Inhibition in Prolonged Work Tasks

A. H. G. S. van der Ven, J. C. Smit, and R. W. T. L. Jansen
University of Nijmegen

A new model is presented that explains reaction time fluctuations in prolonged work tasks. The model extends the so-called Poisson-Erlang model and can account for long-term trend effects in the reaction time curve. The model is consistent with Spearman's hypothesis that inhibition increases during work and decreases during rest. Predictions concerning the long-term trend were tested against data from the Bourdon-Vos cancellation test. The long-term trend in the mean and in the variance was perfectly described by the model. A goodness-of-fit test comparing frequency distributions of observed reaction and simulated reaction times was also supported by the model. *Index terms: concentration, continuous work, distraction, inhibition, prolonged work, reaction time, response time.*

Many factors contribute to the amount of time needed to solve an intelligence test item, and which factor becomes dominant depends on the type of test. Tests requiring access to knowledge may focus on memory search and retrieval, whereas in reasoning tests hypothesis selection and testing will have a random effect upon the response time. Sometimes examinees may be uncertain about the correct answer, and the resulting process of hesitation will take a randomly varying amount of time. If no satisfactory solution is obtained, the examinee must decide whether to continue with the item or

start the next item. Almost all tests require some time before the person adapts to the situation, which also influences the final response time. Clearly, a very complex model is needed to account for all of these factors. Even then, it is doubtful that such a model could be empirically tested.

This study focuses on the speed of *simple, over-learned, continuous reaction* tasks, which are usually referred to as prolonged or continuous work tasks. The typical prolonged work task consists of a series of simple items, which are administered in a self-paced continuous manner. The examinee's response to each item releases the next one in the sequence. No pauses are allowed.

The study of prolonged work tasks has a long history that has been described in Eysenck and Frith (1977, chap. 1). They referred to a study by Oehrman (1895), in which he used a variety of tasks that included counting letters, searching for letters, and adding single numbers. Prolonged work tasks have also been used to measure concentration ability. For example, Binet (1900) reported an extensive study on the measurement of concentration (he referred to it as "la force d'attention volontaire"), in which he used a so-called cancellation test originally proposed by Bourdon (1895).

Prolonged work tasks must be performed for a relatively long period, usually 10 to 30 minutes. Performance is measured by a time series that consists of either a series of *reaction times* (RTs), in which each is the result of a fixed number of responses, or a series of *response numbers* in which

each number is obtained in a fixed amount of time. A well-known example of the former is the Bourdon-Wiersma test (Huiskam & de Mare, 1947; Kamphuis, 1962), which is a cancellation test. A well-known example of the latter type of test is the Pauli test (Arnold, 1975), which is a single-digit addition task. The time series consists of the number of additions per minute during a 30-minute period.

This article focuses on prolonged work tasks that yield a time series consisting of RTs. Because most intelligence tests are prolonged work tasks, especially those that are timed, the results might aid in understanding the nature of intelligence tests.

The Inhibition Model

Pieters and van der Ven (1982) explored the concept of distraction as an explanation for the fluctuations in RT that occurred in prolonged work tasks. They assumed that the manifest RT should be considered as being composed of a relatively constant (over items) real total work time, interrupted by a series of random distraction times. This led to the formulation of the so-called Poisson-Erlang model. The notion of intermediate periods of distraction has already been suggested by many authors, such as Peak and Boring (1926), Bills (1931, 1935, 1964), and Berger (1982). The Poisson-Erlang model accounts for short-term variation in the latencies, but not for any long-term trend effect. The model presented here explains certain long-term trend effects, based on the assumption that distractions are periods of recovery from accumulated fatigue. This idea was also suggested by authors such as Spearman (1927), Bills (1935), and Hull (1951).

The model describes the statistical behavior of the RTs of an examinee over a series of equal tasks executed consecutively. It extends the Poisson-Erlang model (PE model), which assumes that:

1. Each task requires the same processing time A .
2. Work on a task is interrupted by involuntary rest pauses or distractions. The durations of the distractions are independent and exponentially distributed (with the same parameter δ).

3. The number N of distractions during a task is Poisson distributed (with parameter κ).

Hence the reaction time T on a task is composed of the processing time A and the sum of the distractions interrupting it, $D = d_1 + \dots + d_N$, giving

$$T = A + D \quad (1)$$

According to the PE model, the RTs $T_1, T_2, \dots, T_n$ on the successive tasks constitute a sequence of independent random variates with the same distribution. The model does not consider trend phenomena.

To better understand both the old and the new model, the assumptions above may be interpreted as follows. The examinee is alternately in State 1 (processing, work) and State 0 (distraction, rest). Let the transition rates from State 0 to State 1 and from State 1 to State 0 be denoted by λ_0 and λ_1 . When in State i ($i = 0$ or 1), the probability of jumping to the other state in the next infinitesimal interval dt equals $\lambda_i dt$. Thus, Assumptions 2 and 3 above follow from the assumption that these transition rates are constant.

