A Structural Theory of Spatial Abilities

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A cylindrical-wedge model is proposed to represent the correlational structure of a variety of spatial ability tests. The model corresponds to the design of the tests' content, according to three facets: (1) type of rule task, (2) dimensionality of the test items, and (3) need to mentally rotate test objects in space. Additional facets are suggested to refine the theoretical and empirical structure. The model emphasizes regionality for representing interrelationships as an alternative to factor analytic models which seek meaningful reference axes. The axis approach has not supplied an unambiguous theory that unifies content classification with the empirical structure of spatial abilities; it is also technically more awkward and less parsimonious than the regional approach. This paper advances theory and data analysis in the field of spatial ability by providing a unified conceptual framework that can be refined and expanded systematically, and that serves as an actual experimental design that can be easily executed by other workers in the field. Existing data are shown to support the regional cylindrical-wedge model. *Index terms: facet theory, factor analysis, intelligence, mapping sentence, Smallest Space Analysis, spatial ability.*

Anastasi (1983) stated that major trait theories on the structure and organization of intelligence can be subsumed under four types of models: (1) two-factor, (2) multiple-factor, (3) facet, and (4) hierarchical. Numerous critiques of the use of factor analysis in the study of cognitive abilities have appeared in the literature. The most recent is that of Carroll (1988). The present paper deals with one particular group of intellectual abilities within the framework of two of the above-mentioned models. A brief critical review is provided of the respective contributions of fac-

tor analysis and regional analysis to the elaboration of the structure of spatial abilities. A facet design and regional model for spatial abilities is then presented that makes explicit the relationship between theory construction, test design, and subsequent empirical analysis of the data.

The Contribution of Factor Analytic Studies to a Theory of Spatial Abilities

Since the 1920s, when spatial ability was treated by factor analysis as a subtype of intellectual ability (Brown & Stephenson, 1933; El-Khoussy, 1935; Kelly, 1928; Murphy, 1936), numerous efforts have been made to further differentiate spatial ability into at least two or more sub-abilities. Research done in the 1940s and 1950s consistently suggested two possible spatial factors: spatial orientation and spatial visualization (Michael, 1954; Michael, Guilford, Fruchter, & Zimmerman, 1957; Michael, Zimmerman, & Guilford, 1950).

Subsequently, the many later studies based on factor analytic methods resulted in a large and apparently non-convergent collection of articles, each presenting its own definition of spatial sub-factors. Lohman (1988) listed ten spatial factors with specific tests that appeared in Eliot and Smith (1983). Because Lohman gave a synopsis of the enormous factor analytic literature, only some of the factor analytic studies that show the continuing problems in constructing a definitional system for spatial abilities are reviewed here.

Table 1 summarizes major factor analytic studies on the definition and structure of spatial abilities. The table includes some of the tests designed to measure abilities represented by designated factor names. Factor names are capitalized and are generally defined in the authors' words.

Guilford and Lacey (1947) reported that fac-

Table 1
 Factor Analytic Studies of Spatial Abilities

Investigator and Factors Specified	Tests Used
Kelly (1928)	
Sub-factor 1: perception and retention of geometric forms	
Sub-factor 2: a facility for the mental manipulation of shapes	
El-Khoussy (1935)	
K: a group factor for the ability to obtain and the facility to utilize spatial imagery	
Guilford & Lacey (1947) (AAF)	
<i>SPATIAL RELATIONS (SR)</i> : relating different stimuli to different responses, arranged in spatial order	<i>SR</i> : Instrument comprehension, aerial orientation, stick and rudder orientation, flags, figures, cards, dial and table reading, discrimination reaction time, cubes, two-hand coordination, spatial orientation
<i>SPATIAL VISUALIZATION (VZ)</i> : ability to imagine the rotation of depicted objects, the folding or unfolding of flat patterns, the relative changes of position of objects in space, the notion of machinery	<i>VZ</i> : Mechanical principles, mechanical movements, spatial visualization, pattern comprehension, directional plotting, mechanical comprehension, spatial visualization
Thurstone (1950)	
S ₁ : ability to recognize identity of an object when seen from different angles and to visualize a rigid configuration when it is moved into different positions	S ₁ : Flags, cards, figures
S ₂ : ability to visualize a configuration in which there is movement or displacement among its parts	S ₂ : Surface development, mechanical movements, paper puzzles
S ₃ : ability to think about spatial relations in which the body orientation of the observer is an essential part of the problem	S ₃ : Lozenges, cubes
French (1951)	
<i>SPACE (S)</i> : ability to perceive spatial patterns accurately and to compare them with each other	
<i>SPATIAL ORIENTATION (SO)</i> : ability to remain unconfused by the varying orientations in which a spatial pattern may be presented	
V ₁ : ability to comprehend imaginary movement in 3-D space, or to manipulate objects in imagination	
Zimmerman (1954)	
Perceptual speed, spatial relations, visualization and reasoning factors, on continuum of complexity	
Michael et al. (1957)	
<i>SPATIAL RELATION & ORIENTATION (SR-O)</i> : ability to comprehend the nature of the arrangement of elements within a visual stimulus pattern with respect to the examinee's body as the frame of reference	<i>SR-O</i> : Instrument comprehension, directional orientation, cubes, flags, figures and cards, paper puzzles, Guilford-Zimmerman spatial orientation, aerial orientation, etc.
<i>VISUALIZATION (Vz)</i> : mental manipulation of a highly complex stimulus pattern	V _z : Directional plotting, spatial visualization, mechanical movements, pattern comprehension, form board, G-Z spatial visualization

