MORPHOMETRIC CONTROL OF ANNUAL HEAT BUDGETS IN TEMPERATE LAKES¹

Eville Gorham

Department of Botany, University of Minnesota, Minneapolis

ABSTRACT

Annual heat budgets of 71 lakes in the temperate zone are strongly correlated with mean depth and area. Depth and area influences taken together are responsible for an even higher degree of correlation between heat budgets and lake volumes. For lakes of a given volume, the deeper ones of lesser area appear to take up slightly more heat than the shallower ones of greater area. Dimictic lakes exhibit slightly higher heat budgets than warm monomictic lakes of similar size. Heat uptake appears to rise continuously with increasing lake dimensions, though at a decreasing rate, up to the largest dimensions observed.

INTRODUCTION

The annual heat budget of a lake is the quantity of heat required to warm its water from the winter minimum to the summer maximum temperature. It is usually expressed per unit area. According to Birge (1915), normal lakes with dimensions of 10×2 km or more and mean depths of at least 30 m take up the maximum amount of heat possible in a given climate. Hutchinson (1957) modified this statement by suggesting that annual heat budgets of very large lakes such as Lake Baikal and Lake Michigan might be greatly increased by the mixing of deep cold water, exposed upwind during periods of severe wind stress, into the warm epilimnion. Such mixing would later facilitate heat gain at the cooled surface. Fog condensation was also mentioned by Hutchinson as a factor of possible importance in such lakes.

In view of the numerous data compiled by Hutchinson (1957, Table 53, p. 496-500), it seemed likely that the influence of lake morphometry upon heat uptake could be investigated statistically. Such a quantitative formulation would be of intrinsic limnological interest by providing a standard with which to compare individual

heat budgets from lakes of any size. It might also be of value to scientists studying local climatic modification by lakes of different sizes. Data from 43 dimictic and 28 warm monomictic lakes in the temperate zone were used to test relationships between annual heat budget and lake dimensions. Hutchinson's treatise formed the main source of material, with Anneks Sjø, Great Slave Lake, and the Central American and Indonesian lakes being excluded. (Incidentally, mean and maximum depths for Lake Mead are transposed in Hutchinson's Table 53.) Additional data came from Convict Lake (Reimers, Maciolek, and Pister 1955) and Alamogordo Reservoir (Cole 1963). Correlations and regressions were calculated for annual heat budgets and logarithms of lake dimensions, the data being rounded off to two figures for machine calculation. The symbols used in the regression equations are those of Hutchinson (1957).

THE INFLUENCE OF DEPTH

The effect of mean depth upon annual heat budget is shown in Figs. 1 and 2. Fig. 1 employs a logarithmic transformation of mean depth to yield linear regressions, while Fig. 2 shows the regression of heat budget upon mean depth restored to an arithmetic basis.

It is evident from Fig. 1 that annual heat budgets are strongly dependent upon mean depth, presumably because deeper water columns provide greater storage capacity for heat. The correlation coefficient (r)

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FIG. 1. Semilogarithmic correlation between annual heat budget and mean depth in temperate lakes. Solid circles represent dimictic and open circles monimictic lakes. Crosses represent arithmetic means for dimictic (*D*) and monimictic (*M*) lakes.

FIG. 3. Semilogarithmic correlation between annual heat budget and the square root of area in temperate lakes. Symbols as in Fig. 1.

FIG. 2. Regressions of annual heat budget upon mean depth in temperate lakes. The solid line is the zero order regression and the dashed line the first order regression eliminating the influence of variation in area. The cross represents the arithmetic mean for heat budget and the logarithmic mean for mean depth.

FIG. 4. Regressions of annual heat budget upon the square root of area in temperate lakes. The solid line is the zero order regression, and the dashed line the first order regression eliminating the influence of variation in mean depth. The cross represents the arithmetic mean for heat budget and the logarithmic mean for square root of area.

for heat budget and the logarithm of mean depth is +0.77, and the regression of heat budget (in kcal cm⁻² year⁻¹) upon mean depth (in m) is given by the equation

$$\Theta_{ba} = 18.4 \log_{10} \bar{z} - 2.52.$$

The standard error of estimate is 7.2. The highest heat budget in Fig. 1 is for Lake Baikal, and Hutchinson suggests that the value of 66 kcal/cm² may perhaps be too high. His alternative estimate of 57 kcal/cm² is shown by the descending arrow. The crosses in Fig. 1 represent mean values for dimictic (*D*) and warm monomictic (*M*) lakes, and show that annual circulation pattern has only slight influence in the cases under examination, the dimictic lakes taking up on the average a little more and the monomictic lakes a little less heat than would be expected at a given mean depth.

The solid line in Fig. 2, giving the zero order regression of annual heat budget upon mean depth in wholly arithmetic terms, shows that the depth effect is most pronounced in very shallow lakes, as expected. However, contrary to Birge's (1915) statement, the statistical treatment suggests that heat budgets continue to rise as mean depths increase far beyond 30 m, albeit to a steadily lessening degree. Neumann (1958) gives a freehand curve indicating a generally similar relationship between annual heat budgets and mean depths up to 140 m.

