Unidimensionality Versus Statistical Accuracy: A Note on Bejar's Method for Detecting Dimensionality of Achievement Tests

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A simulation investigated use of the difficulty parameter (Bejar, 1980) to evaluate item unidimensionality. Artificial tests were designed to be nonequivalent in both length and dimensionality. Simulated item responses to the tests were analyzed with the LOGIST computer program. Two indices were calculated: the slope of the principal axis between the content-area-based item difficulty estimates and corresponding total-test-based estimates, and the correlation between the two sets of estimates. Results show that the magnitude of the correlation coefficient provides no information about dimensionality of a set of test items. The slope of the principal axis, on the other hand, is sensitive to multidimensionality in the data as well as test length. The size of the slope adequately detects the dimensionality of items for relatively long tests. *Index terms: equating, item difficulty parameters, item response theory, unidimensionality.*

Bejar (1980) proposed using content-area-based and total-test-based item difficulty (b) estimates to detect the dimensionality of achievement tests. Bejar argued that if a set of test items is unidimensional, then grouping test items into subareas to calibrate items will be irrelevant. Parameter estimates for items calibrated within different subareas, aside from sampling errors, should be identical; they should be close to a theoretical axis with a slope of 1 and an intercept of 0. The Bejar procedure fits the principal axis to the content-area-

based b values versus corresponding total-test-based values. The distance between the principal axis and the theoretical axis indicates how well the data satisfy the unidimensionality assumption of the logistic models.

Because b parameters are generally estimated with less sampling error than are ability or item discrimination parameters, Bejar's method uses b estimates to analyze dimensionality. However, the slope and intercept of the theoretical axis selected in the method show that the interpretation is unique for b values calibrated with the total test and with the content areas.

The IRT parameter scale has an indeterminacy problem that is particularly associated with the three-parameter logistic model. The same b parameters calibrated with two separate datasets will scatter around a straight line (Lord, 1980, p. 36) that does not necessarily have a slope of 1 and an intercept of 0. Nevertheless, test equating techniques are often used to generate comparability between the parameters.

When the same examinees' data are involved in both computer runs, as with Bejar's method, factors such as statistical accuracy as a result of test length and the deviation of item responses from the logistic function could also affect the calibration of item and θ parameters. Because these factors function differently in the calibration, the total-test-based and content-area-based data are not likely to have the same meaning even if they are unidi-

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mensional. Consequently, these factors could also affect the sensitivity of Bejar's method in investigating the dimensionality of achievement tests.

In their subsequent study on the dimensionality of achievement tests, Hambleton and Rovinelli (1986) found that Bejar's method was insensitive to multidimensionality in the data except when the correlation between the underlying abilities was low and when a disproportionate number of test items measured one of the abilities. However, their findings were derived from a scale-free measure of association (the correlation) between the content-area-based and total-test-based b values.

In the present analyses, tests of unequal lengths were simulated to measure single and multiple latent traits. The simulated test data were calibrated with LOGIST 5 (Wingersky, Barton, & Lord, 1983). The association between content-area-based and total-test-based b values was evaluated by calculating both their principal axis and correlation. The present simulation results further highlight the sensitivity of Bejar's method to multidimensionality in the data.

Method

The simulation assumed that 1,000 examinees took all the simulated tests. θ_i for each examinee was generated from a standard normal distribution with the IMSL generator GGNPM (International Mathematical and Statistical Libraries, 1982) and was restricted to the interval from -2.5 to $+2.5$. In the first evaluation, a pair of tests with 60 and 45 items were simulated for measuring a general factor. Each test consisted of three content areas. The 45-item test was comprised of items 1–15, 21–35, and 41–55 from the 60-item test. The factor loading ρ_{i0_1} of each item on the general factor was generated in an interval ranging from .6 to .7 by the IMSL generator GGUBS. A standardized latent response y_i was simulated with a process described by Drasgow and Parsons (1983):

$$y_i = (\theta_i)\rho_{i0_1} + e_i(1 - \rho_{i0_1}^2)^{1/2}, \quad (1)$$

where e_i was a random deviate generated from a standard normal distribution. The item threshold values γ for dichotomizing latent responses into

binary scores were generated in an interval ranging from -1 to $+1$. Guessing was simulated by generating item scores and then rescoreing the 0 scores as correct with a probability of .15, which was the hypothetical guessing parameter in the evaluation.