continued on the next page

Table 1, continued
Factor Analytic Studies of Spatial Abilities

<p><i>KINAESTHETIC IMAGERY (K)</i>: one movement to vicariously move an entire configuration through turning or twisting it left or right</p> <p>Werdelin (1961)</p> <p>Spatial orientation: examinees react to entire figure</p> <p>Visualization: examinees react to parts of figure</p> <p>French et al. (1963)</p> <p><i>SPATIAL ORIENTATION</i>: perception of the position and configuration of objects in space (observer as reference point)</p> <p><i>VISUALIZATION</i>: ability to manipulate the stimulus and alter its image</p> <p>Smith (1964)</p> <p><i>SR-O</i>: an ability to manipulate the <i>same</i> shape</p> <p><i>Vz</i>: an ability to perceive, retain, and recognize a <i>whole</i> configuration (i.e., to discriminate between shapes)</p> <p>Pawlik (1966)</p> <p><i>SPATIAL RELATIONS</i>: identification of an object when seen from different angles</p> <p><i>SPATIAL ORIENTATIONS</i>: observer's body orientation as an essential part of the problem</p> <p>Wilson et al. (1975)</p> <p>Spatial Factor</p> <p>Yen (1975)</p> <p><i>SPATIAL ORIENTATION</i></p> <p><i>SPATIAL VISUALIZATION</i></p> <p>Goldberg & Meredith (1975)</p> <p><i>GENERAL SPATIAL FACTOR</i></p> <p>McGee (1979)</p> <p><i>SPATIAL FACTORS</i>: Involve comprehension of the arrangement of elements within a visual stimulus pattern, and the aptitude to remain unconfused by the changing orientation</p> <p><i>VISUALIZATION FACTORS</i>: involve the ability to mentally manipulate, rotate, twist or invert a pictorially-presented stimulus object</p> <p>Ekstrom et al. (1979)</p> <p><i>FIGURAL FLUENCY (FF)</i>: ability to make many different decorations on two common objects</p> <p><i>VISUAL MEMORY (MV)</i>: ability to identify irregular forms previously seen</p> <p><i>SPEED OF CLOSURE (CS)</i>: ability to unite an apparently disparate perceptual field into a single gestalt</p> <p><i>SPATIAL ORIENTATION (S)</i>: ability to perceive spatial patterns or to maintain spatial orientation with respect to objects in space</p>	<p>K: Hands, bolts</p> <p>S: Card rotations, cube comparisons, spatial orientation</p> <p>V: Form board, paper folding, surface development</p> <p>Shepard-Metzler mental rotations, card rotations, hidden patterns, paper form board, Raven's Progressive Matrices</p> <p>Orientation: Primary mental abilities spatial relations test, Shepard-Metzler mental rotations test</p> <p>Visualization: Form board test, paper folding test</p> <p>Cards test, paper form board, paper folding, mental rotations</p> <p>FF: Ornamentation, elaboration, symbols</p> <p>MV: Shape memory, building memory, map memory</p> <p>CS: Gestalt completion, concealed words, snowy pictures</p> <p>S: Card rotations, cube comparisons, spatial aspects</p>
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tor analysis of Army Air Force (AAF) tests administered to Air Force personnel suggested two spatial factors: spatial relations (SR), and spatial visualization (V_z). Thurstone (1950) suggested three spatial factors: S_1 , S_2 , and S_3 . French (1951) proposed his version of spatial factors: space (S) and spatial orientation (SO). French also suggested a visualization factor: V_1 .

In an effort to synthesize "decidedly different notations and somewhat different psychological interpretations of the factors," Michael et al. (1957, p. 187) organized the factor names proposed in previous studies into three more general factor names: spatial relation and orientation (SR-O), visualization (V_z), and kinesthetic imagery (K). To distinguish among these three hypothesized factors, the Michael study considered the psychological aspects of each factor in tests of spatial ability. These aspects included complexity of stimulus, manipulative operations, examinee's bodily orientation to stimulus, movement of parts of stimulus object versus movement of entire object, and speed and power conditions. The Kit of Reference Tests for Cognitive Factors (French, Ekstrom, & Price, 1963) described Spatial Orientation and Visualization, using definitions similar to (but not identical to) those of Michael et al. (1957).

Smith (1964) summarized factor-analytic theories of spatial abilities proposed prior to 1964 and concluded that "a remarkable feature of the American research on spatial ability is the difficulty the American psychologists are finding in clarifying the distinctions between the different spatial factors and especially between SR-O and V_z " (p. 95).

Subsequent studies proposed a "single spatial visualization factor" (Wilson, De Fries, McClearn, & Vandenberg, 1975), and a "general spatial factor" (Goldberg & Meredith, 1975). Yen (1975) used marker tests for the visualization and orientation factors. However, Yen also took into account aspects of the stimulus (dimensionality) and mental processes (rotation). She noticed that although tests of spatial orientation demanded mental rotation to solve the problems,

tests of visualization did not. After analyzing eight spatial ability tests, Poltrock and Brown (1984) proposed a single factor representing broad visualization ability; they attempted by means of path analysis to decompose spatial visualization into cognitive components.

In his review of spatial studies, McGee (1979) concluded that "... after 70 years of psychometric research, there is still vast disagreement about just how best to classify standard tests of spatial abilities" (p. 892). He offered one general definition for all the visualization factors and one for all the spatial orientation factors (see Table 1).

The 1979 monograph of Ekstrom, French, and Harman was a revision of the 1963 Kit of Reference Tests for Cognitive Factors (French et al., 1963). New tests were added to existing test batteries as markers for four factors with spatial ability names "figural fluency, visual memory, speed of closure, and spatial orientation." The 1979 monograph provides an example of a basic problem in the history of factor-analytic studies of spatial abilities: Even though researchers have used marker tests and have hypothesized about their factorial content, there has been no success in building an integrated theory of the organization of spatial abilities based on a priori definitions of specific tests and their roles in the model. Certain factor names, such as visualization and orientation, reappear in several different studies, but the various authors have not been consistent in assigning the same definitions to the same names. Table 1 shows that these studies have not resulted in a theoretical framework for the structure of spatial abilities because the factor names are often for vague and overlapping concepts.

