Correction for Restriction of Range when Both X and Y are Truncated

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The effect of range restriction on one variable in a bivariate normal distribution on the X - Y correlation and the problem of estimating unrestricted from restricted correlations has been widely studied for more than half a century. The behavior of correction formulas under truncation of both X and Y , however, remains largely unresearched. The performance of the correction formula for unidimensional truncation (Thorndike, 1947, Case 2) and an approximation procedure for correcting for bidimensional truncation proposed by Wells and Fruchter (1970) were investigated. The Thorndike correction formula undercorrects in

most circumstances. The Wells and Fruchter procedure performs quite well under most conditions but often results in a slight overcorrection. The performance of the Wells and Fruchter and Thorndike formulas are also compared under truncation on X or Y alone. In these circumstances the Wells and Fruchter correction is either equal or markedly superior to the traditional correction. Based on overall performance in recapturing the unbiased population values under both unidimensional and bidimensional truncation, the Wells and Fruchter correction is recommended as the preferred procedure in many practical settings.

Restriction of range has been widely studied since Karl Pearson's (1903) initial work on the subject. Of particular interest here is the literature that addresses the problem of identifying changes in the correlation, ρ , of a bivariate normal population when one of the two variables is directly truncated or when the two variables are indirectly restricted by truncation on some third variable. Correction formulas for estimating unbiased ρ appeared in the psychological literature by the early 1920's (Kelley, 1923; Otis, 1922), although Thorndike (1947) is often credited with bringing them into general use in the field of psychological testing and measurement. His Cases 1 and 2 deal with direct truncation and his Case 3 with indirect truncation. Recent research interest in this area has focused on the performance of these correction formulas (particularly Case 2) under various violations of the distribution assumptions or on the effects of applying the direct truncation correction formula when indirect truncation has occurred (Greener & Osburn, 1979; Gross, 1982; Gross & Fleischman, 1983; Linn, 1968).

Although the problem of unidimensional truncation (restriction on either X or Y) has a long history of extensive research, the problem of truncation in both X and Y (bidimensional truncation) has been much more resistant to analysis, primarily because of the intractability of the mathematics. While Pearson (1908) presented a partial solution, only relatively recently has it been more completely solved (Birnbaum & Meyer, 1953; Finney, 1962; Tallis, 1961). Formulas now exist for computing population means,

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variances, and correlations under truncation of both X and Y , but explicit formulas for calculating unbiased from biased values have not yet been found (Regier & Hamdan, 1971). Wells and Fruchter (1970) developed a procedure for approximating such corrected values. To date, both the usual correction formula (Thorndike's Case 2) and the Wells and Fruchter procedure remain largely unresearched as to their performance under truncation on both X and Y . This article reports the accuracy of both the Thorndike and Wells and Fruchter correction formulas for estimating unbiased ρ under both unidimensional and bidimensional truncation.

Method

Beginning from an unrestricted bivariate normal X - Y distribution with known means, standard deviations, and correlation ($\mu_x, \mu_y, \sigma_x, \sigma_y, \rho$) and cutting points a on X and b on Y , biased parameters in the restricted distribution ($\mu'_x, \mu'_y, \sigma'_x, \sigma'_y, \rho'$) were computed. These values formed the basis for applying correction formulas to obtain estimates of the unrestricted correlation.

Computation of Restricted Population Parameters

Regier and Hamdan (1971) and Weiler (1959) provide mathematical expressions for calculating the parameters of a bivariate normal distribution truncated such that values outside the range ($a \leq x \leq \infty$, $b \leq y \leq \infty$) are excluded. Two familiar functions of any standardized univariate normal distribution v with cutting point t are:

$$Z(t) = [1/(2\pi)^{1/2}] \exp [(-1/2)t^2] \quad , \quad (1)$$

$$Q(t) = \int_t^{\infty} Z(v) dv \quad . \quad (2)$$

Similarly, for any standardized bivariate normal distribution of u, v with cutting points s and t :

$$Z(s, t) = \{1/[2\pi(1 - \rho^2)^{1/2}]\} \exp \{[-1/(2 - 2\rho^2)](s^2 - 2\rho st + t^2)\} \quad , \quad (3)$$