Assumptions of the Model

The new model (the inhibition model) retains Assumptions 1 and 2. With the parameters of the new model, the constant value of the transition rate λ_0 is assumed to be

$$\lambda_0 = ac \quad (2)$$

Assumption 3, however, is relaxed in the following way. In the new model the transition rate λ_1 , which represents the inclination to stop working for a while, may increase with the level of inhibition. This new element of the model, *inhibition* (denoted by Y), increases during work and decreases during distractions. Therefore, the level of inhibition is similar to fatigue. More precisely, it is assumed (Assumption 4) that inhibition increases during work with rate 1 and decreases during distraction with rate a . Further, the transition rate λ_1 is a linearly increasing function of the inhibition y (restricted to be positive):

$$\begin{aligned} \lambda_1(y) &= c(1 + by)^+ \\ &= \max[0, c(1 + by)] \quad (3) \end{aligned}$$

The inhibition model is completely specified by Assumptions 1 and 4 and Equations 2 and 3. When the parameters A , a , b , and c are fixed and an initial value y_0 of the inhibition is selected, computer simulations of the inhibition process can be generated, as explained in the Appendix. Note that the new inhibition model includes the Poisson-Erlang model as a special case; setting $b = 0$ in Equation 3 results in constant transition rates.

The present model was described in an incomplete form in van Breukelen, Jansen, Roskam, van der Ven, and Smit (1987). In that presentation, a different parameterization proposed by Roskam was used in which the quantities Y and a were replaced by the quantities I and μ_2/μ_1 , respectively, where $I = c(1 + bY)$, $\mu_1 = bc$, and $\mu_2 = abc$.

Parameter Estimation

The empirical part of this article presents a goodness-of-fit procedure that compares observed and simulated RTs. The parameters used to generate the simulated RTs were estimated from the observed data. The Poisson-Erlang model was also applied as a rival model. In both models a modified method of moments was used to estimate the parameters. The term "modified" refers to the fact that the A parameter is estimated not by moments, but by taking the minimum RT. This procedure was previously used by Pieters (1985) and Pieters and van der Ven (1982).

According to the Poisson-Erlang model, the expectation of the RT equals

$$\mu = A + \frac{\kappa}{\delta} \quad (4)$$

and the variance of the RT equals

$$\sigma^2 = \frac{2\kappa}{\delta^2} \quad (5)$$

(Pieters & van der Ven, 1982, Equations 17 and 18).

The following estimators were used (for a derivation see Pieters and van der Ven, 1982, Equations 24 and 25):

$$\delta = \frac{2(m-A)}{s^2} \quad (6)$$

and

$$\kappa = \frac{2(m-A)^2}{s^2}, \quad (7)$$

where m and s^2 are sample estimates for μ and σ^2 , respectively. The inhibition model is, except for the shift parameter A , specified by the following parameters: y_0 , a , b , and c . According to the model, the expectation of the k th RT is

$$E(T_k) = \alpha + \beta\gamma^{k-1} \quad (k = 1, 2, \dots, n) \quad (8)$$

(see Appendix Equation A33). Estimators for y_0 , a , and b were based on Equation A33, resulting in

$$a = \frac{A}{\alpha - A}, \quad (9)$$

$$b = -\frac{\ln \gamma}{A}, \quad (10)$$

and

$$y_0 = \frac{\beta a}{1 - \gamma}. \quad (11)$$

The estimator for c is based on Appendix Equation A35, giving

$$c = \frac{2(1 - \gamma)}{a^2 b \sigma_e^2}, \quad (12)$$

where σ_e^2 is equal to the residual variance after elimination of the long-term trend:

$$\sigma_e^2 = \frac{\sum (T_k - \alpha - \beta\gamma^{k-1})^2}{n - 3}. \quad (13)$$

For both models the shift parameter A was set equal to the smallest T , $-.5$, because the RTs had been recorded to the nearest second.

Empirical Evidence

Several approaches were used to find empirical evidence for the model. First, a goodness-of-fit inquiry was performed based on an overall comparison of the observed and simulated frequency distributions. Second, a factor analysis was carried out to show the predicted long-term trend in the mean, and an analysis of variance was used to show the predicted long-term trend in the variance. Third, the autocorrelations up to the fifth lag were computed.

The Test and Examinees

The results reported below are based on data obtained from a cancellation test developed by Vos (1988). The test is known as the Bourdon-Vos test and is actually a modification for children of the Bourdon-Wiersma test (Huiskam & de Mare, 1947) that was originally proposed by Abelson (1911). A similar test was mentioned by Spearman (1927), who called it "Fours" (crossing out groups of four dots from among groups of three, four, and five dots). The test consisted of 33 lines, with each line containing, in random order, eight groups of three dots, eight groups of four dots, and eight groups of five dots. The average diameter of the groups of dots was about 5 mm. The examinees marked each group of four dots "as quickly and as accurately" as possible. Two lines on the back of the test sheet illustrated the procedure, with the instructor doing the first line to demonstrate how to perform the task, and the examinee completing the second line as practice. At the end of a line, examinees were instructed to start immediately at the beginning of the next line without a break. At the end of each line, the execution time spent was registered in seconds using a stopwatch, resulting in a series of 33 consecutive RTs. The examinees were 516 normal children from 6 to 14 years old (272 boys and 244 girls) who attended school in Venray, Oostrum, or Nijmegen.