A structural theory of spatial abilities, represented by a three-dimensional model based on facet theory and coupled with Smallest Space Analysis (SSA) of regionality, is discussed below. Instead of seeking reference axes (common factors) and attempting to name them a posteriori by the content of tests that correlate highly with them, the tests were classified a priori by facet design of their content. The space of smallest dimensionality for representing the test inter-

correlations was then sought, in which *regions* (not reference axes) corresponded to the facet design. Shye (1988, appendix) elaborates further points of contrast between smallest space and factor analysis, such as parsimony, dimensionality, and clarity of conceptual differentiation. (See also Sternberg & Powell, 1982, pp. 988–989. See Canter, 1985, for a more detailed introduction to facet theory.)

Facet Theory and Regionality

Mapping Sentence and Smallest Space Analysis

The mapping sentence device was introduced by Louis Guttman as a general way to define a universe of observations for a theory. A mapping sentence has two main parts: a formal part made up of its facets, and a less formal part comprising the phrases linking the facets together (see Shye, 1978, pp. 179–180; see also Hans, Bernstein, & Marcus, 1985; Levy, 1976, 1985). The mapping sentence in Figure 1 includes the three basic varieties of facets: population, content, and range. X designates the population of persons, and together with the facets that classify the *con-*

tent of the variables (ABC . . . N), makes up the *domain* of the mapping sentence. The third type of facet, the *range* (R) is the set of response categories specified for the item studied. Each respondent (x) in the population has one response in the range for each question classified by the elements of the juxtaposition ABC of the three content facets.

In essence, the mapping sentence serves to define a priori exactly what is being studied (the population and the content of variables), and the range of the possible responses. The example in Figure 1 is for the design of the Wechsler Intelligence Scale for Children (WISC; Wechsler, 1949). The 12 Wechsler subtests are a subset of the 27 kinds of tests defined by this $3 \times 3 \times 3$ ABC design of content. This mapping sentence and the subsequent discussion are presented here to show how the faceted model of the structure of spatial ability tests is an integral part of the general facet model of intelligence tests.

The facets here are for the design of empirical test items, in contrast to Guilford's (1959, 1967) use of facets for classifying factor names (see Guttman, 1958). Facet A lists elements for

Figure 1

Mapping Sentence for the Wechsler Intelligence Scale for Children (Levy, 1985; Reproduced by Permission)

the method expression, elements of facet B include three languages of communication, and facet C distinguishes among rule tasks imposed on the examinee.

Facet design thus provides the definitional system of the observations specified by the mapping sentence, which in turn suggests a rationale for empirical hypotheses. The set of intercorrelations among observations is the empirical aspect. The set of intercorrelations can be analyzed by SSA, in which a set of variables and their intercorrelations are geometrically portrayed in space (Guttman, 1965, 1968; Lingoes, 1973).

The division of the space into regions comes from boundary curves determined by structuples of the variables. A *structuple* is a profile of scores on each variable, composed by selecting an element from each facet. Each facet element (e.g., a_i is an element of the facet A) is called a *struct*, and structuples are made up of one struct from each facet (Shye, 1978). For instance, an oral vocabulary item on the WISC would have the following structuple: $a_1b_1c_1$.

SSA treats each variable as a point in an Euclidean space, such that the higher the correlation between two variables, the closer are the points in the space. The content facets specified in the mapping sentence can then be studied in conjunction with this empirical space to see if they correspond to regional partitions of the space. Each content facet has its own regional hypothesis—namely, that it corresponds to a partitioning of the SSA space into as many regions as there are elements to the facet.

A region should not be confused with a cluster. A *cluster* is defined as a subset of points that is separated from other points by relatively empty space around the subset. This is only a special kind of region; more generally, a region is a subset of points separated from other points by a boundary, and there need be no empty space or special gradient at all around the boundary. In particular, any uniformly distributed set of points can be partitioned into regions, without the slightest suggestion of clustering. Facet designs such as the one above usually imply a con-

tinuous space, in principle, for the universe of variables. Apparent clustering in sample data may be simply an artifact of unbalanced selection of a sample of variables from the universe of the mapping sentence.

The mapping sentence on the intelligence tests in Figure 1 led first to a two-dimensional representation—a *radex*—of its facet elements, and eventually to the *cylindrex*—a three-dimensional model of intelligence tests suggested by Guttman (1980). The radex, originally developed by Guttman (1965) and extended by Schlesinger and Guttman (1969), is shown in Figure 2. This radex represents a two-dimensional partitioning of the space into three polar and three modular regions. Each polar region corresponds to an element of facet B, Language of Communication—these are verbal, numerical, and figural (or spatial). Polar regions partition a circular space into wedge-like regions, all emanating from a common point. That facet B should play a polar role is to be expected, because its elements are unordered but related; the languages of communication are different, yet can impose the same rule tasks, and can be tested by the same modality of expression. This is an example of how facet roles correspond to specifications (related or not, ordered or unordered) for the facet elements. For a general discussion of the roles of facets (polar, modulating, axial) see Levy (1985).

Rule Task, or facet C, plays a modulating role, which means that elements of this facet are ordered with a maximal (or minimal) element that corresponds to the origin of a polar facet. The modular regions are concentric rings around an origin, each ring corresponding to an element of the modular facet. The order of the rings corresponds to the order of the facet elements. A rationale for the modular order for the facet of Rule Task is that rule application or rule learning does not require inference, whereas rule inference tasks cannot be solved by application or learning alone (for a description of such tasks see below).

The radex here is a two-dimensional space

Figure 2

The Radex of 18 Intelligence Tests (From Schlesinger and Guttman, 1969; Reproduced by Permission)

partitioned according to two facets—one polar and one modular—the two partitions intersecting each other to yield finer subregions than from each facet separately. (For a discussion of the significance of the radex for intelligence theory, and examples of radexes in intelligence research, see Adler & Guttman, 1982; Koop, 1985; Marshalek, Lohman, & Snow, 1983; Shye, 1988; Sternberg & Powell, 1982, p. 988; Tziner & Rimmer, 1984.)