$$Q(s, t) = \int_s^{\infty} \int_t^{\infty} Z(u, v) dudv \quad . \quad (4)$$

Finally, define

$$A = (b - \rho a)/(1 - \rho^2)^{1/2} \quad , \quad (5)$$

$$B = (a - \rho b)/(1 - \rho^2)^{1/2} \quad . \quad (6)$$

In the truncated distribution, then,

$$E(X) = [Z(a)Q(A) + \rho Z(b)Q(B)]/Q(a, b) \quad , \quad (7)$$

$$E(Y) = [Z(b)Q(B) + \rho Z(a)Q(A)]/Q(a, b) \quad , \quad (8)$$

$$E(X^2) = [aZ(a)Q(A) + \rho^2 bZ(b)Q(B) + \rho(1 - \rho^2)Z(a, b) + Q(a, b)]/Q(a, b) \quad , \quad (9)$$

$$E(Y^2) = [bZ(b)Q(B) + \rho^2 aZ(a)Q(A) + \rho(1 - \rho^2)Z(a, b) + Q(a, b)]/Q(a, b) \quad , \quad (10)$$

$$E(XY) = \{\rho[aZ(a)Q(A) + bZ(b)Q(B) + Q(a, b)] + (1 - \rho^2)Z(a, b)\}/Q(a, b) \quad , \quad (11)$$

$$\mu'_x = E(X); \mu'_y = E(Y) \quad , \quad (12)$$

$$\sigma'_x = \{E(X^2) - [E(X)]^2\}^{1/2}; \quad \sigma'_y = \{E(Y^2) - [E(Y)]^2\}^{1/2} \quad , \quad (13)$$

and

$$\rho' = [E(XY) - \mu'_x \mu'_y]/\sigma'_x \sigma'_y \quad . \quad (14)$$

Restriction of Range Corrections

In the present notation, the Thorndike (1947) Case 2 correction formula is given by

$$\rho_T = \rho' / [\rho'^2 + (\sigma'_x/\sigma_x)^2(1 - \rho'^2)]^{1/2} . \quad (15)$$

Wells and Fruchter reasoned that under direct truncation on both X and Y , σ'_x and σ'_y (restricted standard deviations) deviate from their respective unrestricted values (σ_x , σ_y) as a result of both direct and indirect truncation. That is, σ'_x as a function of σ_x is influenced directly by the truncation on X and indirectly by the truncation on Y . Similarly for σ'_y . Their procedure attempts to partition the changes from unrestricted to restricted standard deviations into components due to direct and indirect truncation. Putting their Equations 11 through 17 into the present notation, that procedure is as follows:

$$\rho_1 = \rho' / [\rho'^2 + (\sigma'_x/\sigma_x)^2(1 - \rho'^2)]^{1/2} , \quad (16)$$

$$\sigma_{y1} = \sigma_y [(1 - \rho_1^2)/(1 - \rho'^2)]^{1/2} , \quad (17)$$

$$\sigma_{y2} = \sigma_y + \sigma'_y - \sigma_{y1} , \quad (18)$$

$$\rho_2 = \rho' / [\rho'^2 + (\sigma_{y2}/\sigma_y)^2(1 - \rho'^2)]^{1/2} , \quad (19)$$

$$\sigma_{x1} = \sigma_x [(1 - \rho_2^2)/(1 - \rho'^2)]^{1/2} , \quad (20)$$

$$\sigma_{x2} = \sigma_x + \sigma'_x - \sigma_{x1} , \quad (21)$$

and

$$\rho_1 = \rho' / [\rho'^2 + (\sigma_{x2}/\sigma_x)^2(1 - \rho'^2)]^{1/2} . \quad (22)$$

After the first pass through Equations 16 through 22, additional iterations of Equations 17 through 22 are calculated until the results converge on stable values of σ_{x2} and σ_{y2} . Once stability has been reached, compute

$$\rho_{WF} = \rho_1 / [\rho_1^2 + (\sigma_{y2}/\sigma_y)^2(1 - \rho_1^2)]^{1/2} . \quad (23)$$

Data Analysis

For the analysis under bidimensional truncation, unrestricted standardized bivariate normal populations were defined ($\mu_x = \mu_y = 0$; $\sigma_x = \sigma_y = 1$) for values of ρ from $-.90$ to $+.90$ in increments of $.10$. Cutting points a on X and b on Y were varied from -2.0σ to $+2.0\sigma$ at $.2\sigma$ intervals.