In the second evaluation, another pair of tests were simulated for measuring a general factor and three group factors. It was assumed that the group factors were uncorrelated with each other and with the general factor. Each simulation item also loaded only on a single group factor. The examinees' θ_2 , θ_3 , and θ_4 on the group factors were independently generated from a standard normal distribution. Factor loadings ρ_{i0_2} , ρ_{i0_3} , and ρ_{i0_4} of each item on the corresponding group factor were generated in an interval ranging from .3 to .5. The communalities of the items were simulated to have comparable sizes in the two evaluations. Therefore, a multidimensional counterpart in Evaluation 2 always existed for each item in Evaluation 1. Latent responses to the multidimensional tests were also simulated with the method described by Drasgow and Parsons (1983):

$$y_i = \sum_{j=1}^4 (\theta_j)\rho_{i0_j} + e_i \left(1 - \sum_{j=1}^4 \rho_{i0_j}^2 \right)^{1/2}. \quad (2)$$

The item threshold values and the guessing parameter selected in Evaluation 1 were also used in Evaluation 2.

Item responses for the total test and for the subareas were calibrated in different runs with LOGIST 5 using default convergence criteria. The principal axis for the total-test-based and content-area-based b values was also calculated. The sum of the squared distances was minimized in order to find the principal axis by

$$d^2 = \sum_i [\sin\phi(b_{i1} - \bar{b}_1) - \cos\phi(b_{i2} - \bar{b}_2)]^2, \quad (3)$$

where b_{i1} was the i th item content-area-based difficulty estimate and b_{i2} was the total-test-based estimate. The values $\bar{b}_1$ and $\bar{b}_2$ were corresponding mean difficulty estimates. The rotation of the axis locating the content-area-based estimate was through the angle ϕ .

The slope of the principal axis was calculated from

$$\tan\phi = \frac{(\sigma_1^2 - \sigma_2^2) \pm [(\sigma_1^2 - \sigma_2^2)^2 + 4\sigma_{12}^2]^{1/2}}{2\sigma_{12}} \quad (4)$$

(Bejar, 1980), where σ_1^2 and σ_2^2 were the variances of b estimates and σ_{12} was the covariance between the two sets of estimates. The value of $\tan\phi$ was selected as positive. The mean squared distance (MSD) was calculated by averaging the distances in Equation 3.

Results

The factor loadings and item threshold values used to obtain the response data are presented in Tables 1 and 2. The number of items (N), mean, standard deviation and KR-20 for each simulation test and for its content areas are listed in Table 3. The value of $\tan\phi$ for the principal axis and the correlation r between difficulty parameters are also in Table 3. The slope of the principal axis for the total test was calculated by combining the content-area-based b values. Table 3 also shows the MSD

for each content area from its corresponding principal axis. The plots of the content-area-based b values and corresponding total-test-based values for the simulation tests are shown in Figures 1 and 2.

The 60- and 45-item tests were simulated to be identical in all respects except for their lengths. The simulation results from Evaluation 1 indicate that the values of $\tan\phi$ for the 45-item tests differ from 1 even though the test is unidimensional (e.g., the third content area has a slope of 1.20). The 45-item test also has larger MSD values than the 60-item test.

In Evaluation 2, the pairs of tests were simulated as multidimensional. The slopes for the 60- and 45-item tests both deviate from 1. However, the slope of the 60-item test (.878) is as discrepant as the slope for the 45-item test (1.088) in Evaluation 1. This result implies that the test length and multidimensionality in the data might both account for the deviation. For both evaluations, the correlations between the b values fail to give any interesting pattern (all the correlations are greater than .9).