Figure 3 shows a three-dimensional representation of the variables derived from the entire mapping sentence, including a third facet beyond the polar and modulating ones. Facet A refers to modality of expression, which may be oral, manual, or paper-and-pencil. Facet A plays an axial role. It enlarges the universe of variables, in a way that requires another dimension for the SSA representation of the enlarged correlation matrix. The regional hypothesis now becomes that of a three-dimensional cylindrex. The base of the cylindrex is a radex with the above-described two-dimensional regionality. Each two-

dimensional subregion in the radex becomes the three-dimensional solid in the cylinder rising vertically from that subregion. The vertical solids are divided, in turn, into three subsolids corresponding to three modalities of expression (facet A): oral, manual, and paper-and-pencil. The test items found in the cylindrex in Figure 3 were taken from intercorrelation matrices of WISC tests given to several age groups of both American and Israeli children; these data conformed to the cylindrical regional hypothesis (Guttman & Levy, in press; Levy, 1985).

For example, the middle section of the cylindrex in Figure 3 shows that, according to the regions in which it falls, Test 10 imposes an applicative rule task on the examinee (element 2 of the modulating facet C), deals with a geometrical (spatial) language (element 3 from the polar regions of facet B), and is solved through manual manipulations (element 2 of axial facet A). If the same test were written, it would have appeared in the same radial place but on the bot-

Figure 3
Schematic Representation of the Cylindrical Structure of the Wechsler Intelligence Scale for Children
(Adapted from Levy, 1985)

tom layer of the cylinder, because it would have fallen in the paper-and-pencil (not the manual manipulation) region of axial facet A. Test 10 is the Object Assembly Test from the WISC battery; its characteristics correspond to those depicted by its place in the cylindrex.

A Faceted Model of Spatial Ability Tests

Amir's Study

Amir (1976), Guttman and Shoham (1979), and Shilo (1985), used a faceted design of spatial abilities to assess the structure of the inter-correlations among tests and among items of spatial tests. According to the regional theory, spatial tests generated by the partnership between the facet design and SSA are represented by a

polar wedge in the cylindrical structure of intelligence tests (Geometrical in Figure 3).

A mapping sentence was constructed for designing a test battery in accordance with certain basic facets. In order to construct the mapping sentence that appears in Figure 4, the literature on spatial abilities, tests, and reported factors was surveyed to arrive at content facets that could be defined with clear and unambiguous elements. Facet A, Rule Task, was taken from the radex model of intelligence (Guttman, 1965, 1980; Schlesinger & Guttman, 1969). Presence or absence of Mental Rotation, facet B, appears in the spatial visualization factor of Guilford and Lacey (1947), in Thurstone's (1950) S_2 factor, and has also been described by Bock and Kolakowsky (1973), and by Stafford (1961). The importance of Dimensionality, facet D, in spatial ability was

Figure 4
Mapping Sentence for Spatial Ability Tests

suggested by Vandenberg (1969), Yen (1975), and McGee (1979). Facet C (presence or absence of Reflection) was added by Amir (1976) as an original content facet.

Because Amir's battery included only paper-and-pencil tests, there was no need for a facet of modalities of expression. The first three facets refer to the type of mental processes needed to solve the problem. Rule Task (facet A) specifies the type of reasoning needed: Either the examinee must find the rule necessary to solve the problem (rule inference), or he or she must apply the rules of a given relationship (rule application). For example, items on Raven's Progressive Matrices (Raven, 1960) require rule inference because the examinee must determine the logic of the pattern and then make an inference as to which piece will complete the pattern according to the rule or rules. The Hidden Blocks test (taken from a battery of the Israel Ministry of Education), however, is a rule application test; the examinee is told to find the number of blocks and to apply this "rule" by counting blocks. Rotation (facet B) means that a figure (or its parts) needs to be mentally rotated to reach a solution, and Reflection (facet C) is a recognition of the mirror-image of a figure or part of a figure. The facet of Dimensionality (facet D) classifies the stimulus as being either two- or three-dimensional.

Each of the eight tests was assigned one or

more structuple. The tests, with their structuples, are described below.

Hidden Patterns (French et al., 1963). This test is a rule application test (a_2) and all items are two dimensional (d_1). It was modified for this study by rotating some of the hidden figures. Thus it covers all four possible combinations of facets B and C (i.e., it includes items with and without rotation and/or reflection). Four structuples are represented:

$a_2b_1c_1d_1$, with rotation, with reflection.

$a_2b_2c_1d_1$, without rotation, with reflection.

$a_2b_1c_2d_1$, with rotation, without reflection.

$a_2b_2c_2d_1$, without rotation, without reflection.

Hidden Blocks (taken from a battery of the Israel Ministry of Education). This is also a rule application (a_2) test. All items are three-dimensional drawings (d_2), and no mental rotation (b_2) or reflection (c_2) is involved. It is represented by the structuple $a_2b_2c_2d_2$.

Shepard-Metzler Mental Rotation (Shepard & Metzler, 1971). All items are three-dimensional (d_2) and the test is of rule application (a_2). All items involve rotation (b_1), but no reflection (c_2). Thus the test is represented by the structuple $a_2b_1c_2d_2$.

Thurstone Test of Figure Analogies (Thurstone, Thurstone, & Thurstone, 1947). This is a rule inference (a_1) test in two dimensions (d_1). It is represented by the structuple $a_1b_2c_2d_1$.

Identical Blocks (Stafford, 1961). This is a rule application (a_2) test, and all items are three-dimensional (d_2). All items involve rotation (b_1). The structuple is $a_2b_1c_2d_2$. (This structuple is the same as for the Shepard-Metzler test.)

Raven's Progressive Matrices (Raven, 1960; 15 selected items). This is a well-known rule inference (a_1) test. All items are two-dimensional (d_1), and none involves rotation (b_2) or reflection (c_2). It is represented by the structuple $a_1b_2c_2d_1$.

Differential Aptitude Test (see Guttman & Shoham, 1979). The Differential Aptitude Test (DAT) of abstract reasoning is a rule inference (a_1) test. The items used are two-dimensional (d_1), and do not involve rotation (b_2) or reflection (c_2). The structuple is $a_1b_2c_2d_1$. (Tests 4, 6, and 7 have the same structuple.)

Card Rotation Test (French et al., 1963). This is a two-dimensional (d_1) rule application test (a_1). All items involve rotation (b_1), but not reflection (c_2). It is represented by the structuple $a_2b_1c_2d_1$.