Since all computations were carried out by computer, the partial area under a unidimensional normal curve (Equation 2) was found using the IMSL subroutine MDNOR and the partial volume of a bivariate normal distribution (Equation 4) was found using IMSL subroutine MDBNOR (IMSL, 1979). The remaining Equations 1 through 14 were used to calculate μ'_x , μ'_y , σ'_x , σ'_y , and ρ' for each of the 8,379 combinations of a , b , and ρ (21 values of a by 21 values of b by 19 values of ρ). In order to check the accuracy of the computing algorithms, computed values of ρ' were compared with tabled values in Regier and Hamdan (1971) and values of σ' and μ' with values from Weiler (1959).

Range restriction corrected estimates of ρ were computed as if unidimensional truncation had occurred on X using the Thorndike correction (ρ_T , Equation 15). Equations 16 through 23 were used to compute the Wells and Fruchter approximation (ρ_{WF}) at one to nine iterations.

The percentage error that would result in using the corrected correlation to estimate the unrestricted correlation was calculated by (for example)

$$PE_T = [(\rho_T - \rho)/\rho]100 . \quad (24)$$

The sign of this percent error value provides the direction of error with positive values indicating over-correction and negative values indicating undercorrection.

Finally, the investigation of unidimensional truncation involved repeating the analyses just described for $a = -12.0\sigma$ (i.e., effectively setting the cutting score of X at $-\infty$) and again for $b = -12.0\sigma$ (Weiler, 1959).

Results

Bidimensional Truncation

In order to summarize the overall performance of the Thorndike (Case 2) and Wells and Fruchter procedures for range restriction correction under conditions of truncation on both X and Y , the percentage of error (Equation 24) was grouped into 10 categories: -25% or more, -10% to -25% , -5% to -10% , -1% to -5% , 0% to -1% , 0% to $+1\%$, $+1\%$ to $+5\%$, $+5\%$ to $+10\%$, $+10\%$ to $+25\%$, and $+25\%$ or more. For each value of unbiased ρ , a particular correction was applied to the 441 values of biased ρ (21 cuts on X by 21 cuts on Y). These 441 values were grouped by percentage error category. The frequency of these error categories as a function of the unrestricted correlation (ρ) are presented in Table 1 for the Thorndike Case 2 correction, and in Table 2 for the Wells and Fruchter correction without iteration.

Before discussing these results, two comments are in order. First, since truncation will leave an unrestricted zero correlation ($\rho = 0$) essentially unchanged, the tables do not include values for this case. Second, for increasingly large negative correlations, more severe restriction of range (large positive value of a and b) will result in excessively small remaining partial volumes (Equation 4). In these cases, computations cannot be carried out with sufficient accuracy to provide meaningful results and the values were excluded from computation (cf. Regier & Hamdan, 1971, Table 1). As a result, the frequencies of percentage error in Tables 1 and 2 for negative $\rho \geq .20$ will sum to less than 441.

In their 1970 article, Wells and Fruchter reasoned (but presented no data) that the Thorndike formula would overcorrect. From Table 1 it can be seen that use of the Thorndike Case 2 correction formula under bidimensional truncation results in an undercorrection the majority of the time. Only in the case of $\rho \geq .70$ is overcorrection found, and then the overcorrection never exceeds 1%. On the other hand, the extent of undercorrection can be substantial, often -25% or more. Table 2 shows that the Wells and Fruchter correction performs reasonably well even without iteration. In fact, in the large majority of cases, the percentage of error is within $\pm 5\%$.