Estimation of Model Parameters

To estimate the trend parameters α , β , and γ , the IMSL subroutine RSMITZ (International Mathematical and Statistical Libraries, 1982) was used, with PAR(3) = .0001. (PAR(3) refers to the maximum error allowed in the approximation of the least-squares estimate of γ .) In 204 cases a so-called IER = 33 message was reported that indicated "... the function which computes the error sum of squares for the model was not unimodal due to rounding errors ..." (see program description). To better understand these messages for each of the 204 cases, the following analysis was done. For each γ value in the range [0,1], α and β were

estimated by minimizing the residual sum of squares (SS error):

$$\text{SS error} = \sum_{k=1}^n (T_k - \alpha - \beta\gamma^{k-1})^2 \quad (14)$$

The SS error for each γ was compared with the SS error for $\gamma = 0$ and with the least-squares linear fit SS error, $y = a + bx$. In 44 cases the smallest SS error was found for $\gamma = 0$ and in 13 cases the interior minimum did not differ much from the SS error for $\gamma = 0$. In 85 cases the smallest SS error was found for the linear fit, and in 18 cases the interior minimum did not differ very much from the linear fit SS error. Therefore, 78% of the cases in which RSMITZ produced an IER = 33 message can be explained by the fact that, except for random fluctuations, the curve was either horizontal in the RTs 2 to 33, or linear in the RTs 1 to 33. For the remaining 44 cases, a further examination of the SS error curve revealed that:

1. In 8 cases the SS error for $\gamma = 0$ was equal to the linear fit SS error, with no other lower SS error;
2. In 12 cases the curve was very flat showing no variation in SS error, indicating a flat RT curve; and
3. In 24 cases a distinct global minimum was found.

In 312 cases the program did not produce an IER = 33 message. However, the program yielded reliable estimates of α , β , and γ in only 249 cases. The problems with the other 63 cases were as follows:

1. In 37 out of these 312 cases, the program produced a standard solution of γ equal to .000042. For all of these cases the smallest SS error was found for $\gamma = 0$.
2. In 22 cases the program produced a standard solution of γ equal to .999959; for these cases the smallest SS error was found for the linear fit.
3. Four cases were excluded because of extremely high values of γ ($\gamma > .99$); in these cases a linear fit also would have been justified.

Therefore, 249 cases had reasonable estimates of the trend parameters. The cases with $\gamma = 0$ or with the linear fit superior to the exponential fit do not contradict the model, although they cannot be used to estimate the parameters of the model, especially not the b parameter.

Frequency Distribution of Reaction Time

Reaction time models are usually tested by comparing the observed to the expected frequency distribution. The difference is often evaluated by a goodness-of-fit test such as the χ^2 test that is based on the assumption that the RTs are independent and identically distributed. This assumption generally does not hold in the inhibition model because the trend precludes identical distributions and autocorrelation prevents independent distributions.

In this study, the inhibition model was evaluated for goodness of fit compared to the PE model by using the following procedure. For each participant for whom the inhibition model parameters could be estimated, 33 RTs were simulated according to the inhibition model with parameter values estimated from the observed data. The simulated RTs were then placed in a frequency distribution with the same class limits as in the observed frequency distribution. The class limits of the latter were determined so that the number of observations was about the same in each class and each class contained at least five observations. On the resulting two-way frequency table, χ^2 was calculated as a measure of discrepancy between the observed and simulated distributions.

The same procedure was applied with datasets which were simulated according to the PE model. Thus, two χ^2 values were obtained for each of the 249 examinees—one for the inhibition model and one for the PE model. Finally, the difference between means of the two χ^2 measures was tested by the t test for correlated samples ($r = .18$). The inhibition model yielded a mean of 2.12 with standard deviation (SD) = 2.02; for the PE model the results were a mean of 2.53 and SD = 2.19. The

difference between the two means ($t = -2.39$) was statistically significant ($df = 248$, $p = .018$), which indicates that the inhibition model performed better.

Next, the magnitudes of the respective mean values of the modified χ^2 were evaluated by comparing these values with the results obtained from simulated data. For that purpose the 249 simulated datasets were treated as observed data. In only 175 cases (from the 249 simulated datasets) could the trend parameters α , β , and γ be estimated directly. In the other cases a new set of RTs was simulated (with the same inhibition model parameters) until the trend parameters could be estimated. Table 1 presents the number of times a new set had to be simulated.

The goodness-of-fit analysis based on these completely simulated datasets resulted in mean = 2.00, SD = 2.41 for the inhibition model, and mean = 2.58, SD = 2.30 for the PE model. With $r = .09$, the $t = -2.88$ ($df = 248$) was significant at $p = .004$. The resemblance of these values to the previous results is very strong, which indicates that the magnitude of the respective mean χ^2 values is exactly what should be expected according to the inhibition model.

Table 1
The Number of Times That A
New Simulation Was Necessary

Number of Repeated Simulations	Number of Examinees
0	175
1	41
2	10
3	9
4	2
5	4
6	2
9	1
10	2
15	1
17	1
29	1

Long-Term Trend in Mean Reaction Times

If the initial inhibition differs distinctly from its stationary mean, then an exponential trend effect of the form $y = \alpha + \beta\gamma^{k-1}$ ($k = 1, 2, 3, \dots, n$) emerges (See Equation A33). To show this effect, factor analyses were performed based on the 249 observed datasets and the corresponding 249 simulated datasets. Each analysis was based on the $n \times n$ correlation matrix of the successive RTs (occasions) where n was the number of RTs ($n = 33$) and where the correlations were over examinees ($N = 249$). This specific use of factor analysis has been applied by Kouwer and Hartong (1961) and van der Ven (1964).

The factors were extracted by the principal components method. After carefully examining trial solutions with different numbers of factors, the decision was made to extract three factors. However, because the inhibition model only involves two factors (a general level factor and a trend factor), the third factor should be considered an error factor. The unrotated first three factors are shown in Table 2.