This battery, therefore, comprised 11 structuples out of a possible 16 ($2 \times 2 \times 2 \times 2$).

The tests were administered to 90 students at the Hebrew University. The intercorrelations among 11 variables are shown in Table 2. These are coefficients of monotonicity (μ_2) that are more appropriate than Pearson coefficients for variables with unequal marginals (Guttman, 1986; Raveh, 1978). The Hidden Patterns Test is presented as four subtests, each having a different structuple, as described above. The tests in Table 2 are ordered according to the rotation facet; tests 1 through 5 require rotation, and tests 6 through 11 do not.

The correlation matrix was analyzed by SSA-I (Lingoes, 1973). In SSA, each test score is regarded as a point in the derived geometrical space. The relative distance between two points expresses the relative size of correlations of those two variables. The larger the correlation between two

variables, the smaller the Euclidean distance separating their points. Only relative distances between points are of concern here. Coordinate systems for the (smallest) space, or rotations of axes, have no intrinsic meaning. In a Euclidean space, distances between points do not depend on the choice of any particular coordinate system.

Amir (1976) hypothesized that an SSA of the data from these spatial tests (11 structuples) would result in a space of at least three dimensions, partitioned according to at least four content facets. Actually, she found differentiation in a three-dimensional space for all facets except Reflection. Figure 5 shows a two-dimensional projection that divides the space into two clear regions of Rule Task: three tests involving rule inference, and eight remaining tests of rule application. These results represents a further replication of the Schlesinger-Guttman (1969) radex. The high intercorrelations of the four Hidden Patterns subtests, as can be seen in Table 2, are expressed in their proximity in this projection.

Using three separate projections of Amir's SSA, a composite three-dimensional model was constructed of the structure of spatial ability tests, as represented by the 11 structuples. This model is shown in Figure 6. The model represents the spatial wedge of the facet, Languages of Communication, in the cylindrex of intelligence shown in Figure 3. In Figure 6, the regions into which each spatial test falls can be easily located. For instance, points 3 and 4, representing the Shepard-Metzler and Identical Blocks tests, are placed according to their structuple, $a_2b_1d_2$ (Reflection, facet C, has been omitted). Both tests require rule application; the points are in the outer ring of the Rule Task regions. The tests demand rotation (represented by the left polar region of facet B), and are three-dimensional (the middle region of the axial facet D).

Points 9 (DAT), 10 (Raven), and 11 (Thurstone) each have the same structuple profile. They are all rule-inference tests, they do not require rotation, and they are two-dimensional. The points are located accordingly in the model; the points representing the Hidden Patterns subtests (1, 2,

Table 2
Intercorrelations Among Spatial Subtests (Decimal Points Omitted)

Subtest	Rotation					No Rotation					
	1	2	3	4	5	6	7	8	9	10	11
Rotation											
Hidden Patterns: Rotation and Reflection	—	75	64	39	33	84	83	38	35	36	05
Hidden Patterns: Rotation	75	—	57	56	38	77	83	18	17	50	04
Shepard-Metzler	64	57	—	77	67	44	51	49	39	50	41
Identical Blocks	39	56	77	—	83	54	60	47	60	55	60
Card Rotations	33	38	67	83	—	35	45	44	40	54	52
No Rotation											
Hidden Patterns	84	77	44	54	35	—	92	26	43	46	32
Hidden Patterns: Reflection	83	83	51	60	45	92	—	35	38	48	22
Hidden Blocks	38	18	49	47	44	26	35	—	35	58	23
DAT	35	17	39	60	40	43	38	35	—	51	31
Raven	36	50	50	55	54	46	48	58	51	—	54
Thurstone Analogies	05	04	41	60	52	32	22	23	31	54	—

6, and 7), are close to one another in the two-dimensional application region, but fall in the regions “rotation” or “no rotation,” in accordance with the hypothesized structuples. Test 8 (Hidden Blocks, $a_2b_2c_2$) is represented by a point located on the application side of the no-rotation region of the polar facet B. It is a three-

dimensional, rule application test.

A further examination of Figure 6 shows that the facet of dimensionality conforms to regionality, but not in a simple fashion. The three-dimensional tests are all in the central region of the cylindrex, and the two-dimensional tests are divided into the two extreme regions of the cylin-

Figure 5
Two-Dimensional Projection of Amir's SSA Space Divided Into Regions of Rule Inference and Rule Application Tests

Figure 6
A Schematic Representation of the Eight Spatial Ability Tests (11 Structuples)
Corresponding to Three Facets in Amir's Study (1976)

drex. However, a further facet ("hidden" versus "non-hidden" objects) may be needed to distinguish between the Hidden Patterns structuples and the other two-dimensional tests. Thus, this model represents a preliminary structure based on three hypothesized facets of spatial ability: the rule task, the need for rotation, and dimensionality.

Factor Analysis of Amir's Data

To provide a basis for comparison between SSA and factor analysis of the same data matrix, the matrix presented in Table 2 was also subjected

to factor analysis using SPSS (Nie, Hull, Jenkins, Steinbrenner, & Bent, 1975). The analysis showed one eigenvalue greater than 1, and therefore factor extraction was stopped at one common factor (see Table 3). In normal use of factor analysis, the conclusion would be that the correlation matrix contained a single factor plus error. Three factors were then requested, and a varimax rotation with three common factors is presented in Table 4. This rotation should reveal a simple structure if one exists for the data, but the varimax program lacks an actual test of the simple structure hypothesis. An examination of the

Table 3
Factor Matrix of Amir's Data Using
Principal Factors With Iterations

Spatial Test	Factor 1*	Communality
Shepard-Metzler	.77	.59
Identical Blocks	.94	.89
Card Rotation	.80	.64
Hidden Patterns	.56	.31
Hidden Blocks	.57	.32
DAT	.60	.36
Raven	.73	.54
Thurstone	.60	.40

*Eigenvalue for Factor 1 = 1.40.

highest loadings on the first factor revealed Identical Blocks to be the "purest" test. Two other tests, Shepard-Metzler and Card Rotation, had high loadings on the first factor; all three tests require rotation. Identical Blocks, however, also had among the highest loadings on factor 2. No relationships were found between the loadings and the designs of the tests in factors 2 or 3.