In their own investigation of the correction procedure, Wells and Fruchter found that the estimations of σ_x and σ_y would tend to stabilize at four to five iterations and thus recommended four or five iterations in order to obtain the most accurate estimates of the unrestricted correlation. Comparison of the present results from up to nine iterations found that, for all negative correlations and for positive $\rho \leq .40$, application of their procedure without iteration and at five iterations produces virtually identical results. For positive correlations of .50 or greater, though the procedure at five iterations gives somewhat more accurate results, the procedure without iteration is somewhat more conservative—less likely to overcorrect.

Tables 3 and 4 give a more specific picture of the nature and extent of both the restricted and corrected values of correlations under bidimensional truncation. The restricted correlations, corrected values resulting from application of the Thorndike formula, and corrected correlations from the Wells and Fruchter procedure without iteration are presented for $\rho = .2$ in Table 3 and for $\rho = .6$ in Table 4. In the interest of conserving space, Tables 3 and 4 show values for cuts on X and Y from -2.0σ to $+2.0\sigma$ in increments of $.4\sigma$.

Inspection of Tables 3 and 4 clearly shows the pattern of accuracy in using the Thorndike correction under bidimensional truncation. This correction formula most accurately recaptures the unrestricted correlation when restriction on Y is negligible and restriction on X is substantial (Y cut = -2.0σ , X cut

Table 1
Frequency of Ranges of Percent Error (PE_T) Using the
Thorndike Correction to Estimate ρ

Unrestricted ρ	Range of PE									
	Undercorrection					Overcorrection				
	-25% or More	-10% to -25%	-5% to -10%	-1% to -5%	0 to -1%	0 to +1%	+1% to +5%	+5% to +10%	+10% to +25%	+25% or More
+ .90	0	30	75	98	87	151	0	0	0	0
+ .80	36	109	51	71	82	92	0	0	0	0
+ .70	101	99	45	72	97	27	0	0	0	0
+ .60	150	93	46	72	80	0	0	0	0	0
+ .50	185	93	45	72	46	0	0	0	0	0
+ .40	215	93	45	66	22	0	0	0	0	0
+ .30	242	92	47	56	4	0	0	0	0	0
+ .20	266	91	46	38	0	0	0	0	0	0
+ .10	285	90	50	16	0	0	0	0	0	0
- .10	310	93	36	2	0	0	0	0	0	0
- .20*	319	95	25	0	0	0	0	0	0	0
- .30*	327	89	20	0	0	0	0	0	0	0
- .40*	331	84	20	1	0	0	0	0	0	0
- .50*	329	82	21	4	0	0	0	0	0	0
- .60*	315	81	25	9	0	0	0	0	0	0
- .70*	298	88	29	20	0	0	0	0	0	0
- .80*	254	90	38	45	5	0	0	0	0	0
- .90*	133	98	50	74	70	0	0	0	0	0

Note. For each ρ , 441 values were calculated (21 values of a by 21 values of b)

*These rows will not sum to 441 (see note above) since ρ' cannot be calculated with sufficient accuracy in cases of extreme restriction.

= +2.0 σ). This correction becomes less and less accurate as restriction on X becomes less severe and as truncation on Y becomes more severe. This pattern is as would be expected since the Thorndike formula uses only the change in σ_x to correct for restriction of range.

The pattern of results for the Wells and Fruchter procedure (without iteration) is quite different but similarly systematic. Referring to Table 2 (which provides the frequencies) and to Tables 3 and 4 (particularly Table 4), it is found that for relatively large correlations the corrected values are most often moderate overestimates, with the exception of moderate truncation on X coupled with more severe truncation on Y in which case the correction results in an underestimate. In the case of small correlations, the same pattern holds, except that the frequency of undercorrection becomes quite small and the degree of overcorrection increases.

Unidimensional Truncation

While the results just described provide support for the use of the Wells and Fruchter correction procedure when restriction of range occurs on both variables, the question remains as to the severity of error in using that procedure when only one of the two variables is truncated.