Table 1
Item Threshold Values (γ) and Factor Loadings ($\rho_{i\theta_1}$)
for The Simulation Tests in Evaluation 1

Content Area 1			Content Area 2			Content Area 3		
Item	γ	$\rho_{i\theta_1}$	Item	γ	$\rho_{i\theta_1}$	Item	γ	$\rho_{i\theta_1}$
1	-.200	.601	21	.614	.624	41	.682	.660
2	.154	.620	22	.342	.611	42	.071	.647
3	.653	.620	23	-.173	.678	43	.150	.687
4	-.469	.653	24	-.902	.628	44	.366	.673
5	.766	.621	25	.179	.635	45	.218	.698
6	-.502	.623	26	.441	.641	46	-.844	.627
7	-.555	.687	27	.863	.640	47	-.247	.658
8	-.736	.604	28	-.521	.619	48	-.475	.605
9	.017	.686	29	.663	.661	49	-.694	.665
10	-.602	.695	30	-.477	.634	50	-.340	.661
11	-.962	.620	31	.381	.637	51	.232	.664
12	.030	.690	32	-.633	.662	52	.909	.700
13	.892	.694	33	-.049	.657	53	.549	.631
14	.979	.604	34	-.989	.666	54	-.781	.661
15	.966	.619	35	-.282	.632	55	.373	.627
16	-.296	.687	36	.545	.634	56	.662	.623
17	.810	.643	37	.333	.628	57	-.839	.636
18	-.781	.621	38	-.924	.689	58	.308	.624
19	-.824	.602	39	.413	.620	59	.821	.672
20	.959	.614	40	-.266	.646	60	-.774	.653

Table 2
Item Threshold Values (γ) and Factor Loadings ($\rho_{i\theta_j}$) for The Simulation Tests in Evaluation 2

Content Area 1										Content Area 2					Content Area 3				
Item	γ	$\rho_{i\theta_1}$	$\rho_{i\theta_2}$	$\rho_{i\theta_3}$	$\rho_{i\theta_4}$	Item	γ	$\rho_{i\theta_1}$	$\rho_{i\theta_2}$	$\rho_{i\theta_3}$	$\rho_{i\theta_4}$	Item	γ	$\rho_{i\theta_1}$	$\rho_{i\theta_2}$	$\rho_{i\theta_3}$	$\rho_{i\theta_4}$		
1	-.200	.493	.344	0.0	0.0	21	.614	.500	0.0	.374	0.0	41	.682	.491	0.0	0.0	.441		
2	.154	.416	.460	0.0	0.0	22	.342	.364	0.0	.491	0.0	42	.071	.539	0.0	0.0	.357		
3	.653	.539	.306	0.0	0.0	23	-.173	.493	0.0	.466	0.0	43	.150	.594	0.0	0.0	.346		
4	-.469	.494	.428	0.0	0.0	24	-.902	.536	0.0	.327	0.0	44	.366	.526	0.0	0.0	.419		
5	.766	.485	.388	0.0	0.0	25	.179	.532	0.0	.347	0.0	45	.218	.608	0.0	0.0	.344		
6	-.502	.478	.400	0.0	0.0	26	.441	.424	0.0	.481	0.0	46	-.844	.481	0.0	0.0	.401		
7	-.555	.551	.411	0.0	0.0	27	.863	.466	0.0	.439	0.0	47	-.247	.510	0.0	0.0	.415		
8	-.736	.405	.449	0.0	0.0	28	-.521	.434	0.0	.441	0.0	48	-.475	.477	0.0	0.0	.372		
9	.017	.476	.494	0.0	0.0	29	.663	.507	0.0	.424	0.0	49	-.694	.562	0.0	0.0	.356		
10	-.602	.504	.478	0.0	0.0	30	-.477	.552	0.0	.312	0.0	50	-.340	.461	0.0	0.0	.474		
11	-.962	.511	.352	0.0	0.0	31	.381	.453	0.0	.448	0.0	51	.232	.524	0.0	0.0	.408		
12	.030	.541	.427	0.0	0.0	32	-.633	.496	0.0	.439	0.0	52	.909	.548	0.0	0.0	.436		
13	.892	.543	.433	0.0	0.0	33	-.049	.522	0.0	.399	0.0	53	.549	.389	0.0	0.0	.497		
14	.979	.516	.314	0.0	0.0	34	-.989	.468	0.0	.474	0.0	54	-.781	.572	0.0	0.0	.330		
15	.966	.396	.476	0.0	0.0	35	-.282	.410	0.0	.481	0.0	55	.373	.427	0.0	0.0	.460		
16	-.296	.577	.372	0.0	0.0	36	.545	.519	0.0	.363	0.0	56	.662	.470	0.0	0.0	.409		
17	.810	.567	.304	0.0	0.0	37	.333	.499	0.0	.381	0.0	57	-.839	.477	0.0	0.0	.422		
18	-.781	.499	.369	0.0	0.0	38	-.924	.475	0.0	.500	0.0	58	.308	.541	0.0	0.0	.312		
19	-.824	.487	.353	0.0	0.0	39	.413	.506	0.0	.358	0.0	59	.821	.598	0.0	0.0	.307		
20	.959	.453	.415	0.0	0.0	40	-.266	.478	0.0	.434	0.0	60	-.774	.533	0.0	0.0	.378		