The results clearly show a general level factor (Factor 1) and a long-term trend factor (Factor 2). The contribution of the third factor is very low, although the loadings show a definite long-term trend effect. The three positive loadings on Factor 3 at the end of the test are probably the result of an end-spurt effect. The long-term trend effect is probably caused by the examinees' attitude toward the task during the test. This long-term trend has a positive effect on the autocorrelation and might reduce the negative autocorrelation effect predicted by the model.

A similar factor analysis was also done based on the 249 corresponding simulated datasets. The results are in Table 3. Clearly, the first two factors in both tables are quite similar, which supports the model. The long-term trend in the third factor in Table 2 is not present in the third factor in Table 3, indicating that the effect is not artificial.

The first two factors predicted by the model were compared using the root mean square of the de-

viations to indicate the Procrustean goodness of fit of the loadings,

$$S_p = \left[\frac{\sum_j (a_{jp} - b_{jp})^2}{n} \right]^{1/2}, \quad (15)$$

where a is the loading of a factor based on the observed datasets, b is the loading of a factor based on the simulated datasets, j is a variable (in this case a reaction time), and p is a factor. The stress values for the first two factors were .032 and .086, respectively, which are quite acceptable. Thus, it appears that the results favor the inhibition model.

The question then arose as to whether a similar pattern could be found in a factor analysis based on the remaining 267 datasets. The results are shown in Table 4. The factor loadings show a similar pattern to those in Tables 2 and 3, which tends to support the inhibition model. The trend in the second factor is less salient and is probably caused by a corresponding weaker trend in the observed datasets that may explain why RSMITZ yielded an IER = 33 message for these datasets.

Long-Term Trend in the Variance of Reaction Times

According to the model, not only should a trend in the expectation of RT exist, but a trend in the variance should also occur (see Equation A35). To test this hypothesis, data for examinees with negative β time series ($N = 220$), indicating an increasing trend, were partitioned into 11 successive blocks of three consecutive RTs each. The analysis was based on the residual variance within each block. The residuals were computed by subtracting the exponential trend effect from the observed RT. The mean residual variances of the 11 blocks are shown in Table 5.

The results clearly show an increasing trend in the means of the residual variances. For each examinee, the linear trend contrast was computed. The mean of this contrast was significantly higher than 0 ($F_{1,219} = 36.41, p = .001$). Quadratic and higher polynomial trends were not significant. These results confirm the model.

Table 2
Means and Standard Deviations for Each Line
and Factor Loadings on the First Three Factors
for the 249 Observed Datasets

Line	Mean	SD	Factor		
			1	2	3
1	14.09	4.66	.72	.55	-.08
2	16.26	4.99	.76	.53	-.09
3	18.01	5.29	.76	.39	.11
4	16.18	4.19	.88	.15	.00
5	17.36	4.64	.85	.22	.00
6	17.01	4.90	.84	.25	-.16
7	17.09	4.46	.87	.13	.06
8	17.33	4.96	.86	.21	.10
9	17.51	4.57	.86	.12	.05
10	17.98	4.89	.87	.06	.05
11	17.63	4.75	.85	.01	.26
12	17.51	4.84	.87	.04	.09
13	17.31	4.92	.86	-.08	.07
14	16.71	4.97	.83	-.09	.12
15	18.42	5.52	.88	-.03	.02
16	17.87	4.92	.88	-.05	.06
17	17.68	4.88	.86	-.09	-.03
18	17.95	4.75	.84	-.23	.02
19	18.24	5.10	.86	-.13	.05
20	18.37	5.18	.85	-.11	-.25
21	18.44	5.28	.87	-.15	-.17
22	18.16	5.36	.86	-.16	-.02
23	18.85	5.54	.84	-.15	-.28
24	18.77	5.30	.89	-.17	-.09
25	18.06	5.11	.84	-.06	.01
26	17.90	4.90	.88	-.08	-.08
27	18.29	4.95	.84	-.15	-.27
28	17.63	4.97	.82	-.14	.07
29	18.63	4.93	.83	-.07	-.18
30	17.96	5.05	.83	-.14	-.13
31	18.22	5.46	.86	-.09	.19
32	17.64	4.55	.83	-.19	.21
33	17.57	4.02	.76	-.15	.34
Eigenvalue			23.46	1.31	.68

According to the model, a decreasing trend in the mean should coincide with a decreasing trend in the variance. To test this hypothesis, a similar analysis was performed on the data of the remaining 29 examinees with positive β , indicating a decreasing trend. The trend in the residual variances was also linearly decreasing, but was only significant at $p = .10$ ($F_{1,28} = 3.36$, $p = .077$). For ex-

aminees with a decreasing trend in the mean, a decreasing trend in the variance can also be observed, as the model predicts.

Autocorrelation

The inhibition model states that if the process is stationary, then, starting with the first lag, the au-

Table 3
Means and Standard Deviations for Each Line
and Factor Loadings on the First Three Factors
for the 249 Simulated Datasets