Table 2
Frequency of Ranges of Percent Error (PE_{WF}) Using the Wells and

Fruchter Correction Without Iteration to Estimate ρ

Unrestricted ρ	Range of PE									
	Undercorrection					Overcorrection				
	-25% or More	-10% to -25%	-5% to -10%	-1% to -5%	0 to -1%	0 to +1%	+1% to +5%	+5% to +10%	+10% to +25%	+25% or More
+.90	0	0	57	135	91	158	0	0	0	0
+.80	0	0	40	161	61	179	0	0	0	0
+.70	0	0	0	134	54	194	59	0	0	0
+.60	0	0	0	65	38	142	196	0	0	0
+.50	0	0	0	0	46	109	237	49	0	0
+.40	0	0	0	0	9	88	262	82	0	0
+.30	0	0	0	0	1	70	312	58	0	0
+.20	0	3	2	3	0	69	353	11	0	0
+.10	5	3	3	1	0	158	259	10	2	0
-.10	1	8	8	266	135	4	4	2	3	10
-.20*	7	4	8	378	23	2	2	3	4	8
-.30*	5	6	6	387	12	0	8	4	3	5
-.40*	7	4	13	374	13	3	8	0	6	8
-.50*	14	0	85	299	10	0	7	8	2	11
-.60*	17	2	140	224	18	10	2	8	7	2
-.70*	31	7	172	184	14	6	7	6	8	0
-.80*	44	5	201	157	19	2	4	0	0	0
-.90*	31	10	188	119	69	7	1	0	0	0

Note. For each ρ , 441 values were calculated (21 values of a by 21 values of b)

*These rows will not sum to 441 (see note above) since ρ' cannot be calculated with sufficient accuracy in cases of extreme restriction.

Two possible situations can be distinguished: (1) the restriction is on X and correctly diagnosed as such, and (2) the restriction is on Y and improperly assumed to have been on X . In the first case, applications of the two correction formulas to correlations arising from restriction on X only, show that both result in identical values for the corrected ρ s.

In the situation where restriction has occurred on Y but the Thorndike and Wells and Fruchter corrections have both been incorrectly applied (as though restriction had occurred on X), the Wells and Fruchter procedure is clearly superior. These results are summarized in Table 5 for positive ρ and for restrictions on Y at -2.0σ , -1.0σ , μ , $+1.0\sigma$, and $+2.0\sigma$.

As can be seen from Table 5, the Wells and Fruchter procedure results in slight undercorrections with the largest underestimate for $\rho = .90$ and severe restriction on Y ($b = +2.0\sigma$). Also, the Wells and Fruchter procedure always results in less error in estimating the unrestricted ρ than does the Thorndike Case 2 formula, often substantially so.

These results hold if the role of X and Y in the above discussion are reversed. That is, if the restriction is on Y and correctly diagnosed, then the proper application of the Thorndike and of the Wells and

Table 3
Restricted Population Coefficients (ρ') and Estimated ρ Using the Thorndike
Restriction of Range Correction and the Wells and Fruchter Restriction of Range
Correction for $\rho = .20$

Cuts on X (σ)	Cuts on Y (σ)										
	-2.0	-1.6	-1.2	-0.8	-0.4	0.0	0.4	0.8	1.2	1.6	2.0
Restricted Population Coefficients											
-2.0	179	171	160	147	132	118	105	094	084	075	068
-1.6	171	164	153	141	128	114	102	091	082	073	066
-1.2	160	153	144	133	121	109	097	087	078	070	064
-0.8	147	141	133	123	112	101	091	081	073	066	060
-0.4	132	128	121	112	102	092	083	075	067	061	055
0.0	118	114	109	101	092	084	075	068	061	055	051
0.4	105	102	097	091	083	075	068	061	055	050	046
0.8	094	091	087	081	075	068	061	055	050	045	042
1.2	084	082	078	073	067	061	055	050	045	040	037
1.6	075	073	070	066	061	055	050	045	040	034	034
2.0	068	066	064	060	055	051	046	042	037	034	031
Thorndike Corrected Coefficients											
-2.0	190	182	170	155	140	125	111	099	088	078	071
-1.6	191	182	171	157	141	126	112	099	088	079	071
-1.2	192	184	172	158	143	128	113	100	089	080	072
-0.8	192	185	174	160	145	129	115	102	090	080	072
-0.4	193	186	175	162	147	131	116	103	092	082	073
0.0	194	187	177	164	149	133	118	105	093	083	074
0.4	194	188	179	166	151	135	120	107	095	084	075
0.8	195	189	180	168	153	137	122	108	096	085	078
1.2	196	190	182	170	155	140	125	111	098	087	079
1.6	196	191	183	172	158	142	127	112	099	083	080
2.0	197	192	185	174	160	145	129	116	103	092	082
Wells and Fruchter Corrected Coefficients											
-2.0	201	201	201	202	202	202	202	201	201	201	201
-1.6	201	202	202	203	203	203	203	202	202	202	202
-1.2	201	202	203	204	204	204	204	204	204	203	203
-0.8	202	203	204	205	205	205	205	205	205	204	204
-0.4	202	203	204	205	206	206	206	206	206	206	205
0.0	202	203	204	205	206	207	207	207	207	207	207
0.4	202	203	204	205	206	207	207	207	208	207	207
0.8	201	203	204	205	206	207	207	208	208	208	210
1.2	201	202	204	205	206	207	208	208	208	207	210
1.6	201	202	203	204	206	207	207	208	207	210	211
2.0	201	202	203	204	205	207	207	210	210	211	212