Table 3
Content Area and Total Test Statistics for the 60-Item and 45-Item Tests
in Evaluation 1 and Evaluation 2 for the Simulation Tests

Test and Statistic	Evaluation 1				Evaluation 2			
	Content Area			Total Test	Content Area			Total Test
	1	2	3		1	2	3	
Test 1								
No. of Items	20	20	20	60	20	20	20	60
Mean	11.145	11.277	11.116	33.538	11.178	11.247	11.419	33.844
SD	3.882	4.027	4.262	11.346	3.752	3.953	4.034	9.399
KR20	.745	.741	.764	.904	.728	.731	.737	.861
r	.999	.997	.999	.998	.994	.990	.960	.981
tan θ	.962	1.045	.993	.995	.915	.824	.882	.878
MSD (Principal)	.004	.005	.001		.027	.081	.071	
Test 2								
No. of Items	15	15	15	15	15	15	15	45
Mean	8.230	7.854	8.834	24.918	7.915	7.861	8.965	24.741
SD	3.072	2.997	3.399	8.608	2.774	3.021	3.093	7.024
KR20	.693	.652	.720	.875	.656	.646	.669	.817
r	.998	.994	.993	.993	.991	.990	.981	.986
tan θ	1.075	1.030	1.200	1.088	.995	.904	.859	.925
MSD (Principal)	.012	.008	.048		.011	.031	.053	

Figure 1

Plot of the b Estimates for the 60-Item and 45-Item Tests in Evaluation 1

Figure 2
Plot of the b Estimates for the 60-Item and 45-Item Tests in Evaluation 2

Conclusions

When applying Bejar's method for detecting dimensionality of achievement tests, researchers should be cautious when interpreting the slope of the principal axis. These results indicate that the size of the slope is sensitive to both multidimensionality and test length. With studies using empirical data, factors other than test length might also affect the magnitude of the slope, so that even more stipulations are needed before accepting the theoretical axis proposed by Bejar. On the other hand, Bejar's method worked reasonably well for the relatively long tests, which is illustrated by the fact that the size of the slope for the 60-item test in Evaluation 2 differed from 1 more than the same length test in Evaluation 1.

Bejar's method avoids the computational problem generally encountered with factor analyses of binary data. If the researcher accepts the three-parameter logistic model with some confidence and the content area test has a reasonable length (at least 20 items), the Bejar procedure provides a straightforward check on the unidimensionality of test data. The present results suggest the necessity of using other information from the data (e.g., test length, item-total correlations) in conjunction with

Bejar's method to address the question of item unidimensionality.

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