Line	Mean	SD	Factor		
			1	2	3
1	14.60	4.97	.67	.52	.28
2	16.10	4.67	.77	.50	-.05
3	16.50	5.28	.75	.38	.32
4	16.77	4.38	.83	.23	-.01
5	17.09	4.83	.80	.19	.11
6	17.37	4.98	.82	.17	-.05
7	17.56	5.07	.81	.24	-.28
8	17.33	4.63	.80	.22	-.02
9	17.57	5.13	.85	-.07	.16
10	17.43	4.66	.84	.06	-.20
11	17.60	4.58	.84	.04	-.23
12	18.00	6.15	.82	.13	-.27
13	17.83	5.30	.85	.01	.10
14	17.81	4.92	.85	-.02	-.08
15	17.98	5.09	.84	-.14	.00
16	17.94	5.16	.86	-.18	.05
17	17.53	4.59	.86	-.10	.00
18	18.04	5.19	.83	-.13	.12
19	18.12	5.06	.84	.09	-.14
20	18.02	5.15	.87	-.14	.15
21	18.08	5.21	.86	-.14	-.18
22	18.10	5.44	.81	-.11	.17
23	18.14	4.85	.87	-.17	-.02
24	18.10	5.18	.87	-.10	-.03
25	18.06	4.97	.86	-.18	.12
26	18.39	5.45	.83	.02	-.31
27	18.37	4.91	.85	-.13	.14
28	18.16	5.12	.84	-.14	.01
29	18.14	5.33	.86	-.14	.01
30	18.23	5.00	.84	-.09	.12
31	18.04	4.85	.84	-.16	.01
32	18.20	5.25	.82	-.15	.00
33	18.14	5.19	.83	-.27	.06
Eigenvalue			22.74	1.31	.74

tocorrelations are negative and subsequently rapidly die out (see Equation A29). The process is not stationary if the initial inhibition is outside the range of its stationary value. The autocorrelation function depends on the speed of work (the inverse of A) and the sensitivity to inhibition (the b parameter). However, if the process is not stationary, then the negative effect on the autocorrelation due to the

interdependence of the RTs is masked by a positive effect due to the resulting long-term trend in the RTs.

This effect can be eliminated by fitting the exponential function predicted by the model and subsequently subtracting this effect from the actual data. However, using trend elimination to correct the data artificially generates a negative autocor-

Table 4
Means and Standard Deviations for Each Line
and Factor Loadings on the First Three Factors
for the 267 Remaining Observed Datasets

Line	Mean	SD	Factor		
			1	2	3
1	13.13	3.97	.87	.21	-.05
2	16.02	4.23	.91	.16	.06
3	17.31	5.04	.84	.23	-.21
4	15.22	3.86	.86	.20	.16
5	15.88	4.24	.89	.12	.05
6	15.34	4.17	.88	.19	.06
7	15.52	4.46	.88	.17	.02
8	15.77	4.76	.88	.17	-.02
9	15.63	4.61	.86	.16	.12
10	16.09	4.81	.88	.00	.17
11	15.48	4.58	.85	.20	.04
12	15.63	4.43	.88	.14	.13
13	15.13	4.17	.87	.14	.17
14	14.73	4.23	.88	.08	-.10
15	15.93	4.51	.91	.06	.00
16	15.58	4.56	.92	.02	.09
17	15.78	5.16	.87	.07	-.28
18	15.69	4.66	.89	-.02	-.20
19	16.10	4.19	.88	.03	.06
20	16.53	5.04	.88	.03	-.08
21	16.29	4.84	.86	.01	-.27
22	15.66	4.07	.88	-.07	-.04
23	16.63	4.78	.86	-.11	-.03
24	16.42	4.11	.83	-.15	.13
25	16.10	3.98	.81	-.16	.25
26	15.87	4.51	.87	-.18	-.18
27	16.01	4.00	.85	-.22	.05
28	15.72	4.24	.84	-.24	.00
29	16.78	4.74	.81	-.32	-.05
30	16.10	4.24	.84	-.23	-.06
31	16.60	4.09	.82	-.29	.09
32	15.97	3.68	.80	-.20	-.21
33	15.88	3.95	.76	-.37	.16
Eigenvalue			24.50	1.01	.59

relation. A similar argument holds if examinees who demonstrate the exponential trend are eliminated from the total sample. In the latter case the mean of the autocorrelation over examinees will artificially tend to become more negative. At this time, no statistical methods can separate the positive autocorrelation effect due to the long-term trend from the negative effect caused by the short-

term oscillation. Therefore, only the gross autocorrelation was considered.

The analysis included both the observed and simulated datasets. For each of the 249 observed datasets, the simple and partial autocorrelations were computed up to and including the fifth lag. Subsequently, for each lag the mean and SD of the simple and the partial autocorrelation were com-

Table 5
Mean and Standard Deviation of
the Residual Variance Within
Each Block Based Upon All
Datasets With $\beta < 0$ ($N = 220$)

Block	Mean	SD
1	4.19	8.07
2	4.47	7.18
3	4.99	8.52
4	5.05	6.91
5	8.38	19.49
6	5.64	9.05
7	6.47	12.22
8	6.65	9.39
9	6.43	14.57
10	8.02	15.91
11	7.27	12.15

puted over examinees. The same procedure was followed in the case of the 249 simulated datasets. Table 6 presents the results for the observed and simulated datasets.

With respect to the autocorrelations, the difference between the vectors of means computed from the observed and simulated datasets was significant at the .01 level ($F_{5,244} = 2.81, p = .017$). A sim-

ilar result was found for the difference between the vectors of means of partial autocorrelations ($f_{5,244} = 9.80, p < .001$). In particular, the difference between the two first lag autocorrelation means plays an important part ($r = .51, t = 5.92, df = 258, p < .001$). These results, which contradict the model, may have resulted from the confounding effect of the third factor long-term trend mentioned earlier (see Table 2). The long-term trend causes the autocorrelation to be larger than should be expected from the model.