The dashed line in Fig. 2 represents the first order regression of annual heat budget upon mean depth, with the influence of varying lake area eliminated. The zero order regression has a distinctly steeper slope, the semilogarithmic regression coefficient being 18.4, as compared with a first order coefficient of 12.2. This steeper slope is owing to the positive logarithmic correlation ($r = +0.57$) between area and mean depth, taken together with the positive correlation between heat budget and lake area (to be discussed in the next section).

The first order correlation coefficient for heat budget and the logarithm of mean depth is +0.62.

THE INFLUENCE OF AREA

Figs. 3 and 4 show the effect of lake area upon annual heat budget, with the square root of the area being taken as the unit of measurement. The area correlation shown in Fig. 3 is nearly as strong as the depth correlation, presumably because larger surface areas allow greater opportunities for wind stress to produce a thick epilimnion, through increased wave action and the mixing of cool deep water, exposed upwind, into the epilimnion (*compare with* Hutchinson 1957). The semilogarithmic correlation coefficient is +0.74, and the regression of heat budget upon area is given by the equation

$$\Theta_{ba} = 14.8 \log_{10} \sqrt{A_0} - 28.3,$$

the square root of area being expressed in meters. The standard error of estimate is 7.5. Dimictic lakes tend to take up about the same amount of heat as monomictic lakes of the same area.

There is a steady, though lessening, rise in heat budget with increasing area, as shown in Fig. 4, rather than a climatic maximum level reached at an area of about 20 km², as Birge (1915) thought likely, or an effect only upon the largest lakes, as implied by Hutchinson (1957). The influence of the positive correlation between area and mean depth is shown by a comparison of the solid and dashed lines in Fig. 4, the zero order and first order (semilogarithmic) regression coefficients for heat budget, and the square root of area, being 14.8 and 9.1, respectively. The first order correlation coefficient is +0.58.

THE INFLUENCE OF VOLUME

Since annual heat budget appears correlated to a highly significant degree with both mean depth and area, it would seem reasonable to expect an even higher degree of correlation with their product, lake volume. In this instance, the cube root of volume was subjected to a logarithmic transformation to compare with similar transformations of the square root of area and the linear dimension of mean depth. The semilogarithmic volume correlation co-

FIG. 5. Semilogarithmic correlation between annual heat budget and the cube root of volume in temperate lakes. Symbols as in Fig. 1.

FIG. 7. Logarithmic correlation between the square root of area and mean depth in temperate lakes. Symbols as in Fig. 1. Dashed lines join two series of lakes, one with unusually high and the other with unusually low area/depth ratios (see Fig. 8).

FIG. 6. The regression of annual heat budget upon the cube root of volume in temperate lakes. The cross represents the arithmetic mean for heat budget and the logarithmic mean for cube root of volume.

FIG. 8. The influence of basin shape (area/depth ratio) upon the relation between annual heat budget and lake volume.

efficient did turn out to be higher than the depth and area coefficients, +0.82 as compared to +0.77 and +0.74; and Figs. 5 and 6 show the semilogarithmic and arithmetic regressions, respectively. The regression of heat budget upon volume is given by the equation

$$\Theta_{bu} = 18.8 \log_{10} \sqrt[3]{V} - 29.7,$$

the cube root of volume being expressed in meters. The standard error of estimate is 6.4. On a volume basis, dimictic lakes tend to take up slightly more heat than warm monomictic lakes of similar size.

Fig. 6 demonstrates that there is a steady although lessening rise in heat budget as lake volume increases up to the maximum size represented by Lake Baikal, instead of a climatic maximum reached according to Birge (1915) at about 0.6 km³ ($\sqrt[3]{V} = 0.84$ km).

THE INFLUENCE OF BASIN SHAPE

While a high degree of correlation is exhibited between annual heat budget and lake volume, other factors also influence heat uptake to a minor degree. Fig. 5 shows a slight influence of latitude, insofar as it determines whether circulation will be dimictic or monomictic. Another factor seeming worthy of attention is basin shape, as represented by the ratio of area to mean depth. In order to select basins of widely differing slope (*area/depth* ratio), Fig. 7 was constructed, in which the logarithm of $\sqrt{\text{area}}$ is plotted vs. the logarithm of mean depth. Two groups of lakes were then chosen by inspection of the graph, the first being the upper boundary series (Hula, Hemmelsdorfer, Müritzt, Mendota, Washington, Galilee, Vattern, Ladoga, and Mich-

igan) and the second the lower boundary series (Beloye, Alamogordo, Weisser, Pulvermaar, Hallstätter, Walchen, Walen, Thun, Como, Tahoe, and Baikal). The two series are shown by the dashed lines in Fig. 7. Annual heat budgets of lakes in the two groups were then plotted versus the logarithm of $\sqrt[3]{\text{volume}}$ in Fig. 8. Comparison of mean values for the two groups of lakes shows slightly higher heat budgets for lakes with a low ratio of area to mean depth, despite the distinctly lower mean volume for this group of lakes. In other words, more steeply sloping basins appear to take up a little more heat per unit area than basins of the same volume but with gently sloping sides. The data in Fig. 8, while far from conclusive, suggest that any shape effect is manifested chiefly in the smaller lakes, where the effects of varying area and depth are also most pronounced. Presumably the greater possibilities of heat storage in a deeper water column more than counterbalance the lessened effect, in a lake with a smaller area, of wind stress upon the entry of heat into the water column.

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