Note. Decimal points omitted.

Fruchter procedures (by reversing the roles of X and Y in Equations 16 through 23) will result in identical corrected values. In summary, the proper diagnosis of unidimensional truncation and corrections applied correctly by the two methods will yield identical results. On the other hand, improper diagnosis (i.e., restriction is thought to have occurred on X but actually occurred on Y or vice versa) will result in the Wells and Fruchter correction being superior to the Thorndike correction.

Table 4
 Restricted Population Coefficients (ρ') and Estimated ρ Using the Thorndike
 Restriction of Range Correction and the Wells and Fruchter Restriction of Range
 Correction for $\rho = .60$

Cuts on X (σ)	Cuts on Y (σ)										
	-2.0	-1.6	-1.2	-0.8	-0.4	0.0	0.4	0.8	1.2	1.6	2.0
Restricted Population Coefficients											
-2.0	564	548	523	490	452	412	372	335	301	271	245
-1.6	548	536	514	485	449	410	371	334	301	271	245
-1.2	523	514	498	473	441	405	368	333	300	271	246
-0.8	490	485	473	453	426	395	361	328	298	270	245
-0.4	452	449	441	426	405	378	349	319	292	266	243
0.0	412	410	405	395	378	356	331	305	281	258	238
0.4	372	371	368	361	349	331	309	287	266	246	228
0.8	335	334	333	328	319	305	287	267	248	230	215
1.2	301	301	300	298	292	281	266	248	230	214	200
1.6	271	271	271	270	266	258	246	230	214	198	184
2.0	245	245	246	245	243	238	228	215	200	184	170
Thorndike Corrected Coefficients											
-2.0	590	578	558	530	497	460	423	386	353	321	293
-1.6	593	582	564	537	503	465	426	388	353	322	294
-1.2	596	587	571	546	512	473	432	392	356	323	295
-0.8	598	592	579	557	525	486	443	401	361	327	296
-0.4	599	595	586	567	539	501	458	413	371	333	301
0.0	599	597	591	577	552	517	475	430	385	344	309
0.4	600	599	595	584	564	534	494	448	402	359	320
0.8	600	599	597	590	575	549	512	468	421	376	335
1.2	600	600	598	594	583	562	530	488	442	395	352
1.6	600	600	599	597	589	574	546	509	464	416	370
2.0	600	600	600	598	594	583	561	528	486	438	390
Wells and Fruchter Corrected Coefficients											
-2.0	601	601	599	596	594	591	590	589	589	589	589
-1.6	602	603	602	600	597	594	592	590	589	589	589
-1.2	603	605	605	604	602	599	595	593	591	590	590
-0.8	603	605	607	608	607	604	601	598	595	593	591
-0.4	602	605	608	611	611	610	607	604	600	597	595
0.0	602	604	607	611	613	614	613	610	607	603	600
0.4	601	603	606	610	614	616	617	615	613	610	607
0.8	600	602	604	608	613	617	619	619	618	616	614
1.2	600	601	603	606	611	615	619	621	622	621	620
1.6	600	600	601	604	608	613	618	621	623	624	623
2.0	600	600	601	602	605	610	615	620	624	625	625

Note. Decimal points omitted.