Discussion

This study provides some evidence to support the proposed inhibition model. Specifically, the long-term trend could be explained by the model as a consequence of the convergence to stationarity. The autocorrelation effect described by the model, however, was not found, probably because of the confounding long-term trend effect that appeared as the third factor in the factor analysis. This effect may have been caused by the fact that examinees continuously changed their attitudes toward the speed of work. Examinees know how far they have progressed because of the paper-and-pencil nature of

Table 6
Mean and Standard Deviation of the k th Lag
Autocorrelation and Partial Autocorrelation
Computed From the 249 Observed Datasets and
the Corresponding Simulated Datasets

Lag	Autocorrelation		Partial Autocorrelation	
	Mean	SD	Mean	SD
Observed Datasets				
1	.09	.21	.09	.21
2	.07	.19	.02	.18
3	.07	.17	.05	.15
4	.05	.17	-.01	.15
5	.04	.16	.02	.15
Simulated Datasets				
1	.00	.28	.00	.28
2	.04	.23	-.03	.21
3	.07	.20	.01	.17
4	.05	.19	-.03	.16
5	.04	.17	-.01	.14

the test. Depending on their position in the test, they will work more quickly or more slowly. This effect could be eliminated by administering the test by computer, so that the stimuli (patterns of dots) would be presented separately and each response triggers the next stimulus. In an experiment of van Breukelen and Jansen (van Breukelen et al., 1987), such a presentation was used. As a consequence, the extraneous long-term trend effect disappeared and some evidence was found for the negative autocorrelation effect predicted by the model.

The present study only tested the inhibition model on the Bourdon-Vos test. Clearly, further empirical testing using a variety of qualitatively different tasks is needed to confirm the underlying theory. However, to do the model justice, the task has to be

overlearned in advance, which has not always been the case even for apparently simple tasks such as the inverted-alphabet printing test (Kientzle, 1946). In that task, examinees are required to print letters of the alphabet upside-down. In one experiment (see Eysenck & Frith, 1977, p. 7), the test was administered to 2,560 boys and 2,679 girls between 10 and 11 years old. A very clear learning effect was found at the beginning of the test. If learning still plays a part, then the increasing trend resulting from a low inhibition at the beginning of the test is masked by the decreasing trend due to learning. The best way to control for learning is to administer the test repeatedly. However, more research is needed to develop an experimental setting in which learning is completely under control.

Appendix

The Mathematics of the Inhibition Model

Let $Y(t)$ denote the inhibition after t units of working time. Assumption 1 implies that the successive tasks ("lines" in the Bourdon-Vos test) are completed at the moments at which the examinee has spent amounts of time $A, 2A, 3A, \dots, nA$ in State 1 (the working state), where n refers to the last task or line. Thus, it is convenient to adopt the time spent in State 1—not real time—as the time parameter for the inhibition process.

According to this convention, and assuming that inhibition increases during work with rate 1 and decreases during distraction with rate a (Assumption 4 above), the increase of inhibition during the first task (line) equals

$$Y(A) - Y(0) = A - aD_1, \quad (A1)$$

and in general the increase of Y over the k th task is equal to

$$Y(kA) - Y[(k-1)A] = A - aD_k, \quad (A2)$$

where D_k denotes total distraction time during task k . Because the k th RT T_k is equal to $A + D_k$, it follows that

$$T_k = A + a^{-1}A - a^{-1}\{Y(kA) - Y[(k-1)A]\}. \quad (A3)$$

In particular, $T_1 = A + a^{-1}A - a^{-1}[Y(A) - Y(0)]$. Therefore the behavior of the RTs T_k is determined by increments of the process $Y(t)$ over the successive (worktime) intervals of length A .

The process $Y(t)$ will now be analyzed and described in such a way that it will be clear how to perform computer simulations of it. $Y(t)$ is a kind of "sawtooth" process. During work intervals $Y(t)$ increases linearly with slope 1. At the end of each work interval, a distraction starts during which the workclock stops and the inhibition decreases with an amount ad , where d is the duration of the distraction (following Assumption 4). This causes a downward jump for $Y(t)$ of size $j = ad$.

Because $\lambda_0 = ac$, a distraction being exponentially distributed with mean $(ac)^{-1}$ may be generated by $d = -(ac)^{-1} \ln U$, where U is the usual uniform random number. Thus the corresponding downward jump j is generated by

$$j = -c^{-1} \ln U. \quad (A4)$$

Now consider work intervals. During a work interval starting at t_0 with $Y(t_0) = y$, inhibition rises (with rate 1). Therefore $Y(t_0 + t) = Y(t_0) + t = y + t$ if no interruption has occurred in the ensuing t units of time. According to Equation 3, the transition rate λ_1 is then equal to $\lambda_1[Y(t_0 + t)] = \lambda_1(y + t) = c[1 + b(y + t)]^+ = bc(b^{-1} + y + t)^+$. The duration w of this work interval is a random variable with hazard function $h(t) = \lambda_1(y + t) = bc(b^{-1} + y + t)^+$ and cumulative hazard

$$H(t) = \int_0^t h(s)ds = \frac{1}{2}bc\{[(b^{-1} + y + t)^+]^2 - [(b^{-1} + y)^+]^2\} \quad (A5)$$