A more direct comparison would be to do an SSA on the observed variables and (rotated) factors combined—that is, on the $(n + 3)(n + 3)$ matrix, instead of the original $n \times n$ correlation matrix. Then the factors would also appear as (three) points in the SSA space, and the regions into which these points fall could be seen. This was done for other mental test data by Schlesinger and Guttman (1969) and others (see references in Guttman, 1982), and showed immediately the superiority of the regional approach in obtaining correspondence with the definitional facets. The factor analytic results here were so poor that it was not deemed necessary to proceed to a more detailed analysis of their failure, and the SSA results provided a good correspondence with the hypothesized structure (see appendix in Shye, 1978).

The Guttman-Shoham Study

In a second, larger study, an attempt was made to validate the structural model presented in Figure 5. In accordance with Amir's results, the original mapping sentence was modified by omit-

ting the facet of reflection and adding a new facet to represent the test format. Because this was a behavior-genetic study and the sample consisted of families (see Guttman & Shoham, 1979), a facet of the examinees' family role was also added. The modified mapping sentence appears in Figure 7. (This mapping sentence has also appeared in Eliot & Smith, 1983.)

The new test battery included seven of the eight tests used by Amir. The rotation items were omitted from the Hidden Patterns Test. A new test was designed by Ruth Guttman and Ilana Shoham (Ilana Rotations) to incorporate the need for rotation in a two-dimensional test of rule inference. The structuple of this test is $b_1c_1d_1e_8$. A sample item of this test appears in Figure 8. As in Thurstone's Test of Figure Analogies (Thurstone et al., 1947), the examinee needs to infer the relationship between the two rotated sample figures and then find which of the four examples given relates in the same way to the figure in the box.

The battery of eight tests was administered to 260 families in Jerusalem (Guttman & Shoham, 1979), and the intercorrelation matrix of the test scores was analyzed by SSA-I. This matrix intercorrelated test scores from fathers, mothers, and each of two children; it was thus a 32×32 variables matrix. The matrix appears in full in Guttman and Shoham (1979).

The results of the SSA again revealed a regional type of correspondence between facet elements and correlation coefficients. It was found that several two-dimensional projections related sys-

Table 4
Varimax Rotated Factor Matrix for Amir's Data

Spatial Test	Factor 1	Factor 2	Factor 3
Shepard-Metzler	.69	.28	.35
Identical Blocks	.83	.52	.19
Card Rotation	.78	.23	.31
Hidden Patterns	.26	.55	.16
Hidden Blocks	.27	.17	.65
DAT	.25	.58	.25
Raven	.26	.51	.63
Thurstone	.42	.36	.20

Modified Mapping Sentence for Spatial Ability Tests (from Guttman & Shoham, 1979; Reproduced by Permission)

Figure 9 shows a two-dimensional projection of the SSA space corresponding to Rule Task that divides the space into rule application and rule

inference regions. In Figure 10, the space is divided into regions of rotation versus no rotation, according to facet E (Type of Test). For different family roles, the same tests tend to fall, with few exceptions, into the same subregions; the tests have essentially the same structure of interrelationship for father, mother, first child, and second child, respectively. In fact, Figure 11

Figure 8
An Example of the Ilana Rotations Test

represents an internal replication of the regionality of the Rule Task facet *within* each family group. Each family member—father, mother, older child, and younger child—has his or her own region in which the points of the eight tests are located. Within each family member's region, there is a clear partitioning of rule inference versus rule application tests. This shows that the tests in this study have essentially the same interrelationship for each of the family members, thus providing further evidence that two- and three-dimensional projections, and not individual reference axes, can lead to empirical lawfulness.

Again, this analysis did not give a clear partitioning of the SSA space for elements of facet C (Dimensionality). Therefore, the role of dimensionality in spatial tests remains unclear. Except for this facet, this SSA structure replicated the structure of the Amir data. The two-dimensional projections in Figures 9, 10, and 11 can fit into the three-dimensional model presented in Figure 6. An SSA of 51 selected items from the eight tests of the Guttman-Shoham battery also replicated the spatial structure according to the facets of

Rule Task and Rotation (Guttman & Shoham, 1982).

The Shilo Study

In a further study by Shilo (1985), two new tests were designed that required the “completion” and “construction” of figures with Lego blocks. In the first test the rule was given, and in the second test the examinee was required to find the rule for solving the problem. These tests were added to the Guttman-Shoham spatial battery and given to 80 pairs of school-aged twins as part of another behavior-genetic project. Shilo's intercorrelation matrix is given in Table 5. His SSA analysis replicated the results by Amir, and by Guttman and Shoham, with regard to the facets Rule Task and Rotation. Both new tests fell into a separate region of Manual Manipulation, with Completion in the rule application region, and Construction in the rule inference region. The model presented in Figure 6 could therefore be extended to include these two tests of manual manipulation, and could fit as a spatial wedge into two levels (paper-and-pencil, manual man-

Figure 9
Projection of the SSA Space Divided Into Regions of Rule Inference and Rule Application Tests
(From Guttman & Shoham, 1979; Reproduced by Permission)

Figure 10
Projection of the SSA Space Divided into Regions of Rotation and No-Rotation Tests
(From Guttman & Shoham, 1979; Reproduced by Permission)

Figure 11
Projection of the SSA Space Corresponding to Facet A: Family Role (Adapted from Guttman & Shoham, 1979)

Table 5
Intercorrelations Among Spatial Subtests in Shilo's Study
(Decimal Points Omitted)

Subtest	1	2	3	4	5	6	7	8
1. Hidden Patterns	—	78	78	72	68	71	72	80
2. Shepard-Metzler	78	—	65	85	44	73	62	79
3. Thurstone	78	65	—	87	76	75	77	73
4. Identical Blocks	72	85	87	—	56	82	71	81
5. Raven	68	44	76	56	—	50	46	56
6. Ilana	71	73	75	82	50	—	72	71
7. Completion of Figures	72	62	77	71	46	72	—	80
8. Construction of Figures	80	79	73	81	56	71	80	—

ipulation) of the Guttman-Levy cylinder of intelligence, as shown in Figure 3.