Discussion

Before proceeding, a point of clarification may be needed. Throughout this article, the Thorndike Case 2 correction is referred to primarily. It should be kept in mind that the results described are identical for his Case 1 correction since the only difference between them has to do with whether knowledge of the unrestricted variance is available on the directly (Case 2) or indirectly (Case 1) truncated variable.

Table 5
Estimated ρ Using the Thorndike Correction (ρ_T) and Wells and Fruchter
Correction Without Iteration (ρ_{WF}) for Range Restriction on Y Only but
Corrected Assuming Restriction on X Only, or Restricted on X only but Corrected
Assuming Restriction on Y Only.

Unrestricted ρ	Cuts on Y (σ)									
	-2.0		-1.0		0.0		+1.0		+2.0	
	ρ_T	ρ_{WF}	ρ_T	ρ_{WF}	ρ_T	ρ_{WF}	ρ_T	ρ_{WF}	ρ_T	ρ_{WF}
.90	898	899	891	893	873	880	841	862	796	845
.80	793	797	771	789	722	775	649	763	565	757
.70	689	697	652	690	580	682	489	677	403	677
.60	585	598	539	594	458	590	369	588	294	589
.50	483	499	433	497	355	495	277	495	216	496
.40	383	400	336	399	267	398	204	398	157	399
.30	285	300	246	300	192	300	144	300	110	300
.20	189	200	161	200	124	200	092	200	070	200
.10	094	100	080	100	061	100	045	100	034	100

Note. ρ_T and ρ_{WF} are given to three decimal places, with the decimal point omitted in the table.

Research on restriction of range, the change from unrestricted to restricted correlations, and the performance of correction formulas has been confined, until recently, almost entirely to the case either of direct truncation on one (but not both) of the variables of interest or to restriction on some third (indirect) variable. Unidimensional truncation has been assumed in two practical measurement settings where bidimensional truncation is a more likely assumption.

The first measurement setting relates to reliability. Lord and Novick (1968) discussed the effects of relative homogeneity/heterogeneity of the sample on empirical estimates of reliability. Their discussion was based on the mathematics of direct unidimensional truncation. A moment's reflection, however, would suggest that in a test-retest or parallel forms reliability setting, restriction of range on "X" necessarily implies a similar restriction on "Y." Thus, reliability coefficients in such instances, if corrected using the Thorndike correction formula, may still be inappropriately low.

A second practical situation is that of predictive validation studies. In most such cases, it is assumed that X (the predictor) is restricted due to explicit selection. In practice, not all of those admitted by the selection procedure remain available for measurement on the criterion. At the very least (since prediction is imperfect and is likely to result in some number of false positives), a subset of those will have left the organization due to failure to achieve some minimal level of success (Barrett, Phillips, & Alexander, 1981).

These results suggest several areas for further research. The present study has dealt with the characteristics of correction formulas for population values. The relative sampling behavior of these corrections remains to be investigated. Since little research has been done on the distribution of sample r s that have arisen from truncated populations (Alexander, Carson, Alliger, & Barrett, 1983; Gajjar & Subrahmaniam, 1978), and even less on the sampling distribution of corrected r s (Gullikson & Hopkins, 1976), generalization from the present results to sample values is necessarily tenuous. In addition, the methods of

this study assume strict truncation on the population: the same assumption made by all restriction of range research. The behavior of biased correlations and correction formulas under violation of that assumption are still largely a matter of conjecture (Alexander, Barrett, Alliger, & Carson, 1983).

Based on these results, it is concluded that in situations where the possibility of bidimensional truncation is suspected, the correction procedure of choice appears to be that of Wells and Fruchter. In addition, if the Wells and Fruchter procedure is applied when the situation is actually one of unidimensional truncation, then it will, at worst, equal the Thorndike result; and, if the wrong variable is assumed to be truncated, then it will perform better than the Thorndike correction.

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