The distribution of w , therefore, is given by

$$P(w > t) = \exp[-H(t)] = \exp\left[-\frac{1}{2}bc\{[(b^{-1} + y + t)^+]^2 - [(b^{-1} + y)^+]^2\}\right] \quad (A6)$$

where

$$P(w > t) = \exp(-ct) \quad \text{if } b = 0 \quad (A7)$$

To generate values of w (by the inversion method), equate the right side of Equation A6 to U and solve for t ($= w$), giving

$$w = -(b^{-1} + y) + \{[(b^{-1} + y)^+]^2 - 2b^{-1}c^{-1} \ln U\}^{1/2} \quad (A8)$$

where

$$w = -c^{-1} \ln U \quad \text{if } b = 0 \quad (A9)$$

It should now be clear how to simulate the process $Y(t)$. Given an initial value $Y(0) = y_0$, the first work interval w_1 is generated by Equation A8 with $y = y_0$. Then a distraction d_1 follows. The corresponding downward jump j_1 is generated by Equation A4. The second work interval is then generated by Equation A8 with $y = y_0 + w_1 - j_1$. Continue until the m th work interval makes the accumulated work time exceed kA (i.e., $w_1 + \dots + w_{m-1} \leq kA$, $w_1 + \dots + w_m > kA$); then $Y(kA) = y_0 + kA - (j_1 + \dots + j_{m-1})$. When this is done for $k = 1, 2, \dots, n$, the RTS $T_1, \dots, T_n$ are determined by Equation A3.

Moments of the Process

Because by Equation A3 the moments of the RTS T_k depend on those of the process $Y(t)$, it is useful to study the functions $g_k(t) = E[XY(t)^k]$, where X is some function of $Y(0)$. The functions g_1 and g_2 obey the following differential equations:

$$g_1'(t) = -bg_1(t) \quad (A10)$$

and

$$g_2'(t) = 2c^{-1}E(X) + 2bc^{-1}g_1(t) - 2bg_2(t) \quad (A11)$$

These equations are obtained as follows. The increment $\Delta = Y(t + h) - Y(t)$ of the inhibition over a small interval $(t, t + h)$ equals $\Delta = h - Ij$, where I takes values 0 or 1, the latter if a jump occurs which happens with probability $\lambda_1[Y(t)]h + o(h)$. Therefore the conditional expectation of Δ given the evolution of the process up to t is equal to

$$E_t(\Delta) = h - P(I = 1)E(j) = h - c[1 + bY(t)]hc^{-1} + o(h) = -bY(t)h + o(h) \quad (A12)$$

The notation $E_t(\Delta)$ is used to express the dependency of Δ (and thus its expectation) on t . Then

$$g_1(t + h) - g_1(t) = E(X\Delta) = E[XE_t(\Delta)] = -bE[XY(t)h + o(h)] = -bg_1(t) + o(h) \quad (A13)$$

In the same way, the conditional expectation of Δ^2 equals

$$E_t(\Delta^2) = P(I = 1)E(j^2) + o(h) = c[1 + bY(t)]2c^{-2}h + o(h) = 2c^{-1}[1 + bY(t)]h + o(h) \quad (A14)$$

because

$$\Delta^2 = h^2 - 2hIj + I^2j^2 = I^2j^2 + o(h) = Ij^2 + o(h) \quad . \quad (\text{A15})$$

Hence

$$\begin{aligned} g_2(t+h) - g_2(t) &= E\{X[(Y(t) + \Delta)^2 - Y(t)^2]\} \\ &= E\{X[2Y(t)E_c(\Delta) + E_c(\Delta^2)]\} \\ &= E\{X[-2bY(t)^2h + 2c^{-1}][1 + bY(t)h]\} + o(h) \\ &= [-2bg_2(t)h + 2c^{-1}E(X) + 2bc^{-1}g_1(t)]h + o(h) \quad . \end{aligned} \quad (\text{A16})$$

The solution of Equations A10 and A11 is given by

$$g_1(t) = g_1(0)\exp(-bt) \quad (\text{A17})$$

$$g_2(t) = (bc)^{-1}E(X) + 2c^{-1}g_1(0)\exp(-bt) + K \exp(-2bt) \quad (\text{A18})$$

with

$$K = g_2(0) - (bc)^{-1}E(X) - 2c^{-1}g_1(0) \quad . \quad (\text{A19})$$

From this the following conclusions may be drawn:

1. With initial values $g_1(0) = 0$ and $g_2(0) = (bc)^{-1}E(X)$, the constant solutions $g_1(t) = 0$ and $g_2(t) = (bc)^{-1}E(X)$ for all $t \geq 0$ can be obtained. In particular, selecting $X = 1$ illustrates that the process may show stationary behavior (with regard to moments of order 1 and 2) with constant mean and variance given by

$$E[Y(t)] = 0, \text{Var}[Y(t)] = (bc)^{-1} \quad (t \geq 0) \quad . \quad (\text{A20})$$

2. With $X = Y(0)^k$, Equation A17 gives $E[Y(0)^k Y(t)] = E[Y(0)^{k+1}]\exp(-bt)$, or shifting the time parameter by s :

$$E[Y(s)^k Y(s+t)] = E[Y(s)^{k+1}]\exp(-bt) \quad (t \geq 0) \quad . \quad (\text{A21})$$

In particular for $k = 1$:

$$E[Y(s)Y(s+t)] = E[Y(s)^2]\exp(-bt) \quad (t \geq 0) \quad . \quad (\text{A22})$$

Notice that in deriving the differential Equations A10 and A11, $\lambda_1[Y(t)]$ was set equal to $c[1 + bY(t)]$ instead of $c[1 + bY(t)]^+$. The error in making this substitution is negligible if b is small. In fact, further analysis of the process reveals that $Y(t)$ has a stationary distribution with density function

$$f(y) = B \exp\left(-\frac{1}{2}bcy^2\right) \quad \text{if } y \geq -b^{-1} \quad (\text{A23})$$

$$f(y) = B \exp\left[c\left(y + \frac{1}{2}b^{-1}\right)\right] \quad \text{if } y < -b^{-1} \quad , \quad (\text{A24})$$

where B is a norming constant. Apart from the exponential tail to the left of $-b^{-1}$, this is a normal distribution with a mean of 0 and variance $(bc)^{-1}$. Hence if b^{-1} is larger than three times the SD [i.e., $b^{-1} > 3(bc)^{-1/2}$, or $b < c/9$], then $Y(t)$ will almost never be below $-b^{-1}$, so that for all practical purposes $c(1 + by)$ accurately represents the transition rate $\lambda_1(y)$. Thus, it is possible to assume that $b < c/9$.