Discussion and Conclusions

Replicated faceted regional analyses of the structure of intercorrelations among spatial ability tests have been presented above. The lawfulness that is established was compared with results published in the factor-analytic literature. A summary of that literature in Table 1 shows a lack of consistency in assigning the same definitions to factors with the same names, and a subsequent absence of a unifying theory of framework.

The analysis of specific tests of spatial ability resulted in a preliminary model of the structure of spatial ability tests as part of the general model of intelligence tests; this clearly designates an *a priori* choice of tests and tasks to use, according to a definition of each test on the basis of its structuple. The universe of spatial tests generated by three facets—rule inference versus rule application, presence versus absence of need for rotation, and tests of paper-and-pencil versus manual manipulation—can be viewed in terms of a three-dimensional structure that represents a wedge in the cylindrex of intelligence tests proposed by Guttman (1980; Guttman & Levy, *in press*; Levy 1985).

The mapping sentence used here served as a definitional system to stipulate and organize into facets the variables being studied. Thus, hypotheses were made before tests were selected and empirical data collected. Also, three original tests were designed to combine rule inference

with the need for rotation, and two tests of manual manipulation were designed that distinguished between inference and application.

The facets of spatial ability tests can now be incorporated into the original Guttman-Levy mapping sentence (Figure 1), as shown in Figure 12. Rotation becomes an expansion and refinement of facet B, and gives an improved partition of the space. Dimensionality of Test Items apparently requires at least one additional facet. This study showed this facet to be problematic and to require further conceptualization and item construction.

Much more research is necessary for replication and expansion of the model proposed here. Tests need to be designed to represent the strutures theoretically possible in the mapping sentences, but missing from the particular studies relating to this research. For instance, two- and three-dimensional rule inference tests, with or without rotation, need to be designed. There is a need for many more tests requiring manual manipulation with elements of the other facets. Additional facets relating to spatial ability tests can be designated. The facet of Speed of Performance, for example, might be added, with elements of this facet comprising different time periods allowed for solution of the problem.

Lohman (1988) stated that "although the number of potentially identifiable spatial factors is quite large (perhaps even unbounded), a rational analysis of the factors in [his] Table 6.1 suggests that the number of distinct psychological processes required by spatial tasks may be much

Figure 12
Modified Guttman-Levy Mapping Sentence

smaller" (p. 188). In his "faceted spatial task," he lists processes seemingly required by marker tests that include different types of speed: in perception, closure, visualization, and in activities the examinee must perform, memorize, synthesize, relate, and match. Another facet (or set of facets) suggested by Lohman refers to the stimuli involved in the spatial task.

The approach suggested in this paper, of combining facet theory construction with a regional analysis, should facilitate fruitful progress in the classification and design of spatial ability tests, and in the conceptualization of spatial and other abilities.

References

- Adler, N., & Guttman, R. (1982). The radex structure of intelligence: A replication. *Educational and Psychological Measurement*, 42, 739-748.
- Amir, M. (1976). *A faceted analysis of patterns of spatial ability*. Unpublished master's thesis. Hebrew University of Jerusalem, Jerusalem, Israel.
- Anastasi, A. (1983). Evolving trait concepts. *American Psychologist*, 38, 175-184.
- Bock, R. D., & Kolakowsky, D. (1973). Further evidence of sex-linked major-gene influence on human spatial visualizing ability. *American Journal of Human Genetics*, 25, 1-14.
- Brown, W., & Stephenson, W. A. (1933). A test of the theory of two factors. *British Journal of Psychology*, 23, 352-370.
- Canter, D. (Ed.). (1985). *Facet theory: Approaches to social research*. New York: Springer-Verlag.
- Carroll, J. B. (1988). Cognitive abilities, factors, and processes. *Intelligence*, 12, 101-109.
- Ekstrom, R. B., French, J. W., & Harman, H. H. (1979). Cognitive factors: Their identification and replication (pp. 3-84). *Multivariate Behavioral Research Monographs*, No. 79-2.
- El-Khoussy, A. A. H. (1935). *The visual perception of space*. British Journal of Psychology Monograph Supplement No. 20.
- Eliot, J., & Smith, I. M. (1983). *International directory of spatial tests*. Windsor, England: NFER-Nelson.
- French, J. W. (1951). *The description of aptitude and achievement tests in terms of rotated factors*. Psychometric Monographs No. 5.
- French, J. W., Ekstrom, R. B., & Price, L. A. (1963). *Kit of reference tests for cognitive factors*. Princeton NJ: Educational Testing Service.
- Goldberg, J., & Meredith, W. (1975). A longitudinal

