0.77</i>	<i>0.56</i>
Korea	NBER	0.78	-0.37	0.85	0.40
Mexico	NBER	0.91	-0.14	0.89	0.85
South Afr.	NBER	0.86	0.09	0.85	0.31
<i>Emerging</i>		<i>0.85</i>	<i>-0.05</i>	<i>0.86</i>	<i>0.52</i>
USA	ES	0.89	-0.30	0.77	0.36
Canada	ES	0.79	0.30	0.82	0.40
U.K.	ES	0.88	0.13	0.85	0.62
France	ES	0.91	0.32	0.88	0.70
Australia	ES	0.28	0.16	0.33	0.34
<i>Developed</i>		<i>0.75</i>	<i>0.12</i>	<i>0.73</i>	<i>0.48</i>
Korea	ES	0.43	0.27	0.55	0.44
Mexico	ES	0.80	-0.65	0.91	0.99
South Afr.	ES	0.75	0.07	0.76	0.39
<i>Emerging</i>		<i>0.66</i>	<i>-0.10</i>	<i>0.74</i>	<i>0.61</i>

Table C.6: Correlation with own GDP (Full Sample)

		Consumption	Government	Imports	Exports
USA	BM	0.83	-0.75	0.84	0.78
Canada	BM	0.84	-0.40	0.70	0.77
U.K.	BM	0.86	-0.21	0.72	0.52
France	BM	0.68	-0.53	0.85	0.85
Australia	BM	0.49	-0.14	0.67	-0.32
<i>Developed</i>		<i>0.80</i>	<i>-0.41</i>	<i>0.76</i>	<i>0.52</i>
Korea	BM	0.44	0.04	0.89	0.44
Mexico	BM	0.89	-0.11	0.87	0.86
South Afr.	BM	0.88	0.05	0.82	0.55
<i>Emerging</i>		<i>0.74</i>	<i>-0.01</i>	<i>0.86</i>	<i>0.62</i>
USA	NBER	0.93	-0.80	0.92	0.82
Canada	NBER	0.88	-0.48	0.80	0.83
U.K.	NBER	0.94	-0.48	0.82	0.69
France	NBER	0.82	-0.63	0.93	0.89
Australia	NBER	0.65	0.19	0.75	-0.26
<i>Developed</i>		<i>0.84</i>	<i>0.44</i>	<i>0.84</i>	<i>0.59</i>
Korea	NBER	0.85	-0.00	0.91	0.48
Mexico	NBER	0.91	-0.14	0.89	0.85
South Afr.	NBER	0.93	-0.08	0.87	0.69
<i>Emerging</i>		<i>0.90</i>	<i>-0.07</i>	<i>0.89</i>	<i>0.67</i>
USA	GM	0.97	0.06	0.83	0.07
Canada	GM	0.90	0.05	0.77	0.65
U.K.	GM	0.93	0.07	0.88	0.53
France	GM	0.99	0.46	0.89	0.86
Australia	GM	-	-	-	-
<i>Developed</i>		<i>0.95</i>	<i>0.16</i>	<i>0.84</i>	<i>0.53</i>
Korea	GM	0.89	0.60	0.76	-0.71
Mexico	GM	0.80	-0.65	0.91	0.99
South Afr.	GM	0.86	-0.12	0.67	0.63
<i>Emerging</i>		<i>0.85</i>	<i>-0.06</i>	<i>0.78</i>	<i>0.30</i>

Table C.7: Correlation with own GDP (1985-2011)

		USA	Canada	U.K.	France	Australia	Korea	Mexico	South Africa
USA	BM	1.00	0.76	0.66	0.58	0.26	0.27	0.87	0.03
Canada	BM		1.00	0.58	0.56	0.52	0.11	0.62	0.43
U.K.	BM			1.00	0.69	0.32	-0.03	0.51	0.22
France	BM				1.00	0.28	0.34	0.60	0.38
Australia	BM					1.00	-0.17	0.25	0.48
Korea	BM						1.00	0.35	-0.04
Mexico	BM							1.00	0.26
South Afr.	BM								1.00
USA	NBER	1.00	0.81	0.75	0.36	0.39	0.38	0.75	0.11
Canada	NBER		1.00	0.68	0.46	0.63	0.21	0.61	0.48
U.K.	NBER			1.00	0.51	0.48	-0.01	0.50	0.30
France	NBER				1.00	0.35	0.10	0.68	0.48
Australia	NBER					1.00	-0.01	0.42	0.54
Korea	NBER						1.00	0.18	-0.05
Mexico	NBER							1.00	0.37
South Afr.	NBER								1.00
USA	CS	1.00	0.64	0.70	0.37	0.36	0.29	0.61	0.22
Canada	CS		1.00	0.74	0.38	0.29	0.51	0.13	0.13
U.K.	CS			1.00	0.50	0.35	0.50	0.29	0.16
France	CS				1.00	0.21	0.55	0.60	0.23
Australia	CS					1.00	-0.02	0.14	0.50
Korea	CS						1.00	0.15	-0.19
Mexico	CS							1.00	0.01
South Afr.	CS								1.00

Table C.8: Business Cycle: Correlation with Foreign GDP (All dates)

		USA	Canada	U.K.	France	Australia	Korea	Mexico	South Africa
USA	BM	1.00	0.86	0.83	0.74	0.69	0.21	0.79	0.63
Canada	BM		1.00	0.88	0.68	0.78	0.37	0.65	0.74
U.K.	BM			1.00	0.62	0.80	0.48	0.50	0.81
France	BM				1.00	0.50	0.14	0.63	0.67
Australia	BM					1.00	0.12	0.39	0.65
Korea	BM						1.00	-0.08	0.50
Mexico	BM							1.00	0.31
South Afr.	BM								1.00
USA	NBER	1.00	0.86	0.83	0.74	0.69	0.21	0.82	0.63
Canada	NBER		1.00	0.88	0.68	0.78	0.37	0.66	0.74
U.K.	NBER			1.00	0.62	0.60	0.48	0.52	0.81
France	NBER				1.00	0.45	0.14	0.68	0.67
Australia	NBER					1.00	0.12	0.44	0.65
Korea	NBER						1.00	-0.08	0.50
Mexico	NBER							1.00	0.35
South Afr.	NBER								1.00
USA	GM	1.00	0.94	0.86	0.56	-	0.25	0.18	0.50
Canada	GM		1.00	0.76	0.57	-	0.16	0.12	0.31
U.K.	GM			1.00	0.56	-	0.37	0.31	0.63
France	GM				1.00	-	0.86	0.18	-0.01
Korea	GM						1.00	0.16	0.05
Mexico	GM							1.00	-0.03
South Afr.	GM								1.00

Table C.9: Business Cycle: Correlation with Foreign GDP (1985-2011)