These results will now be used to derive the moments of the RTs T_k .

Moments of the Reaction Times

In the stationary case, the mean and variance of $Y(t)$ are given by Equation A20. Using Equation A3 for T_k gives

$$E(T_k) = A + a^{-1}A \quad . \quad (\text{A25})$$

By Equations A20 and A22 it is evident that

$$\text{Var}(T_k) = 2a^{-2}(bc)^{-1}[1 - \exp(-bA)] = 2a^{-2}(bc)^{-1}(1-r) \quad (\text{A26})$$

with $r = \exp(-bA)$. In the same way the autocovariance is found to be

$$\text{Cov}(T_k, T_{k+j}) = -a^{-2}(bc)^{-1}r^{j-1}(1-r)^2 \quad (\text{A27})$$

In particular for lag 1:

$$\text{Cov}(T_k, T_{k+1}) = -a^{-2}(bc)^{-1}(1-r)^2 \quad (\text{A28})$$

Dividing by the variance (Equation A26) gives the autocorrelation

$$\rho_j = -\frac{1}{2}r^{j-1}(1-r) \quad (\text{A29})$$

with $r = \exp(-bA)$. In particular the autocorrelation (lag 1) is given by

$$\rho_1 = -\frac{1}{2}(1-r) = -\frac{1}{2}[1 - \exp(-bA)] \quad (\text{A30})$$

Thus, the model predicts negative correlation between successive RTs somewhere between 0 and $-.5$ according to Equation A30. In actual RT sequences this negative autocorrelation was hardly discernible. One reason may be the variability in real processing time on the successive tasks. Suppose that contrary to the assumption that each task requires the same processing time A , the real processing times $A_1, A_2, \dots, A_n$ on successive tasks are independent and normal with mean A and variance σ^2 . Then $E(T_k) = A + a^{-1}A$, but Equation A26 is replaced by

$$\text{Var}(T_k) = 2a^{-2}(bc)^{-1} \left[1 - \exp\left(-bA + \frac{1}{2}b^2\sigma^2\right) \right] + (1+a^{-1})^2\sigma^2 \quad (\text{A31})$$

and Equation A28 becomes

$$\text{Cov}(T_k, T_{k+1}) = -a^{-2}(bc)^{-1} \left[1 - \exp\left(-bA + \frac{1}{2}b^2\sigma^2\right) \right]^2 \quad (\text{A32})$$

Hence with smaller covariance and larger variance, the autocorrelation is weaker. For example, with typical values of the parameters $A = 11.717$, $a = 1.935$, $b = .032$, $c = .945$, the following first-lag autocorrelations are obtained for σ^2 varying from 0 to 3: $\rho_1(\sigma^2 = 0) = -.156$, $\rho_1(\sigma^2 = 1) = -.110$, $\rho_1(\sigma^2 = 2) = -.085$, and $\rho_1(\sigma^2 = 3) = -.069$.

The Nonstationary Case

If $E[Y(0)] = y_0 \neq 0$, then $E[Y(t)] = y_0 \exp(-bt)$, so that $E[Y(kA)] = y_0 r^k$ with $r = \exp(-bA)$. By Equation A3 it is possible to obtain

$$E(T_k) = A + a^{-1}A + a^{-1}y_0(1-r)r^{k-1} \quad (\text{A33})$$

Consequently the model predicts exponential trend in the sequence of RTs $T_1, T_2, \dots, T_n$. This trend may decrease (if $y_0 > 0$) or increase (if $y_0 < 0$).

The b parameter, which according to Equation A33 determines the speed of the trend, should satisfy $0 \leq b \leq c/9$. If $b = 0$, the inhibition model coincides with the Poisson-Erlang model.

If $E[Y(0)] = y_0 \neq 0$, then by Equation A18

$$E[Y(t)^2] = (bc)^{-1} + 2c^{-1}y_0 \exp(-bt) + K \exp(-2bt) \quad (\text{A34})$$

This, together with Equations A20 and A22, may be used to obtain an expression for $\text{Var}(T_k) = a^{-2}\text{Var}[Y(kA)] - Y[(k-1)A]$. Omitting terms with r^{2k} , which tend to move toward 0 more rapidly,

$$\text{Var}(T_k) = 2a^{-2}(bc)^{-1}(1-r) + 2a^{-2}c^{-1}y_0(1-r)r^{k-1} \quad (\text{A35})$$

Therefore $\text{Var}(T_k)$ will show a trend with the same direction as $E(T_k)$.

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Author's Address

Send requests for reprints or further information to A. H. G. S. van der Ven, Department of Mathematical Psychology, University of Nijmegen, P.O. Box 9104, 6500 HE Nijmegen, The Netherlands.