- study of spatial ability. *Behavior Genetics*, 5, 127-135.
- Guilford, J. P. (1959). Three faces of intellect. *American Psychologist*, 14, 469-479.
- Guilford, J. P. (1967). *The nature of human intelligence*. New York: McGraw-Hill.
- Guilford, J. P., & Lacey, J. I. (1947). *Printed classification tests A.A.F.* (Aviation Psychological Progress Research Report No. 5). Washington DC: U.S. Government Printing Office.
- Guttman, L. (1958). What lies ahead for factor analysis? *Educational and Psychological Measurement*, 18, 497-515.
- Guttman, L. (1965). *The structure of interrelations among intelligence tests*. Proceedings of the 1964 Invitational Conference on Testing Problems. Princeton NJ: Educational Testing Service.
- Guttman, L. (1968). A general nonmetric technique for finding the smallest coordinate space for a configuration of points. *Psychometrika*, 33, 469-506.
- Guttman, L. (1980). Integration of test design and analysis: Status in 1979. *New Directions for Testing and Measurement*, 5, 93-98.
- Guttman, L. (1982). Facet theory, smallest space analysis, and factor analysis. *Perceptual and Motor Skills*, 54, 491-493.
- Guttman, L. (1986). Coefficients of polytonicity and monotonicity. In S. Katz & N.L. Johnson (Eds.), *Encyclopedia of statistical science* (Vol. 7), (pp. 80-86). New York: Wiley.
- Guttman, L., & Levy, S. (in press). Two structural laws for intelligence tests. *Intelligence*.
- Guttman, R., & Shoham, I. (1979). Intrafamilial invariance and parent-offspring resemblance in spatial abilities. *Behavior Genetics*, 9, 367-368.
- Guttman, R., & Shoham, I. (1982). The structure of spatial ability items: A faceted analysis. *Perceptual and Motor Skills*, 54, 487-493.
- Hans, S., Bernstein, V., & Marcus, J. (1985). Some uses of the facet approach in child development. In D. Canter (Ed.), *Facet theory: Approaches to social research* (pp. 151-172). New York: Springer-Verlag.
- Kelly, T. (1928). *Crossroads in the mind*. Palo Alto CA: Stanford University Press.
- Koop, T. (1985). Replication of Guttman's structure of intelligence. In D. Canter (Ed.), *Facet theory: Approaches to social research* (pp. 237-244). New York: Springer-Verlag.
- Levy, S. (1976). Use of the mapping sentence for coordinating theory and research: A cross-cultural example. *Quality and Quantity*, 10, 117-125.
- Levy, S. (1985). Lawful roles of facets in social theories. In D. Canter (Ed.), *Facet theory: Approaches to social research* (pp. 59-96). New York: Springer-Verlag.
- Lingoes, J. C. (1973). *The Guttman-Lingoes nonmetric program series*. Ann Arbor MI: Mathesis Press.
- Lohman, D. F. (1988). Spatial abilities as traits, processes and knowledge. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 4), (pp. 181-248). Hillsdale NJ: Erlbaum.
- Marshalek, B., Lohman, D. F., & Snow, R. E. (1983). The complexity continuum in the radex hierarchical models of intelligence. *Intelligence*, 7, 107-128.
- McGee, M. G. (1979). Human spatial abilities: Psychometric studies and environmental, genetic, hormonal, and neurological influences. *Psychological Bulletin*, 86, 889-918.
- Michael, W. B. (1954). Suggested research approach to the identification of psychological processes associated with spatial visualization factors. *Educational and Psychological Measurement*, 14, 401-406.
- Michael, W. B., Guilford, J. P., Fruchter, B., & Zimmerman, W. S. (1957). The description of spatial-visualization abilities. *Educational and Psychological Measurement*, 17, 185-199.
- Michael, W. B., Zimmerman, W. S., & Guilford, J. P. (1950). An investigation of two hypotheses regarding the nature of the spatial relations and visualization factors. *Educational and Psychological Measurement*, 10, 187-243.
- Murphy, L. W. (1936). The relation between mechanical ability tests and verbal and non-verbal intelligence tests. *Journal of Psychology*, 2, 353-366.
- Nie, N. H., Hull, H. C., Jenkins, J. G., Steinbrenner, K., & Bent, D. H. (1975). *SPSS: Statistical package for the social sciences, Second Edition*. New York: McGraw-Hill.
- Poltrock, S. E., & Brown, P. (1984). Individual differences in visual imagery and spatial ability. *Intelligence*, 8, 93-138.
- Raveh, A. (1978). Finding periodic patterns in time series with monotone trend: A new technique. In S. Shye (Ed.), *Theory construction and data analysis in the behavioral sciences*. San Francisco CA: Jossey-Bass.
- Raven, J. C. (1960). *Guide to the standard progressive matrices*. London: H. D. Lewis.
- Schlesinger, I. M., & Guttman, L. (1969). Smallest space analysis of intelligence and achievement tests. *Psychological Bulletin*, 71, 95-100.
- Shepard, R. W., & Metzler, J. (1971). Mental rotation of three-dimensional objects. *Science*, 171, 701-703.
- Shilo, S. (1985). *Spatial ability in twins*. Unpublished master's thesis. Hebrew University of Jerusalem, Jerusalem, Israel.
- Shye, S. (Ed.). (1978). *Theory construction and data analysis in the behavioral sciences*. San Francisco CA: Jossey-Bass.
- Shye, S. (1988). Inductive and deductive reasoning: A

- structural reanalysis of ability tests. *Journal of Applied Psychology*, 73, 308-311.
- Smith, I. M. (1964). *Spatial ability: Its educational and social significance*. San Diego CA: Knapp.
- Stafford, R. E. (1961). Sex differences in spatial visualization as evidence of sex-linked inheritance. *Perceptual and Motor Skills*, 13, 428.
- Sternberg, R. J., & Powell, J. S. (1982). Theories of intelligence. In R. J. Sternberg (Ed.), *Handbook of human intelligence* (pp. 975-1005). Cambridge: Cambridge University Press.
- Thurstone, L. L. (1950). *Some primary abilities in visual thinking* (Rep. No. 59). Chicago IL: University of Chicago, Psychometric Laboratory.
- Thurstone, L. L., Thurstone, T. H., & Thurstone, G. (1947). *Psychological examination for college freshmen*. Washington DC: American Council on Education.
- Tziner, A., & Rimmer, A. (1984). Examination of an extension of Guttman's model of ability tests. *Applied Psychological Measurement*, 8, 59-69.
- Vandenberg, S. G. (1969). A twin study of spatial ability. *Multivariate Behavioral Research*, 4, 273-277.
- Wechsler, D. (1949). *Manual for the Wechsler Intelligence Scale for Children*. New York: Psychological Corporation.
- Wilson, J. R., De Fries, J. C., McClearn, G. E., & Vandenberg, S. G. (1975). Cognitive abilities: Use of family data as a control to assess sex and age differences in two ethnic groups. *International Journal of Aging and Human Development*, 6, 261-276.
- Yen, W. M. (1975). Sex-linked major-gene influences on selected types of spatial performance. *Behavior Genetics*, 5, 127-135.

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